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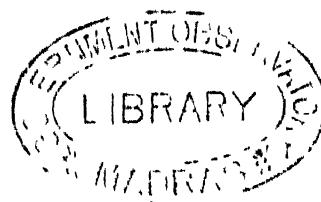
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RESEARCHES  
ON THE  
EVOLUTION OF THE STELLAR SYSTEMS

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VOLUME I



ON THE UNIVERSALITY OF THE LAW OF GRAVITATION AND ON THE  
ORBITS AND GENERAL CHARACTERISTICS OF BINARY STARS

BY

T. J. J. SEE, A.M., PH.D., (BERLIN)

ASTRONOMER AT THE LOWELL OBSERVATORY IN CHARGE OF A SURVEY OF THE SOUTHERN HEAVENS  
FOR THE DISCOVERY AND MEASUREMENT OF NEW DOUBLE STARS AND NEBULAE,  
FELLOW OF THE ROYAL ASTRONOMICAL SOCIETY, MITGLIED DER  
ASTRONOMISCHEN GESELLSCHAFT, ETC, ETC

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DEDICATED

TO

DR. BENJAMIN APTHORP GOULD,

*THE ARGELANDER OF THE SOUTHERN HEAVENS,*

IN TESTIMONY OF A HIGH APPRECIATION OF LIFE-LONG SERVICES  
CONSECRATED TO THE ADVANCEMENT OF

AMERICAN SCIENCE.



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*“L’un des phénomènes les plus remarquables du système du monde est celui de tous les mouvements de rotation et de révolution des planètes et des satellites dans le sens de la rotation du soleil et à peu près dans le plan de son équateur. Un phénomène aussi remarquable n’est point l’effet du hasard, il indique une cause générale qui a déterminé tous ces mouvements*      \* \* \*

*“Un autre phénomène également remarquable du système solaire est le peu d’excentricité des orbites des planètes et des satellites, tandis que ceux des comètes sont très allongés*      \* \* \*

*“Quelle est cette cause primitive ? J’exposerai sur cela, dans la note qui termine cet ouvrage (Système du Monde) une hypothèse, qui me paraît résulter avec une grande vraisemblance des phénomènes précédents, mais que je présente avec la défiance que doit inspirer tout ce qui n’est point un résultat de l’observation ou de calcul”*

LAPLACE

# INTRODUCTION.

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ONE hundred years ago LAPLACE published an outline of the nebular hypothesis, which has since been confirmed and developed by the labors of astronomers. His physical explanation of the evolution of the planets and satellites, under the gradual operation of the laws of nature, was the logical outcome of his profound study of the mechanism of our system, and rested mainly on the common direction of motion and the small eccentricities and mutual inclinations of the orbits. From the concurrence of such remarkable phenomena in a great number of bodies the author of the *Mécanique Céleste* was led to conceive that at a remote epoch in the past, the matter now constituting the planets and satellites was expanded into a vast rotating fiery nebula, which slowly contracted with the radiation of its heat into surrounding space. According to the mechanical principle of the conservation of areas, the contraction accelerated the rotation and thereby increased the oblateness; when the centrifugal force at the equator became equal to the force of gravity the particles ceased to fall towards the centre, and the nebula shed successive rings or zones of vapor from its equatorial periphery. The condensation of the several rings thus abandoned by the contracting mass eventually gave rise to the bodies of the planetary system.

LAPLACE observed that the comets, unlike the planets and satellites, have every degree of inclination and very high eccentricities, and hence he concluded that they were originally foreign to the solar system; accordingly, in the nebular hypothesis, the comets are regarded as small nebulae which have been drawn to the sun in its secular motion among the fixed stars.

The above hypothesis, based on sound dynamical principles and worked out in detail by the philosophic judgement and imaginative genius of LAPLACE, has merited and received the attention of subsequent natural philosophers. Owing to the brief duration of human history compared to the immense ages required for appreciable cosmogonic changes, probably the evolution of the heavenly bodies can never be observed, but must be inferred from a compara-

tive study of existing phenomena; and hence the sublime discovery of the essential process involved in the formation of the planetary system would necessarily mark an epoch in the history of science. The boldness and profound physical insight with which LAPLACE attacked this problem have justly ranked his effort among the greatest achievements of the human intellect. The germ of the general theory of evolution, which has so powerfully influenced the thought of the nineteenth century, may be traced to the recondite speculations of this great geometer.

The strikingly analagous cosmogonic views advanced by KANT in the *Naturgeschichte und Theorie des Himmels* preceded those of LAPLACE by forty-one years, and hence some priority is claimed for the great metaphysician of Königsberg, but since the real vitality of the nebular hypothesis springs from LAPLACE, whose scientific eminence gave it authority commensurate with the development of Physical Astronomy in the eighteenth century, this great cosmogonic speculation is justly dated from the publication of the *Système du Monde* in 1796.

SIR WILLIAM HERSCHEL'S observations on the different types of stars and nebulae led him to consider them of different ages, and to compare the heavenly bodies in such various stages of development to the mixture of growth and decay presented by the trees of an aged forest. The combination of HERSCHEL'S studies on actual phenomena of the heavens with LAPLACE'S dynamical speculations relative to the solar system gave the nebular hypothesis both an observational and a theoretical basis, and hence it soon became an integral part of scientific philosophy. SIR JOHN HERSCHEL'S survey of the entire heavens supplied new and important observations relative to the appearances of the stars and nebulae, and confirmed the general validity of the nebular hypothesis. When, however, LORD ROSSE'S great Reflector resolved certain clusters previously classed as nebulae, the question naturally arose whether with sufficient power all nebulae might not be resolved into discrete stars. Fortunately, the invention of the Spectroscope about 1860, and HUGGINS'S application of it to the heavenly bodies, showed that many of the nebulae are masses of glowing gas gradually condensing into stars, and, so far as possible, realized the postulates laid down by LAPLACE. JOULE'S discovery of the mechanical equivalent of heat and HELMHOLTZ'S application of the resulting laws of thermodynamics to the heat of the sun, established the contraction of the solar nebula, while the subsequent researches of LANE, NEWCOMB, KELVIN and DARWIN have shown the theoretical possibility of most of the development outlined in the *Système du Monde*.

Notwithstanding the general confirmation of the essential parts of LAPLACE'S speculation, some doubt still remains whether the planets and satellites separated as rings or as lumpy masses, and whether rings of anything like regularity could ever condense into single bodies. The most recent investigations of this question indicate that instead of separating as rings or zones which afterwards condensed, the planets and satellites, like the double stars, assumed originally the form of lumpy or globular masses.

In the time of LAPLACE it was supposed that the figures of equilibrium of rotating masses of fluid, whose particles attract one another according to the Newtonian law, are of necessity surfaces of revolution about the axis of rotation, and therefore that a separation could take place only in the form of a ring or zone. But the investigations of JACOBI showed that a homogeneous mass of fluid in the form of an ellipsoid of three unequal axes rotating about its shortest axis could be maintained in equilibrium by the pressure and attraction of its parts, the figure of such a mass is no longer one of revolution, although it is still symmetrical with respect to the axis of rotation. POINCARÉ'S recent investigation of the stability of the equilibrium of the Jacobian ellipsoid showed that when the oblateness has become about two-fifths the equilibrium in this form becomes unstable, and another figure is developed; the body assumes the form of a pear or an hour-glass with two unequal bulbs, and finally breaks up into two comparable, though unequal, masses. Starting from an entirely different point of view, DARWIN made an independent and almost simultaneous investigation of the form assumed by the mass after the Jacobian ellipsoid becomes unstable. Taking two separate masses of fluid revolving as a rigid system in such close proximity that the tidal distortions of figure cause them to coalesce, he determined the resulting figure of equilibrium, and found a dumb-bell form corresponding very closely to the Apoid discovered by POINCARÉ. Though both of these investigations relate to homogeneous masses, and therefore are not strictly applicable to the cases which arise in nature, yet they agree entirely in proving the existence of unsymmetrical forms of equilibrium; and a comparison of these figures with the drawings of double nebulae made by SIR JOHN HERSCHEL leaves no doubt that the process of separation into unequal but comparable masses indicated by these recondite mathematical researches is abundantly illustrated in the evolution of double stars from double nebulae. If this process has played such a prominent part in the genesis of the stellar systems, it is highly probable that the planets and satellites originated in a similar manner, notwithstanding the abnormally rapid increase in density towards the centres of the solar nebula implied by the separation of such inconsiderable masses.

When NEWTON established the law of universal gravitation he also discovered the true *cause* of the tides of the sea, and outlined some of the principal phenomena which follow from the perturbing action of the sun and moon upon the waters which cover the terrestrial spheroid. After the lapse of more than a century LAPLACE attacked this problem from the dynamical point of view, and developed his celebrated analytical theory of oceanic tides, which has been generally adopted in the subsequent researches of astronomers. About two centuries after NEWTON established the cause of the tides, DARWIN was led to consider not only the tides in the mass of fluid spread over the earth's surface, but also those which arise in the body of the globe, owing to its imperfect rigidity. He inquired whether the earth's mass might not be a fluid of great viscosity, and proceeded to develop the theory of *bodily tides*, and to discuss the bearing of these researches on the cosmogonic history of the earth and moon. When the investigation was subsequently extended to other parts of our system, it was found that while LAPLACE's hypothesis as a whole remained unshaken, some appreciable modifications were rendered necessary, especially in the case of the earth and moon, where the relatively large mass-ratio of the component bodies sensibly increased the efficiency of tidal friction. It seemed clear that in the development of the lunar-terrestrial system, the action of tidal friction had been of paramount importance, but that elsewhere the effects had been much less considerable, owing chiefly to the small masses of the attendant bodies.

When we reflect that the planetary system is made up of a great number of very small bodies revolving in almost circular orbits about large central masses, and is therefore different from all other known systems in the heavens, although other systems like it may exist unobserved, it is remarkable that previous investigators have almost invariably approached the problems of Cosmogony from the point of view of the planets and satellites, and that no considerable attempt has been made to inquire into the development of the great number of systems observed among the fixed stars. The short period of time which has elapsed since the explorations of the Telescope have made known the general state of the heavens, with the impossibility of observing any considerable changes, except in the case of double stars, may perhaps account for the natural tendency to focus all effort upon the development of the planets and satellites. But the peculiar character of our system, compared to other known systems in space, renders this procedure incapable of giving us any general law of nature. It is only from a study of the systems of the universe

at large that we may hope to throw light upon the general problems of Cosmogony, among these systems the binary stars are eminently suited for such an investigation

In the present work we propose to investigate the evolution of the stellar systems. The problem is difficult and the observations are incomplete, and hence in this arduous undertaking we may beg the indulgence of astronomers for such imperfections as the discussion of the subject will necessarily exhibit. The present volume is devoted mainly to the *facts* as made known by the labors of double-star observers since the time of SIR WILLIAM HERSCHEL, the more theoretical inquiry into the Secular Effects of Tidal Friction and the Processes of Cosmogony is reserved for subsequent treatment

It would seem that the micrometrical measures discussed in this work establish for the first time, on a secure observational basis, the general shape of the real orbits of double stars. It follows from the results here brought to light that the most probable eccentricity among double stars is over 0.45, and since this mean value is deduced from the consideration of forty orbits, which future observations will not alter materially, we see that such high eccentricities are characteristic of the stellar systems. In the solar system the mean eccentricity for the great planets and their satellites does not surpass 0.0389, and hence we see that the *average eccentricity among double stars is about twelve times that found in our own system*. The great number of binary stars and the practical certainty that the properties deduced from forty of the best orbits now available will be confirmed by the stellar systems in general, justifies us in raising this remarkable induction, relative to the eccentricities, to the dignity of a fundamental law of nature. The binary stars are therefore distinguished from the planets and satellites by two striking characteristics

1. *The orbits are highly eccentric*

2. *The stars of a system are comparable, and frequently almost equal, in mass*

The first of these remarkable properties is traced mainly to the condition stated in the second, high eccentricities probably did not belong to these systems originally, but have been *developed* by the secular action of tidal friction, which is a physical cause affecting all cosmical systems.

In developing the theory of gravitation mathematicians have very generally assumed that the attracting masses are rigid solids, and hence it has been easy to overlook the fact that nearly all the bodies of the visible universe are really *fluid*. The stars and nebulae are self-luminous masses of a gaseous, liquid or

semi-solid nature, and hence it is apparent that in such systems enormous bodily tides will necessarily arise from the mutual gravitation of the particles. Tides are cosmic phenomena as universal as gravitation itself; and since tidal friction will operate in every system of fluid bodies which is endowed with a relative motion of its parts, we see that the general agency of bodily tides gives rise to most important secular changes in the figures and motions of the heavenly bodies. The tidal alterations of figure, which modify the attraction on neighboring bodies, will become especially marked in the case of double stars and double nebulae, where two large fluid masses in comparative proximity are subjected to their mutual gravitation, and hence if the bodies of such a system be rotating as well as revolving the secular working of tidal friction becomes an agency of great and indeed of paramount importance. The general theory of all the secular changes which follow from the double tidal action arising in a binary system remains to be developed, but meanwhile the work of DARWIN in connection with the extension which I have given his researches, makes known some of the more important effects.

From our previous investigations it seems exceedingly probable that the great eccentricities now observed among double stars have arisen from the action of tidal friction during immense ages, that the elongation of the real orbits, so unmistakably indicated by the apparent ellipses described by the stars, is the visible trace of a physical cause which has been working for millions of years. It appears that the orbits were originally nearly circular, and that under the working of the tides in the bodies of the stars they have been gradually expanded and rendered more and more eccentric.

Some simple considerations will enable us to see how these general results arise from the secular action of tidal friction. Suppose the two stars of a system to be spheroidal fluid masses of small viscosity, and let us assume, conformably to the motions observed in the solar system and to those which would result from the division of a double nebula, that the two bodies are rotating about axes nearly perpendicular to the plane of orbital motion, and in the same direction as the revolution about the common centre of gravity, also let the angular velocity of rotation considerably surpass that of orbital revolution. Then, as the fluid is viscous, the tides raised in either mass by the attraction of the other will lag, and hence the major axes of the tidal ellipsoids will point in advance of the tide-raising bodies, and the tidal elevations will exercise on them tangential disturbing forces which tend to accelerate the instantaneous velocities and thereby increase the mean distance. The reaction of the revolving bodies upon the tidal protuberances will retard the axial rotations; for the

moment of momentum of the whole system is constant, and the moment of momentum of axial rotation lost by the stars must be just equal to the gain in moment of momentum of orbital motion. Thus the rotations of the stars are diminished, while the mean distance is correspondingly increased.

But the tangential disturbing force is found to vary inversely as the seventh power of the distance, and hence when the orbit is eccentric the accelerating force at periastron is very much greater than at apastron. The result is that at periastron the disturbing force increases the apastron distance by an abnormally large amount, while at apastron it increases the periastron distance by a very small amount. Thus while the ellipse is being gradually expanded, the apastron is driven away so rapidly compared to the slight recession of the periastron that the orbit grows more and more eccentric. When the axial rotations are sufficiently reduced by the transfer of axial to orbital moment of momentum this change of the system will finally cease, under conditions different from those mentioned above the eccentricity and major axis may decrease, and various other changes take place.

The causes here briefly sketched appear to be sufficient to account for the development of double stars, and the tidal theory might therefore be regarded as satisfactory, yet if the explanation be deemed incomplete it is easy to adduce considerations which exclude other conceivable hypotheses. Let us imagine the  $x$ -axis to represent the region of eccentricity, and divide this line into convenient parts, making the intervals, say, 0.1, then we may erect ordinates denoting the number of orbits falling in a given region, and thus illustrate the distribution of orbits as regards the eccentricity. The irregular line which results from connecting the points determined by a finite number of orbits would become a smooth curve if the number were indefinitely increased. In case of the double stars we obtain what is essentially a probability curve with the maximum near 0.45; the slope on either side appears to be somewhat gradual, but the curve vanishes at zero and unity.

If we make a similar representation for the orbits of comets, we shall find a very high maximum at the eccentricity *unity*; in this case both slopes are extraordinarily steep, though perhaps the curve descends with less rapidity on the side towards the origin, on account of the considerable number of periodic comets which have been gradually accumulated by the perturbing action of the planets. The corresponding curve for the planets and satellites has a high maximum near 0.0389, and while both slopes are steep, that on the side from the origin is the more gradual by virtue of the somewhat unusual eccentricities of *Hyperion*, the Moon and *Mercury*.



If we inquire into the physical meaning of these illustrations, it is easy to see that the distribution of the cometary orbits about the parabolic eccentricity indicates, as LAPLACE first pointed out, that the comets have been drawn to our system from the regions of the fixed stars. The curve for the planets and satellites proves merely that the eccentricities were originally small, and that, under the minimized effects of tidal friction resulting from such inconsiderable masses, they have never been much increased. The curve for the orbits of double stars is of such a nature that we cannot, as in the case of comets, assign to these systems a fortuitous origin, for in this event the eccentricities would surpass, equal or approximate unity, and the periods of revolution, if finite, would be of immense duration, nor could any cause be assigned for the reduction of the eccentricity and period if it be admitted that anything which might properly be called a system could arise from the approach of separate stars. On the other hand the stellar orbits have no close analogy with those of the planets and satellites, for they are densest in the region of mean elliptic eccentricity, and thus almost equally removed from the two extremes presented in the solar system. They were therefore of this mean form originally, or have been made so by a cause which has left a distinct impress upon the nature of the systems. The secular alteration in the figure of equilibrium of a greatly expanded mass like a double nebula would of necessity be very gradual, and hence it follows that the mass cut off under the increased centrifugal force incident to slowly accelerated rotation would begin to revolve in an orbit of comparatively small eccentricity. Indeed, were the initial eccentricity considerable the two nebulae would come into grazing collision at periastron, and in consequence of the resistance encountered the system would rapidly degenerate into a single mass. When at length the bodies are separated, each mass will contract and gain correspondingly in velocity of axial rotation, and tidal friction will begin expanding and elongating the orbit, nothing but this secular process would be adequate to develop the mean eccentricities observed in the immensity of space. If then tidal friction be sufficient to account for the elongation of the real orbits of double stars, we shall be justified in concluding that it is the true cause of the phenomenon. Accordingly, it does not seem probable that the conclusions reached in the *Inaugural Dissertation* which I presented to the Faculty of the University of Berlin will be materially altered, but some of the many problems connected with the general theory of tides still need additional elucidation. If we shall be able to explain the origin and development of double stars, the abundance of such systems will raise a presumption that the agencies and processes involved are more or less general throughout the universe, and no inconsiderable light

will be thrown upon the laws of Cosmogony. By extending our researches to the various classes of nebulae and clusters, additional knowledge will be gained, and in the course of time it will be possible to approach the general problem of cosmical evolution.

For more than two centuries Celestial Mechanics has been occupied with the confirmation of the Newtonian law, and with the development of theories for the precise determination of the figures and motions of the heavenly bodies. In the writings of NEWTON and LAPLACE the attracting masses are essentially solid spheroids covered by a fluid in equilibrium. The theories of the orbital motions and perturbations of the planets, and of the figures and rotations of these bodies about their centres of gravity, are treated mainly from the point of view of rigid dynamics, and little account is taken of the fact that so far as known the heavenly bodies are masses of viscous fluid. The work of DARWIN on the precession of a viscous spheroid and on the secular effects of bodily tidal friction marks an epoch in the history of Celestial Mechanics, which will eventually become a science of the equilibrium and motion of fluids, and must take account of not only the attractions due to undisturbed figures, but also the forces arising from tidal deformation, with the resulting secular changes in the motions of the heavenly bodies.

Physical Astronomy has been devoted heretofore to first approximations under the law of universal gravitation, in particular, to the development of methods for tracing the exact paths of the heavenly bodies through past and future centuries; the theories thus developed are applicable to all periods of recorded history and are justly considered the most imposing monuments yet reared by the human intellect. But the ultimate aim of Astronomy is not only to explain and to predict phenomena which the course of time will make known to observers, but also to determine the secular effects of cumulative causes, and, by approaching the primitive condition of the universe, to discover the origin and to trace the evolutionary history of the stars. As the slow processes of cosmical development are forever withheld from the direct vision of the astronomer, and can be discovered only by the investigation of the continued effects of laws and causes now at work in the heavens, the solution of this sublime problem will be an achievement not unworthy of the human mind.

HAWLEY HOUSE,  
5326 Washington Ave., Chicago,  
May 6, 1896



# CHAPTER I.

## ON THE DEVELOPMENT OF DOUBLE-STAR ASTRONOMY, AND ON THE MATHEMATICAL THEORIES OF THE MOTIONS OF BINARY STARS.

### § 1 *Historical Sketch of Double-Star Astronomy from Herschel to Burnham.*

THE suggestive relation of certain prominent stars, in contrast with the irregular manner in which the multitude are strewn over the surface of the celestial sphere, presented to the minds of the ancients the appearance of arrangement or classification, the more or less obvious constellations thus invented for bright and widely-separated objects were of various sizes, and frequently of an arbitrary character. The condensation of the stars into natural groups, such as the *Pleiades*, *Coma Berenices*, and the clouds in the Milky Way, must have attracted early attention, but no one attempted a philosophical inquiry into the cause of such arrangement until MITCHELL took up the question in 1767, and showed from the theory of probability that a real physical connection was strongly indicated. Further considerations of a similar character led him to predict in advance of observation that compound stars would be found revolving about their common centres of gravity. LAMBERT had surmised the existence of possible stellar systems in 1761, and GIORDANO BRUNO, CASSINI, and MAUPERTUIS had advanced even earlier conjectures of the same kind. The argument for physical connection of closely associated stars, based on the theory of probability, has since been greatly extended by WILLIAM STRUVE, and a practical verification of theory is furnished by the evidence of orbital motion in about 500 out of the 5000 interesting double stars catalogued by modern observers.

The designation *double-star* (διπλούς) was first employed by PTOLEMY in describing the appearance of  $\nu$  *Sagittarii*. The first object of the kind ever discovered with the Telescope was probably  $\zeta$  *Ursae Majoris*, which appeared double to RICCIOLI about the middle of the seventeenth century. The quadruple system  $\theta$  *Orionis* was detected by HUYGHENS in 1656, and the wide pair  $\gamma$  *Arietis* by HOOKE some eight years later. While observing a comet at Pondicherry, India, in December, 1689, FATHER RICHAUD separated the com-

ponents of  $\alpha$  *Centauri*, and thus secured the first record of a star, which has proved to be binary. The duplicity of  $\gamma$  *Virginis* was accidentally discovered by BRADLEY and POUND in 1718, and subsequently re-discovered by CASSINI and MESSIER, while observing occultations, with a view of finding evidence of an atmosphere surrounding the moon.

$\alpha$  *Geminorum* was resolved in 1719,  $\delta$  *Cygni* in 1753, and  $\beta$  *Cygni* in 1755, but although these sporadic discoveries had been made, no systematic search for double stars was attempted until 1777, when CHRISTIAN MAYER, of Mannheim, began to collect a list of these remarkable objects. Having reached the conclusion that faint stars near larger ones are essentially revolving planets, he searched the heavens attentively with an eight-feet mural circle, by BIRD, and discovered in all some seventy-two pairs, including  $\gamma$  *Andromedae*,  $\zeta$  *Canceri*,  $\alpha$  *Herculis*,  $\epsilon$  *Lyrae* and  $\beta$  *Cygni*. Unfortunately, the wide objects within the reach of such a telescope seldom have any appreciable relative motion, and hence the stars discovered by MAYER give very little evidence of the physical connection which he expected.

The real history of double-star discovery and measurement, dates from the explorations begun by SIR WILLIAM HERSCHEL in 1779. This indefatigable observer sought to grapple with the unsolved problem of stellar parallax, which had engaged the attention of astronomers since the time of COPERNICUS. Rejecting the methods recommended by GALILEO, FLAMSTEED and BRADLEY, he proposed one of his own, depending on the measurement of position-angles of two stars of unequal magnitudes from opposite sides of the earth's orbit. HERSCHEL supposed the double stars to be mere groups of perspective, and hence he hoped to detect the relative parallax due to the orbital motion of the earth. He resolved to examine every star in the heavens with the utmost attention under a very high power, the superiority of his telescope gave him an advantage over previous observers, and moreover, his improved optical appliances were supplemented by great energy and boundless enthusiasm. During the interval from 1779 to 1784 he made an extensive catalogue of double stars, some of which he hoped would ultimately prove to be suitable for measurement of parallax. In 1782 he communicated to the Royal Society a catalogue of 269 double stars, 227 of which were new, and followed it three years later by a second catalogue containing 434 such objects. For the next fifteen years the attention of the great observer was devoted to, among other things, the measurement of these pairs, with a view of finding those best adapted to parallax determination. Slight changes were observed from the first, but in most cases the shifting of the relative positions of the objects was attributed

either to the proper motions of the stars, or to the secular motion of the sun in space. The motions were so slow that it took the observations of many years to prove conclusively that certain double stars are moving in regular orbits. This unexpected and astonishing result was finally announced by HERSCHIEL in 1802, and demonstrated during the following year by his elaborate memoirs on binary stars. These investigations supplied the first satisfactory evidence that some of the double stars constitute genuine stellar systems maintained by the action of universal gravitation. HERSCHIEL's celebrated papers dealt with the motions of such objects as  $\xi$  *Ursae Majoris*,  $\eta$  *Ophiuchi*,  $\gamma$  *Virginis*,  $\alpha$  *Geminorum*,  $\eta$  *Coronae Borealis*,  $\xi$  *Bootis*,  $\eta$  *Cassiopeae*,  $\zeta$  *Herculis*,  $\mu^2$  *Bootis*; and in some cases assigned rough estimates of the periods of revolution. The interest in an announcement which opened up fields of inquiry of the widest scope, was fully commensurate with the inherent importance of the discovery, and yet, notwithstanding the splendor of the achievement, double stars were little observed during the first twenty years of this century.

SIR JOHN HERSCHIEL began some preliminary work on double stars in 1816, and was soon joined by SIR JAMES SOUTH. During the next ten years these two observers published several series of observations made either conjointly or separately, and when SIR JOHN HERSCHIEL made his survey of the Southern Hemisphere, over 2000 pairs were discovered and roughly measured. The conscientious records which he has left us in the *Results* of his observations at the Cape of Good Hope, as well as the catalogues since published, and his elegant researches on the orbits of double stars, ensure to him a distinguished place among those astronomers who have labored to advance our knowledge of binary systems.

The systematic survey of the part of the heavens between the north pole and fifteen degrees south declination, executed by WILLIAM STRUVE between the years 1824 and 1836, will long remain the most important contribution to double-star Astronomy ever made by one man. The instrument used was the Dorpat 9.9-inch refractor by FRAUNHOFER, the results furnished the material of the *Mensurae Micrometricae* which includes careful observations of 3112 double and multiple stars, besides records of his previous work with smaller instruments. The labors of WILLIAM STRUVE abolished HERSCHIEL's cumbersome method of referring position-angles to the quadrants, and reduced double-star Astronomy to a scientific basis by reckoning the angle continuously from  $0^\circ$  to  $360^\circ$ . Out of this extensive work grew other reforms, such as the superior classification and arrangement of the results, and in this way STRUVE laid the foundations of the subsequent development of the science.

Among the other observers who contributed to this branch of Astronomy prior to 1850, we may mention especially MADLER, BESSEL, and DAWES. The measures of DAWES take high rank for quality and serve as an example of what may be done by private observers with limited appliances. Other deceased observers especially deserving of mention for important contributions to the records of double-star Astronomy are SECCHI, KAISER, KNOTT, ENGLEMAN, JEDRZEJEWICZ, and, above all, BARON DEMBOWSKI.

Though the last-mentioned observer worked privately and with a small instrument, his measures are more extensive and perhaps more accurate than those of any other observer either living or dead. Covering the period from 1854 to 1878, the work included measures of all the pairs in the *Mensuræ Micrometricæ* accessible to his 7-inch glass, besides numerous observations of pairs more recently discovered by himself, OTTO STRUVE, BURNHAM and ALVAN CLARK. The twenty thousand precise measures executed by this great astronomer were collected after his death, edited by OTTO STRUVE and SCHIAPARELLI, and published in two large quarto volumes by the *Academia dei Lincei* of Rome.

Beginning prior to 1840 and extending over the next fifty years, the double-star work of the illustrious OTTO STRUVE furnishes by far the longest and most homogeneous set of observations yet made by any astronomer. Besides records of the numerous stars discovered by himself and by his father, OTTO STRUVE's work includes reliable data for the most important stars discovered by other previous and contemporary observers. Many of his own stars are close and have proved to be comparatively rapid, and hence will soon yield satisfactory orbits.

Among living observers the names of OTTO STRUVE, HALL, DUNÉR, SCHIAPARELLI, TARRANT, BIGOURDAN, MAW, GLASENAPP, TEBBUTT, STONE, COMSTOCK, KNORRE, SEABROKE, DOBERCK, PERROTIN, HOUGH, and BURNHAM will be familiar to the reader. Each has contributed important material for the study of the stellar systems, but the work of STRUVE, HALL, SCHIAPARELLI, and BURNHAM is especially important to the computer, as covering a long series of years and thus supplying homogeneous material for the determination of the orbits of revolving binaries.

Prior to 1870 it had been generally held by such authorities as DAWES that the subject of double stars was practically exhausted by the discoveries of the HERSCHELS and the systematic surveys of the STRUVES. As the latter had swept over all the brighter stars in the northern heavens, including about 140,000 objects, we may refer with a certain pleasure to the epoch-making discoveries since made by BURNHAM, who has detected nearly 1300 important pairs which had escaped all previous observers. BURNHAM's stars are either very close or

the companion is very faint, and their high importance lies in their rapid orbital motion. This characteristic of BURNHAM's stars has already enabled us to deduce a number of most interesting orbits. It is probable that during the next half century his stars will yield more good orbits than all the other stars previously discovered put together. When we remember that the aim of the observer is to determine the paths of the stars with a view of throwing light upon the character of the stellar systems, it is clear that the measurement of these close objects, which will yield a large number of orbits within a reasonable time, is the most pressing duty of the observer of the future. Many distinguished observers have devoted their attention to the sidereal studies begun by the HERSCHELS and developed by the STRUVES, but none have labored more devotedly or achieved more splendid discoveries than the illustrious BURNHAM.

## § 2 *Laplace's Demonstration of the Law of Gravitation in the Planetary System*

SUPPOSE we denote by  $X$  and  $Y$  the forces which act on a planet, resolved along the coordinate axes, and directed towards the origin at the centre of the sun; let the plane of the orbit be taken as the plane of  $xy$ . Then we have, as the equations of motion,

$$\frac{d^2x}{dt^2} + X = 0 \quad , \quad \frac{d^2y}{dt^2} + Y = 0 \quad (1)$$

If we multiply the first equation by  $-y$ , and the second by  $x$ , and add the results, we find

$$\frac{d(xy - yx)}{dt^2} + xY - yX = 0 \quad (2)$$

But  $\frac{xy - yx}{dt}$  is the double areal velocity, and by KEPLER'S law the areas described by the radius-vector of the planet are proportional to the time. Therefore we have

$$xY - yX = 0, \quad (3)$$

or the forces  $X$  and  $Y$  are related as the coordinates  $x$  and  $y$ ; which indicates that the attractive force is directed to the origin of coordinates. Therefore we conclude that the force which retains the planets in their orbits is directed to the centre of the sun.

We may now investigate the law of this force at different distances. On multiplying the first of (1) by  $dx$ , and the second by  $dy$ , adding and integrating, we have

$$\frac{dx^2 + dy^2}{dt^2} + 2\int(Xdx + Ydy) = 0 \quad (4)$$



If we denote the double areal velocity by  $c$ , we shall have

$$dt = \frac{x dy - y dx}{c},$$

and hence the last equation gives

$$\frac{c^2(dx^2 + dy^2)}{(x dy - y dx)^2} + 2f(X dx + Y dy) = 0 \quad (5)$$

In polar coordinates,

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r = \sqrt{x^2 + y^2},$$

and we find

$$dx^2 + dy^2 = r^2 d\theta^2 + dr^2, \quad x dy - y dx = r^2 d\theta$$

If we now denote by  $R$  the central force which acts on the planet, we shall have

$$X = R \cos \theta, \quad Y = R \sin \theta, \quad R = \sqrt{X^2 + Y^2}$$

Hence we get

$$X dx + Y dy = R \cos \theta (\cos \theta dr - r \sin \theta d\theta) + R \sin \theta (\sin \theta dr + r \cos \theta d\theta) = R dr$$

Therefore

$$c^2 \frac{(r^2 dr^2 + r^4 d\theta^2)}{r^4 dr^2} + 2 \int R dr = 0, \quad (6)$$

and we find

$$dr = \frac{cd\theta}{r \sqrt{-c^2 - 2r^2/R}} \quad (7)$$

If the force  $R$  were a known function of  $r$ , we might find  $r$  by the process of quadrature. But since the force is unknown, although the species of curve it causes the planets to describe is known, we may differentiate equation (6), and obtain

$$R = \frac{c^2}{r^3} - \frac{c^2}{2} \frac{d \left\{ \frac{dr^2}{r^4 d\theta^2} \right\}}{dr} \quad (8)$$

KEPLER found from observation that the planets and comets respectively move in ellipses and parabolas, which are conic sections. The polar equation of a conic may be written

$$\frac{1}{r} = \frac{1 + e \cos(\theta - \omega)}{a(1 - e^2)}, \quad (9)$$

whence we find

$$\frac{dr}{r^2 d\theta} = \frac{e \sin(\theta - \omega)}{a(1 - e^2)},$$

$$\text{or} \quad \frac{dl^2}{r^4 dr^2} = \frac{e^2 - e^2 \cos^2(n-\omega)}{a^2(1-e^2)^2} \quad (10)$$

If we reduce the second member by (9),

$$e \cos(n-\omega) = -1 + \frac{a(1-e^2)}{r},$$

we shall easily find

$$\frac{dl^2}{r^4 dr^2} = \frac{2}{a(1-e^2)} - \frac{1}{r^2} - \frac{1}{a^2(1-e^2)},$$

and hence we get

$$\frac{d \left\{ \frac{dl^2}{r^4 dr^2} \right\}}{dr} = -\frac{2}{a^2(1-e^2)} + \frac{2}{r^3} \quad (11)$$

Thus equation (8) becomes

$$R = \frac{e^2}{a(1-e^2)} - \frac{1}{r^2} \quad (12)$$

Therefore we conclude that the force which causes the planets and comets to move in conic sections about the sun varies inversely as the square of the distance from the sun's centre. Such is the demonstration by which LAPLACE was led to the law of universal gravitation; it rests solely on phenomena, and is independent of any hypothesis. The original demonstration by NEWTON was based on geometrical methods, and is given in the *Principia*, Lib. I, Sec. III, Prop. XI.

The laws of KEPLER made use of in these demonstrations are taken as fundamental facts discovered from observation, but planetary observations in the time of KEPLER were not sufficiently exact to ensure entire rigor in these laws, and besides no account was taken of the mutual gravitation of the planets. Hence it will be seen that the accuracy of the laws of KEPLER, even in the time of NEWTON, could be maintained only within given limits.

It is never possible to realize the conditions of undisturbed motion assumed by KEPLER, and hence the problem presented to astronomers can be solved only by successive approximations. Assuming that the facts embodied in KEPLER's laws are strictly true, NEWTON's reasoning shows that the law of gravitation is mathematically exact; if on the other hand we assume the accuracy of the law of NEWTON, we are led directly to the laws of KEPLER as phenomena which would arise under the operation of gravitation. The laws of KEPLER are sensibly correct, and on the admissible supposition that they are entirely rigorous,\* astronomers have applied the law of gravitation to the disturbed motions of the planets, with a view of explaining observed inequalities, and of discovering from theory other perturbations which have been subse-

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\* The third law is here supposed to be corrected for the planetary masses neglected by KEPLER.

quently verified by observation. This development of the planetary theories has occupied the attention of astronomers for over two centuries, and in every case where doubt has arisen the accuracy of the Newtonian law has been verified.

The range of possible inaccuracy has been gradually narrowed, until at present the data of Astronomy show that if the law of nature departs at all from that given by NEWTON, the deviation must be extremely slight. Indeed, the law of gravitation, taken in connection with its simplicity, is so thoroughly established as to authorize the belief that it is rigorously the law of nature. Its brilliant confirmation and extension since the time of NEWTON, especially by LAPLACE, leaves but few, and generally insignificant, motions yet unexplained, and since we know that the slightest deviation from the law of inverse squares would become very perceptible in the motions of the perihelia of the orbits of the planets and the periplaneta of the orbits of the satellites, and no such outstanding phenomena have been disclosed by observation, except in the case of the perihelion of the orbit of *Mercury*, which may be explained in a different manner, it is hardly possible to doubt that the few anomalous phenomena yet remaining will finally be explained in perfect accord with the law of NEWTON.

The strongest proof of the rigor of this law is to be found in the fact that it accounts for both the regular and the irregular motions of the heavenly bodies, and in the hands of LAPLACE and his successors has become a means of discovery as real as observation itself.

A law which explains satisfactorily the figures, the secular variations, and the delicate long-period inequalities of the planets, and above all the numerous perturbations to which the moon is subjected, certainly has a strong claim to be regarded as a fundamental law of nature, and is incontestably the sublimest discovery yet achieved in any science.

### § 3 *Investigation of the Law of Attraction in the Stellar Systems*

The labors of NEWTON and LAPLACE on the mechanism of the solar system established the law of gravitation with all the rigor which modern observations could demand, but neither of these two great geometers attempted to apply this law to other systems existing in space. The close of the career of LAPLACE, just a century after that of NEWTON, marks an epoch in the verification of the Newtonian law, since in this year SAVARY devised the first method for determining the orbits of double stars, he justly based his theory on the principle

of gravitation which the author of the *Mécanique Céleste* had recently tested with such thoroughness for the regions about the sun traversed by the planets and comets. The method developed by SAVARY has been improved and rendered more practical by the labors of subsequent geometers, and consequently at the present time there is no considerable body of phenomena which appear to be irreconcilable with the law of NEWTON. Indeed, when proper allowance is made for the large but inevitable errors of our micrometrical measures, all modern observations of binary stars may be explained either by the theory of two bodies revolving under the law of gravitation, or by the action of unseen bodies perturbing the regular elliptical motion. This accordance of observation with theory, while it increases enormously the probability of the Newtonian law, does not furnish an independent criterion, and therefore it is desirable to ascertain the most general form of the expressions which will cause a particle to describe a conic, so that we may determine whether any other law can explain the phenomena. In the case of double stars, micrometrical measures enable us to study only the apparent orbits, which are projections of the real orbits upon the plane tangent to the celestial sphere. The apparent orbits are ellipses, and therefore we may conclude that the real orbits are also conics of the same species. When the orbit is projected the centre of the real ellipse will fall upon the centre of the apparent ellipse, but the positions of the projected foci are not determinate unless the position of the real ellipse is known. Astronomers are accustomed to assume that Newtonian gravitation is the attractive force, and as this requires that the principal star shall be in the focus of the real ellipse, it then becomes easy to deduce the corresponding node, inclination and other elements. It is observed that the principal star is not in the centre of the ellipse, and therefore we infer that the force does not vary directly as the distance. But since the areas swept over by the radius vector are proportional to the times, we may conclude that the force is central, and since the apparent motion of 42 *Comæ Berenices* is rectilinear, it is clear that the orbit is a plane curve, or conic section. As other forces besides gravitation could cause a particle to describe a conic, BERTRAND proposed the following problem to the Paris Academy of Sciences: "*Knowing that a material particle under the action of a central force always describes a conic, it is required to find the expression of this force.*"\*

Before presenting the solutions developed by DARBOUX and HALPHEN, we shall give an exposition of the geometrical method by which NEWTON treated the same problem.

In the Scholium to Proposition XVII, Liber I, of the *Principia*, NEWTON

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\* *Comptes Rendus*, April 9, 1887

derived the general expression for the force which will cause a particle to describe a conic section, the centre of force occupying any internal point. The demonstration given by NEWTON depends upon several preceding propositions, a more direct but similar solution of the same problem has been published by PROFESSOR GLAISIER in the *Monthly Notices*, Vol XXXIX.

This investigation is as follows. Let  $C$  be the centre of the ellipse,  $P$  any point occupied by the particle,  $Q$  the point occupied by the particle at the next instant,  $PZ$  the tangent at  $P$ ,  $PG$  the diameter through  $P$ ,  $CD$  the semi-conjugate diameter to  $PG$ ,  $O$  the centre of attraction,  $QS$  a right line parallel

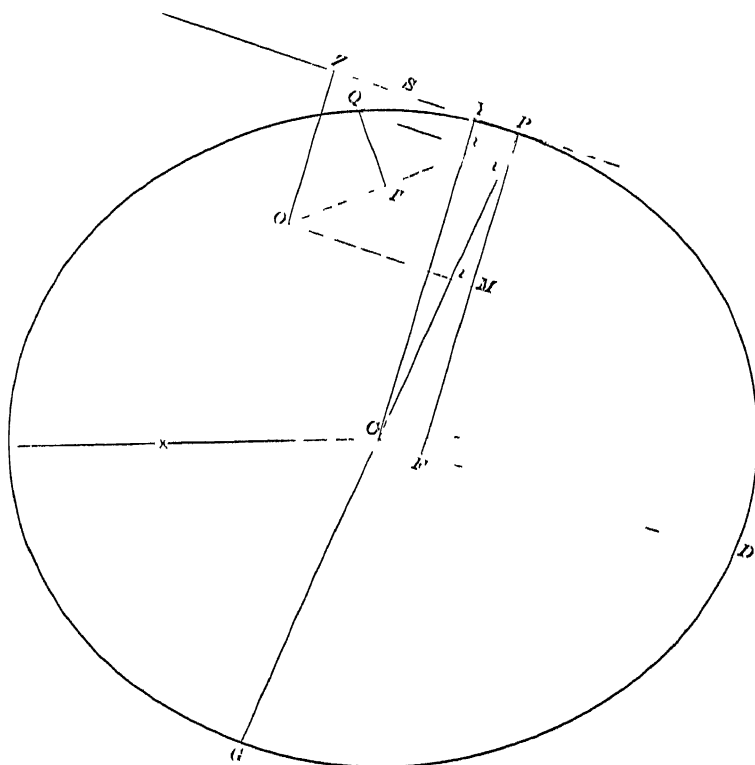


Fig. 1.

to  $OP$ ,  $OZ$  and  $CY$  perpendiculars on the tangent from  $O$  and  $C$ ,  $PF$  the perpendicular on  $CD$  from  $P$ ,  $QT$  the perpendicular from  $Q$  on  $OP$ ,  $Qr$  and  $OM$  perpendiculars on  $PF$  from  $Q$  and  $O$ ,  $r$  the intersection of  $Qr$  with  $OP$ ,  $z$  the intersection of  $OM$  with  $CP$ , and  $R$  the required force tending to  $O$ .

Then we shall have

$$R = \frac{2h^2}{OP^2} \frac{QS}{QF^2}, \quad (1)$$

where  $h$  denotes the areal velocity

By the similar triangles  $QTx$  and  $PMO$ ,

$$\frac{QT}{Qx} = \frac{PM}{OP} \quad (2)$$

By conic sections,

$$\frac{\overline{Qv}^2}{Pv \cdot Gv} = \frac{\overline{CD}^2}{CP^2} \quad (3)$$

And from the figure,

$$\frac{Pv}{QS} = \frac{Pv}{Px} = \frac{Pi}{OP} = \frac{CP}{PF} = \frac{PM}{OP} \quad (4)$$

Therefore by (3) and (4) we find

$$\frac{\overline{Qv}^2}{QS \cdot Gv} = \frac{\overline{CD}^2}{CP \cdot PF} = \frac{PM}{OP} \quad (5)$$

In the limit  $Qx = Qv$ , and hence (2), (3) and (5) give

$$\frac{\overline{QT}^2}{QS} = \frac{2\overline{CD}^2}{PF} \left( \frac{PM}{OP} \right)^3 \quad (6)$$

Substituting in (1), we obtain

$$R = \frac{h^2}{OP^2} \cdot \frac{PF}{\overline{CD}^2} \left( \frac{OP}{PM} \right)^3 = \frac{h^2}{a^2b^2} \left( \frac{PF}{PM} \right)^3 OP = \frac{h^2}{a^2b^2} \left( \frac{CY}{OZ} \right)^3 OP, \quad (7)$$

which is the required law of force.

#### § 4 *Analytical Solution of Bertrand's Problem Based on that Developed by Darboux ; Solution of Halphen*

The equations of acceleration are,

$$\frac{d^2x}{dt^2} = -R \frac{x}{r} = -R \cos \theta, \quad \frac{d^2y}{dt^2} = -R \frac{y}{r} = -R \sin \theta, \quad (1)$$

where  $R$  is the attractive force, at unit distance. Multiplying the first by  $-y$  and the second by  $x$ , and adding, we get

$$x \frac{d^2y}{dt^2} - y \frac{d^2x}{dt^2} = 0 \quad (2)$$

On integrating we obtain

$$x \frac{dy}{dt} - y \frac{dx}{dt} = h \quad (3)$$

In polar coordinates this equation becomes

$$r^2 \frac{d\theta}{dt} = h = \text{double areal velocity} \quad (4)$$

Let us now put  $u = \frac{1}{r}$ , and then

$$x = \frac{\cos \theta}{u}, \quad \frac{dx}{dt} = - \frac{u \sin \theta + \cos \theta \frac{du}{d\theta}}{u^2} \frac{d\theta}{dt} \quad (5)$$

By equation (4) this becomes

$$\frac{dx}{dt} = -h \left( u \sin \theta + \cos \theta \frac{du}{d\theta} \right), \quad (6)$$

$$\text{and} \quad \frac{d^2 x}{dt^2} = -h^2 u^2 \left( u \cos \theta + \cos \theta \frac{d^2 u}{d\theta^2} \right) \quad (7)$$

From (7) and (1) we get

$$R = \frac{h^2}{r^2} \left( u + \frac{d^2 u}{d\theta^2} \right), \quad (8)$$

where the centre of force is at the origin

This equation is perfectly general for the determination of  $R$  when the equation of the path is known. To get the central force,  $R$ , which will cause a particle to describe any given path, we find the value of  $\left( u + \frac{d^2 u}{d\theta^2} \right)$  for that path, and multiply it by  $\frac{h^2}{r^2}$ . Therefore, to find the law of  $R$ , when the path is a conic section, we have the general equation,

$$ax^2 + 2bxy + cy^2 + 2dx + 2fy = g \quad (9)$$

Putting  $r = \frac{1}{u}$ , and transforming to polar coordinates, we have

$$\frac{u \cos^2 \theta}{u^2} + \frac{2b \sin \theta \cos \theta}{u^2} + \frac{c \sin^2 \theta}{u^2} + \frac{2d \cos \theta}{u} + \frac{2f \sin \theta}{u} = g,$$

from which we obtain

$$u = \frac{f \sin \theta + d \cos \theta}{g} + \frac{1}{g} \sqrt{(f^2 + cg) \sin^2 \theta + 2(fd + bg) \sin \theta \cos \theta + (d^2 + ag) \cos^2 \theta} \quad (10)$$

This equation reduces to the form

$$u = A \sin \theta + B \cos \theta + \sqrt{C \sin 2\theta + D \cos 2\theta} + H, \quad (11)$$

where

$$A = \frac{f}{g}, \quad B = \frac{d}{g}, \quad C = \frac{fd + bg}{g^2}, \quad D = \frac{d^2 + ag - f^2 - cg}{2g^2}, \quad H = \frac{d^2 + ag + f^2 + cg}{2g^2}.$$

From (11) we derive

$$\frac{d^2 u}{d\theta^2} = \frac{-A \sin \theta - B \cos \theta - C^2 - D^2 - (C \sin 2\theta + D \cos 2\theta)^2 - 2H(C \sin 2\theta + D \cos 2\theta)}{(C \sin 2\theta + D \cos 2\theta + H)^{3/2}} \quad (12)$$

Therefore by (8) we get

$$R = \frac{h^2}{r^2} \frac{(H^2 - C^2 - D^2)}{(C \sin 2\theta + D \cos 2\theta + H)^{3/2}} \quad (13)$$

This is the general expression for  $R$  whatever be the constants  $a, b, c, d, f$  and  $g$

Since by (11) we have

$$u - A \sin \theta - B \cos \theta = \sqrt{C \sin 2\theta + D \cos 2\theta + H},$$

we may write (13)

$$R = \frac{h^2}{r^2} \frac{(H^2 - C^2 - D^2)}{\left(\frac{1}{r} - A \sin \theta - B \cos \theta\right)^3}, \quad (14)$$

which is another general expression for  $R$

When the cone is an ellipse with the origin at the centre, equation (9) takes the form  $ax^2 + cy^2 = ac$ , and from (13) or (14) we find after reduction

$$R = \frac{h^2}{ac} \quad (15)$$

The force varies directly as  $r$ , which is the well-known law

When the centre of force is on the  $x$ -axis between the centre and one of foci at a distance  $m$  from the centre, equation (9) becomes

$$ax^2 + 2amx + cy^2 = a(c - m^2),$$

and we find from (13)

$$R = \frac{h^2}{r^2} \frac{(a)^{1/2}}{[(a - c + m^2) \cos^2 \theta + c - m^2]^{3/2}} \quad (16)$$

Since  $a - c + m^2$  is always negative, the force at unit distance is a maximum in the direction of the apsides and is a minimum when  $\theta = \frac{\pi}{2}$ . We have from (14), in this case,

$$R = \frac{h^2 c^2 r}{a(c - m^2 - m^2)^3} \quad (17)$$

This expression can readily be transformed into (16)



When the origin is at one of the foci (13) or (14) gives

$$R = \frac{h^2}{r^2} \frac{e^{1/2}}{a}, \quad (18)$$

which is the Newtonian law

This is also deducible from (16) by putting  $m' = e - a$

When the centre of force is on the  $x$ -axis between one of the foci and the nearest apse, at a distance  $n$  from the centre, we obtain from (13)

$$R = \frac{h^2}{r^2} \frac{(ae)^{1/2}}{[(a - e + n^2) \cos^2 \theta + e - n^2]}, \quad (19)$$

Since  $a - e + n^2$  is always positive, the force at unit distance is a maximum when  $\theta = \frac{\pi}{2}$ , and a minimum at the apsides. From (14) it is easy to obtain

$$R = h^2 \frac{e^{1/2}}{a(a - n^2 - n^2)^{3/2}}, \quad (20)$$

which may be transformed into (19).

When the centre of force is on the minor axis at a distance  $k$  from the centre, equation (13) gives

$$R = \frac{h^2}{r^2} \frac{(ae)^{1/2}}{[(a - e - k^2) \cos^2 \theta + e]^2} \quad (21)$$

Since  $a - e - k^2$  is always negative the force at unit distance is a maximum when  $\theta = 0$ , and a minimum when  $\theta = \frac{\pi}{2}$ . In this case we obtain from (14)

$$R = \frac{a^2}{e} \frac{e^{1/2}}{(a - k^2 - k^2 y)^2} \quad (22)$$

When the centre of force is within the ellipse, at a distance  $p$  from the  $y$ -axis, and  $q$  from the  $x$ -axis, we get from (13)

$$R = \frac{h^2}{r^3} \frac{(ae)^{1/2}}{[2pq \sin \theta \cos \theta + (a - e - q^2 + p^2) \cos^2 \theta + e - p^2]}, \quad (23)$$

which becomes (19) when  $q = 0$ , and (21) when  $p = 0$ . We also obtain from (14)

$$R = \frac{h^2 a^2 e^{1/2}}{(a - ap^2 - eq^2 - epy - ap^2)^2}, \quad (24)$$

which becomes (20) when  $q = 0$ , and (22) when  $p = 0$ .

The foregoing values of  $R$  are real and positive, and represent all the laws consistent with the observed motions of binary stars

It may be interesting to note that when the centre of force is at one of the apsides or at one end of the minor axis, our general formulae (13) and (14) give indeterminate results. In this case we take the equation of the ellipse with the origin at the end of one of the axes, and calculate  $R$  by (8). When the centre of force is at the apse, we obtain after reduction

$$R = \frac{h^2}{r^3} \frac{\sqrt{c}}{a \cos^3 \theta} \quad (25)$$

When the centre of force is at the end of the minor axis, we find

$$R = \frac{h^2}{r^3} \frac{\sqrt{a}}{c \sin^3 \theta} \quad (26)$$

In both of these cases the origin is taken in the positive direction from the centre of the ellipse, if the other ends of the axes be chosen the signs of (25) and (26) will be reversed

When  $c = a$  in (25) or (26) the conic becomes a circle, and the expression reduces to the well-known law

$$R = \frac{8h^2 a^{5/2}}{r^6} \quad (27)$$

The expression for the force at external points may be derived in a manner entirely similar to that for points within

### *Solution of Halphen \**

Let  $m$  be the mass of the central body, and  $R$  an unknown function of  $x$  and  $y$ . Then we have the equations

$$m \frac{d^2 x}{dt^2} = -R \frac{x}{r}, \quad m \frac{d^2 y}{dt^2} = -R \frac{y}{r} \quad (28)$$

$R$  is to be determined by the condition that the orbit of the particle is a conic section. Let

$$\frac{dx}{dt} = x', \quad \frac{dy}{dt} = y', \quad R = -m u r, \quad (29)$$

where  $u$  is an unknown function of  $x$  and  $y$

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\* TISSERAND'S *Mécanique Céleste*, Tome I, Chap. I, where the original solution has been somewhat modified

From (28) and (29) we obtain

$$\frac{dx'}{dt} = ux', \quad \frac{dy'}{dt} = uy' \quad (30)$$

By this equation we have

$$\frac{d}{dt} F(x, y, x', y') = x' \frac{\partial F}{\partial x} + y' \frac{\partial F}{\partial y} + u \left( x \frac{\partial F}{\partial x'} + y \frac{\partial F}{\partial y'} \right) \quad (31)$$

We now proceed to find the differential equation which is common to all conics. The general equation of a conic has the form,

$$Ax^2 + 2Bxy + Cy^2 + 2Fx + 2Gy + H = 0, \quad (32)$$

in which there are five arbitrary constants. Taking  $x$  as the independent variable and differentiating five times in succession we have, in LAGRANGE'S notation,

$$\left. \begin{aligned} Cyy' &+ B(xy' + y) + Ax + Gy' + F = 0 \\ C(yy'' + y'^2) &+ B(xy'' + 2y') + Ax + Gy'' = 0 \\ C(yy''' + 3y'y'') &+ B(xy''' + 3y'') + Gy''' = 0 \\ C(yy^{iv} + 4y'y''') + B(xy^{iv} + 4y'') &+ Gy^{iv} = 0 \\ C(yy^v + 5y'y^{iv} + 10y''y''') + B(xy^v + 5y^{iv}) &+ Gy^v = 0 \end{aligned} \right\} \quad (33)$$

We now have to eliminate the five constants in (32) and (33). We notice that the last three equations of (33) are homogeneous, containing only the three constants  $C$ ,  $B$  and  $G$ , and we can eliminate them by equating to zero the determinant

$$\Delta = \begin{vmatrix} yy''' + 3y'y'' & xy''' + 3y'' & y''' \\ yy^{iv} + 4y'y''' + 3y''^2 & xy^{iv} + 4y'' & y^{iv} \\ yy^v + 5y'y^{iv} + 10y''y''' & xy^v + 5y^{iv} & y^v \end{vmatrix} \quad (34)$$

By elementary principles of Determinants equation (34) reduces to

$$\Delta = \begin{vmatrix} 0 & 3y'' & y''' \\ 3y'' & 4y''' & y^{iv} \\ 10y''' & 5y^{iv} & y^v \end{vmatrix} \quad (35)$$

Expanding (35) and returning to the differential notation, we have

$$9 \left( \frac{d^2y}{dx^2} \right)^2 \frac{d^5y}{dx^5} - 45 \frac{d^2y}{dx^2} \frac{d^3y}{dx^3} \frac{d^4y}{dx^4} + 40 \left( \frac{d^3y}{dx^3} \right)^2 = 0 \quad (36)$$

This is the general differential equation of a conic section. We now calculate  $\frac{d^2y}{dx^2}$   $\frac{d^3y}{dx^3}$  from the relations expressed in (29), (30) and (31). We have

$$\frac{dy}{dx} = \frac{y'}{x'},$$

therefore

$$x' \frac{d^2 y}{dx^2} = \frac{x' u y - y' u x}{x'^2},$$

or

$$x'^3 \frac{d^2 y}{dx^2} = (x' y - y' x) u \quad (37)$$

Since the force is central, by the law of areas,  $(x' y - y' x)$  is constant. Therefore we derive

$$\left. \begin{aligned} x'^5 \frac{d^2 y}{dx^3} &= (x' y - y' x) \left( x' \frac{du}{dt} - 3u^2 x' \right) \\ x'^7 \frac{d^4 y}{dx^4} &= (x' y - y' x) \left( x'^2 \frac{d^2 u}{dt^2} - 10u x' \frac{du}{dt} - 3u^2 x'^2 + 15u^3 x'^2 \right) \\ x'^9 \frac{d^6 y}{dx^6} &= (x' y - y' x) \left[ x'^3 \frac{d^3 u}{dt^3} - 15u x' x'^2 \frac{d^2 u}{dt^2} - 10x x'^2 \left( \frac{du}{dt} \right)^2 \right. \\ &\quad \left. + \frac{du}{dt} (105u^2 x'^2 x' - 16u x'^3) + 45u^3 x' x'^2 - 105u^4 x'^3 \right] \end{aligned} \right\} \quad (38)$$

Substituting these values in (36) and reducing, we obtain

$$9u^2 \frac{d^3 u}{dt^3} - 45 \frac{du}{dt} \frac{d^2 u}{dt^2} + 40 \left( \frac{du}{dt} \right)^3 = 9u^3 \frac{du}{dt} \quad (39)$$

Putting  $u = w^{-3/2}$ , in which  $w$  is a function of  $x$  and  $y$ , (39) reduces to

$$\frac{d^3 w}{dt^3} = w^{-3/2} \frac{dw}{dt} \quad (40)$$

When we remember that

$$\frac{dx'}{dt} = x' w^{-3/2} \quad \text{and} \quad \frac{dy'}{dt} = y' w^{-3/2},$$

and that  $w$  is a function of  $x$  and  $y$ , we get

$$\left. \begin{aligned} \frac{dw}{dt} &= x' \frac{\partial w}{\partial x} + y' \frac{\partial w}{\partial y} \\ \frac{d^3 w}{dt^3} &= x'^3 \frac{\partial^3 w}{\partial x^3} + 3x'^2 y' \frac{\partial^3 w}{\partial x^2 \partial y} + 3x' y'^2 \frac{\partial^3 w}{\partial x \partial y^2} + y'^3 \frac{\partial^3 w}{\partial y^3} \\ &\quad + w^{-3/2} \left( x' \frac{\partial w}{\partial x} + y' \frac{\partial w}{\partial y} \right) + 3w^{-3/2} (x' y + y' x) \frac{\partial^2 w}{\partial x \partial y} \\ &\quad + 3w^{-3/2} \left( x x' \frac{\partial^2 w}{\partial x^2} + y y' \frac{\partial^2 w}{\partial y^2} \right) \\ &\quad - \frac{3}{2} w^{-5/2} \left( x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) \left( x' \frac{\partial w}{\partial x} + y' \frac{\partial w}{\partial y} \right) \end{aligned} \right\} \quad (41)$$

Substituting these values in (40), we obtain

$$\left. \begin{aligned} 0 &= x'^3 \frac{\partial^3 w}{\partial x^3} + 3x'^2 y' \frac{\partial^3 w}{\partial x^2 \partial y} + 3x' y'^2 \frac{\partial^3 w}{\partial x \partial y^2} + y'^3 \frac{\partial^3 w}{\partial y^3} \\ &+ \frac{3}{2} x' w^{-5/2} \left[ 2w \left( x \frac{\partial^2 w}{\partial x^2} + y \frac{\partial^2 w}{\partial x \partial y} \right) - \frac{\partial w}{\partial x} \left( x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) \right] \\ &+ \frac{3}{2} y' w^{-5/2} \left[ 2w \left( y \frac{\partial^2 w}{\partial y^2} + x \frac{\partial^2 w}{\partial x \partial y} \right) - \frac{\partial w}{\partial y} \left( x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) \right] \end{aligned} \right\} \quad (42)$$

This equation holds true whatever be the value of  $t$ , and hence when  $t = 0$ , in which case  $x, y, x', y'$  may be any four quantities mutually independent of one another. Then (42) gives the following equations

$$\frac{\partial^3 w}{\partial x^3} = 0, \quad \frac{\partial^3 w}{\partial x^2 \partial y} = 0, \quad \frac{\partial^3 w}{\partial x \partial y^2} = 0, \quad \frac{\partial^3 w}{\partial y^3} = 0 \quad (43)$$

$$\left. \begin{aligned} 2w \left( x \frac{\partial^2 w}{\partial x^2} + y \frac{\partial^2 w}{\partial x \partial y} \right) - \frac{\partial w}{\partial x} \left( x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) &= 0 \\ 2w \left( y \frac{\partial^2 w}{\partial y^2} + x \frac{\partial^2 w}{\partial x \partial y} \right) - \frac{\partial w}{\partial y} \left( x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) &= 0 \end{aligned} \right\} \quad (44)$$

We obtain from (43), when we denote the arbitrary constants by  $a, b, c, f, g, h$ ,

$$w = ax^2 + 2bx + cy^2 + 2fx + 2gy + h \quad (45)$$

Forming the differentials and substituting in (44), we obtain

$$\left. \begin{aligned} (bf - ag)xy + (cf - bq)y^2 + (f^2 - ah)x + (fg - bh)y &= 0 \\ (bg - cf)xy + (ag - bf)x^2 + (fg - bh)x + (g^2 - ch)y &= 0 \end{aligned} \right\} \quad (46)$$

Since these equations hold true for all values of  $x$  and  $y$ , we find

$$ag - bf = 0, \quad bq - cf = 0 \quad (47)$$

$$f^2 - ah = 0, \quad g^2 - ch = 0, \quad fg - bh = 0 \quad (48)$$

From (48) we have

$$fh(ag - bf) = 0, \quad gh(bq - cf) = 0 \quad (49)$$

Then, if none of the quantities  $f, g, h$  vanishes, (47) follows from (48), and it is sufficient to verify the latter.

We may write (45) in the form

$$w = \frac{1}{h} [(fx + gy + h)^2 - (f^2 - ah)x^2 - (g^2 - ch)y^2 - 2(fg - bh)xy], \quad (50)$$

which, in consequence of (48), becomes

$$w = \frac{(fx + gy + h)^2}{h} \quad (51)$$

Therefore, since  $u = w^{-3/2}$ , we have by (29)

$$R_1 = mh^{3/2} \frac{r}{(fx + gy + h)^3}, \quad (52)$$

which is an expression for the force sought. When  $h = 0$ , (48) leads to  $f = 0$  and  $g = 0$ . In this case we have

$$w = ax^2 + 2bxy + cy^2, \quad (53)$$

from which we get

$$R_2 = m \frac{r}{(ax^2 + 2bxy + cy^2)^{3/2}} \quad (54)$$

This is another expression for the force, whatever be the constants  $a$ ,  $b$  and  $c$ .

When  $f = 0$ , (47) and (48) give  $ag = bg = ah = bh = 0$ ,  $g^2 = ch$ , from which  $a = b$ .

In this case we get from (50)

$$w = \frac{(gy + h)^2}{h}, \quad (55)$$

which gives the same result as (52), when  $f = 0$ .

Thus there are two laws of force, and only two, which answer the question, but the forces  $R_1$  and  $R_2$  contain both the radius vector  $r$ , and the polar angle

$$\theta = \tan^{-1} \frac{y}{x}$$

If the forces depend upon  $r$  alone, as is natural to suppose, we should have in  $R_1$ ,  $f = g = 0$ ; and in  $R_2$ ,  $a = c$  and  $b = 0$ . Then we find

$$R_1 = mr, \quad R_2 = \frac{m}{r^2} \quad (56)$$

The first of these laws is excluded by observation, the second is the law of Newtonian gravitation.

§ 5 *Theory of the Determination, by Means of a Single Spectroscopic Observation, of the Absolute Dimensions, Parallaxes and Masses of Stellar Systems whose Orbits are Known from Micrometrical Measurement \**

Recent researches on the orbits of double stars have led me to develop the suggestion, first thrown out by FOX TALBOT in 1871† and since somewhat varied by others,‡ for determining the absolute dimensions, parallaxes and masses of stellar systems by spectroscopic observation of the relative motion of the companion in the line of sight. A simple and general theory of this motion may be derived from the application of the hodograph of the ellipse, and hence we shall now investigate the nature of this curve.

Let  $x, y$  be the coordinates of a point in the ellipse,  $x' y'$  those of the corresponding point in the hodograph, then we shall have

$$x' = \frac{dx}{dt} \quad , \quad y' = \frac{dy}{dt} \quad (1)$$

Suppose  $M$  to be attracting the mass in the focus of the ellipse, and let  $r$  and  $\theta$  be the polar coordinates of the particle moving in the orbit, and we have

$$\frac{d^2x}{dt^2} = -\frac{Mr}{r^3} = -\frac{M}{r^2} \cos \theta \quad , \quad \frac{d^2y}{dt^2} = -\frac{My}{r^3} = -\frac{M}{r^2} \sin \theta \quad (2)$$

By the principle of the conservation of areas resulting from central forces, we have the equation

$$r^2 \frac{d\theta}{dt} = \text{double areal velocity} = C ,$$

or

$$r^2 = C \frac{dt}{d\theta} ,$$

and hence

$$\frac{d^2x}{dt^2} = -\frac{M}{C} \cos \theta \frac{d\theta}{dt} \quad , \quad \frac{d^2y}{dt^2} = -\frac{M}{C} \sin \theta \frac{d\theta}{dt} \quad (3)$$

If we integrate we obtain

$$\frac{dx}{dt} + a = -\frac{M}{C} \sin \theta \quad , \quad \frac{dy}{dt} + b = +\frac{M}{C} \cos \theta , \quad (4)$$

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\* *Astronomische Nachrichten*, No 3314

† Report of British Association, 1871, Part II p 34, CLERKE's "System of the Stars," p 201, and "History of Astronomy during the 19th Century," third edition, p 407

‡ RAMBAUD, *M N*, March, 1890, WILKING, *A N*, 3198, also a paper on the determination of orbits from spectroscopic observation of the velocity-components in the line of sight, by LEHMAN-FRIEDL, *A N*, 3242

where  $a$  and  $b$  are the arbitrary constants of integration. But since

$$\sin \theta = \frac{y}{r} \quad , \quad \cos \theta = \frac{x}{r} \quad ,$$

we find

$$\frac{dx}{dt} + a = -\frac{M}{C} \frac{y}{r} \quad , \quad \frac{dy}{dt} + b = +\frac{M}{C} \frac{x}{r} \quad (5)$$

By means of equation (1) we have

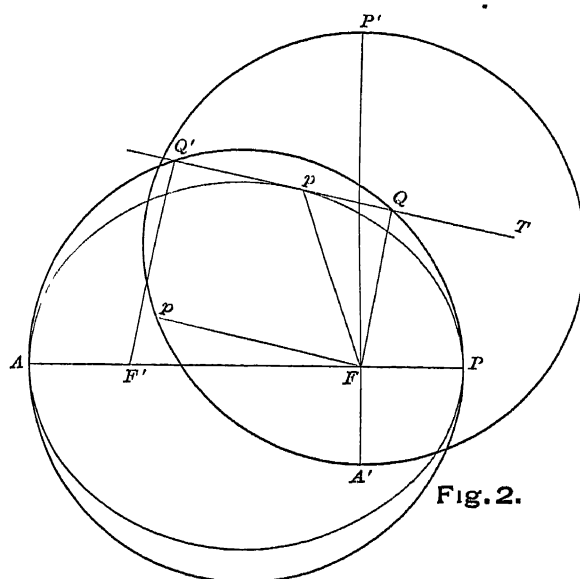
$$x' + a = -\frac{M}{C} \frac{y}{r} \quad , \quad y' + b = +\frac{M}{C} \frac{x}{r} \quad ,$$

and on squaring and adding we obtain

$$(x' + a)^2 + (y' + b)^2 = \frac{M^2}{C^2} \quad , \quad (6)$$

which shows that the hodograph of the ellipse is a circle of radius  $\frac{M}{C}$

The following geometrical proof will render the application somewhat more intelligible



In the figure let  $PpA$  be the ellipse described by the particle  $p$ ,  $PA$  being the major axis, and  $F$  and  $F'$  the two foci. Let  $pT$  be the tangent to the ellipse at  $p$ , and let the perpendicular from the focus upon the tangent be denoted by  $FQ$ . Then by definition the radius vector of the point in the hodograph is parallel to the tangent  $pT$  and proportional to the velocity at



the point  $p$ . It is well known from the law of the conservation of areas that this velocity is always inversely as the perpendicular  $FQ$ , or directly proportional to the length of  $F'Q'$ . But the locus of  $Q$  or  $Q'$  is known to be the auxiliary circle described upon the major axis as a diameter. Therefore we see that the hodograph is of the same form as the locus of  $Q'$ , but since the point  $p'$  in the hodograph is on a radius vector parallel to  $pT$ , its situation relative to the focus  $F$  will always be  $90^\circ$  in advance of  $Q$ .

The shape and situation of the hodograph relative to the ellipse is shown in the figure. Thus, when  $p$  is in periastron the point of the hodograph is in the direction perpendicular to the major axis, and at a distance proportional to  $F'Q'$ , which is then equal to  $F'P$ , and similarly for other points of the orbit. For the sake of clearness we have made the hodograph in the figure of the same size as the auxiliary circle of the ellipse, but if the radius vector in the hodograph is to represent the velocity in the ellipse the scale of the hodograph ought in reality to be greatly reduced.

If the orbit of a double star is given we may at once construct the form of the hodograph, the position relative to the ellipse being the same as in the preceding figure. Moreover if the velocity of the companion about the central star is known in absolute units for any point of the orbit, we may determine the velocity for any other point by means of the hodograph. For the magnitude of the velocity will be the length of the radius vector of the hodograph which is parallel to the tangent of the orbit at the point in question, and can easily be computed or measured graphically directly from the diagram.

When the elements of a binary are known, we may determine the component of the velocity in the line of sight as follows. Suppose  $\rho$  to be the radius vector of the point in the hodograph, and  $\omega$  to be the angle made by the radius vector  $\rho$  with the ascending node, and therefore identical with the angle made by the tangent to the orbit with the line of nodes, and let  $z$  be the inclination of the plane of the orbit to the plane tangent to the celestial sphere. Then we evidently have, as the component towards the earth,

$$\kappa = \rho \sin \omega \sin z \quad (7)$$

The angle  $z$  is an element of the star's orbit and is known, the angle  $\omega$  can be computed from the theory of the ellipse, or can be determined directly from the diagram; and when  $\rho$  is known in absolute units the component in the line of sight is perfectly determined.

We shall now show how to compute  $\omega$  and  $\rho$  for any given orbit. The

radius vector of the star  $r$  and the true anomaly  $v$  must be computed by the usual process, and then we find the radius vector with respect to the other focus

$$r' = 2a - r,$$

and we have the angle  $\gamma$  by means of the equation

$$\sin \gamma = \frac{r \sin v}{r'} \quad (8)$$

The angle  $\psi$  between the radii vectors drawn to the two foci is evidently equal to  $v - \gamma$ , and hence

$$\frac{\psi}{2} = \frac{v - \gamma}{2} \quad (9)$$

It is also easy to see that  $\varphi$ , the angle made by the tangent with the latus rectum of the ellipse, is given by

$$\varphi = v - \frac{1}{2}\psi \quad (10)$$

When the value of  $\varphi$  is determined, it is clear that

$$\omega = \lambda + 90^\circ + \varphi = v + \lambda + 90^\circ - \frac{1}{2}\psi, \quad (11)$$

so that we easily find the angle of the radius vector  $\rho$  from the ascending node

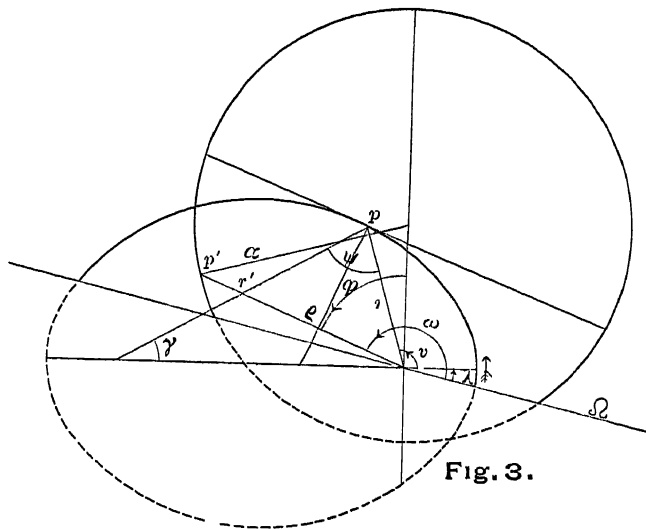


Fig. 3.

We may compute the length of this radius vector in the hodograph in the following manner. Let the radius of the circle be denoted by  $a$ , its value being

supposed known in absolute units, the linear eccentricity will be  $\alpha e$ , and we shall have

$$\alpha^2 = \rho^2 + \alpha^2 e^2 - 2\rho \alpha e \cos \varphi,$$

on solving for  $\rho$  we find

$$\rho = \alpha [e \cos \varphi + \sqrt{1 - e^2 \sin^2 \varphi}] \quad (12)$$

Thus when  $\alpha$ , the radius of the hodograph, is known in absolute units, we are enabled by means of (11) and (12) to predict the motion in the line of sight for any instant whatever

Now suppose we determine the relative motion of the companion in the line of sight by means of a modern Spectrograph such as that at Potsdam, this will give us results freed from the effect of the proper motion of the system in space, as well as the secular motion of the sun and the orbital motion of the earth. Then by equation (7) we have

$$\rho = \frac{\kappa}{\sin \omega \sin i}, \quad (13)$$

in which  $\kappa$  is furnished by spectroscopic measurement, and  $\omega$  and  $i$  are found from the orbit deduced from micrometrical measures.

A single observation therefore gives us the absolute velocity in the orbit, and this fixes the scale of the hodograph. For since we have

$$\rho = \alpha [e \cos \varphi + \sqrt{1 - e^2 \sin^2 \varphi}],$$

and  $e$  and  $\varphi$  are known, we may determine the radius of the hodograph by

$$\alpha = \frac{\rho}{[e \cos \varphi + \sqrt{1 - e^2 \sin^2 \varphi}]} \quad (14)$$

Having determined  $\kappa$  by observation, we get the absolute value of  $\rho$  by (13) and of  $\alpha$  by (14), and we may then predict the value of  $\kappa$  in absolute units for any time whatever. In practice it will be desirable to measure the motion in the line of sight when the function  $\kappa$  is a maximum, in order that an error in  $\kappa$  may have a minimum effect upon the radius of the hodograph.

When  $\alpha$  is thus determined in absolute units, the problem arises to find the absolute dimensions of the system, the masses of the stars, and their distance from the earth. Suppose we choose two epochs separated by a convenient interval of time, say a year or a fractional part of a year, when the companion is near apastron, and the velocity changes slowly. We shall denote the radii vectors by  $r_1$  and  $r_2$ , and the interval of time by  $t_2 - t_1$ . The length of the included elliptic arc can be expressed rigorously only by means of an elliptic

integral, but as the evaluation of this integral would be inconvenient in practice and for a short arc unnecessarily exact, we shall determine the length of the arc by mechanical quadrature. Thus we have

$$\text{arc} = \int_{t_1}^{t_2} \rho \, dt = \bar{\rho} (t_2 - t_1),$$

where  $\bar{\rho}$  is the average velocity of the interval, easily deduced from the hodograph. If the interval is short compared to the time of revolution, so that the arc may be put equal to its sine, we shall have approximately

$$\frac{r_1 + r_2}{2} \sin(v_2 - v_1) = \bar{\rho} (t_2 - t_1),$$

or

$$r_1 + r_2 = \frac{2\bar{\rho} (t_2 - t_1)}{\sin(v_2 - v_1)}$$

Now  $v_2$  and  $v_1$  are known true anomalies, and  $r_1$  and  $r_2$  are given in units of the major axis by the polar equation

$$\frac{r}{a} = \frac{(1 - e^2)}{1 + e \cos v}$$

Hence, with  $r_1$  and  $r_2$  thus expressed numerically, we find

$$a = \frac{2\bar{\rho} (t_2 - t_1)}{(r_1 + r_2) \sin(v_2 - v_1)} \quad (15)$$

Here the interval  $t_2 - t_1$  must be expressed in the same units as  $\bar{\rho}$ , preferably in kilometres per second. The length of the major semi-axis of the orbit is thus found in kilometres, and the absolute dimensions of the system are determined.

The parallax of the system is equal to the major semi-axis of the orbit in seconds of arc divided by the major semi-axis in astronomical units; or the distance of the system from the earth is equal to the major semi-axis in astronomical units divided by the sine of the angle subtended by the major semi-axis in seconds of arc; thus

$$d = \frac{a}{\sin \omega''} \quad (16)$$

If  $M_1 + M_2$  denote the combined mass of the system,  $M + m$  the combined mass of the sun and earth,  $a$  the major semi-axis of the orbit of the companion, and  $P$  the period of revolution,  $R$  the distance of the earth from the sun, and

$T$  the length of the sidereal year, we have, by the well known extension of KEPLER'S law

$$M_1 + M_2 = \frac{a^3}{R^3} \frac{T^2}{P^2} (M+m) \quad (17)$$

If as usual we put  $M+m=1$ ,  $R=1$ , and  $T=1$ , and express  $a$  and  $P$  in these units, we find

$$M_1 + M_2 = \frac{a^3}{P^2}, \quad (18)$$

where the mass of the system will be expressed in units of the combined mass of the sun and earth. The mass of the system is thus determined absolutely.

In conclusion it seems proper to add that this investigation was stimulated by an elegant proof of MR. F. R. MOULTON, that the aberrational orbit of a fixed star is the hodograph of the ellipse in which the earth moves, and therefore a circle. The idea brought out in MR. MOULTON'S proof caused me to revert to the motion of binaries in the line of sight, and hence no small part of the credit is due to him for the interesting application of SIR W. R. HAMILTON'S hodograph given above.

### § 6 *Rigorous Method for Testing the Universality of the Law of Gravitation\**

It remains to consider how we may use the foregoing results to test the law of NEWTON. It is evident that the law of gravitation can be tested by comparing the observed with the theoretical motion of the companion in the line of sight. We may choose a system whose orbit is accurately known and whose stars are suitable for exact spectroscopic measurement of the component  $\kappa$ ; we then determine from one or more observations at a suitable epoch the absolute dimensions of the orbit, as explained in the preceding theory, and predict the motion in the line of sight for other parts of the orbit, perhaps for a whole revolution. If we then determine by spectroscopic measurement the value of the component  $\kappa$  independent of any theory, and find that the theoretical results are confirmed by actual observations, we may consider the result a direct observational proof that the force which retains the companion in its orbit is Newtonian gravitation.

For we know from micrometrical measures that the areas described by the radius vector of the companion are proportional to the time, and therefore that

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\* *Astronomische Nachrichten* No. 3314

the force is central, and the observations of 42 *Comae Berenices*, whose motion happens to be in the plane of vision, indicate that the orbit is a plane curve. The motion being in a plane and the force being central, we must be able to show that the principal star is in the focus of the real ellipse. This can be done if we can show by spectroscopic observations that the inclination and node resulting from the theory of gravitation account perfectly for the motion in the line of sight.

We therefore assume the law of gravitation in deriving the elements of the orbit and in predicting the motion in the line of sight, as heretofore explained; spectroscopic observation will enable us to test the results of theory experimentally. If the theoretical results are confirmed by observation throughout a revolution — thus showing that the node and inclination are identical with those resulting from the theory of gravitation — we may regard the observations as giving a direct and incontestible proof of the validity of the law of NEWTON in the stellar systems.

If we desire to ascertain whether any other inclination and node — in other words, any other law of force — could give rise at every point of the orbit to a relative motion in the line of sight identical with that resulting from the law of gravitation, we may proceed as follows. Suppose that some other inclination and node and orbital velocity be possible; they will differ by unknown quantities from those values resulting from the theory of gravitation, and we shall have the relation

$$\rho \sin i \sin \omega = \rho' \sin (i + \gamma) \sin (\omega + \delta)$$

By expanding and reducing we find

$$\rho = \rho' \{ \cos \gamma \cos \delta + \cos \gamma \cot \omega \sin \delta + \sin \gamma \cot i \cos \delta + \sin \gamma \sin \delta \cot i \cot \omega \}$$

But we observe that  $\omega$  is a variable angle depending on the position of the body in the orbit; and since  $\omega = 0$ , or  $\omega = \pi$  would render the cotangent infinite, and  $\rho$  is known to be finite for every point, (the two bodies never come into contact but are always separated by a certain distance), it follows that those terms depending on  $\cot \omega$  must vanish, or  $\delta = 0$ , and the line of nodes becomes the same as that resulting from the theory of gravitation. Our expression thus takes the form

$$\rho = \rho' (\cos \gamma + \sin \gamma \cot i) = \rho' K,$$

where  $K$  is a constant

Therefore, if the inclination differs by  $\gamma$  from the value given by the theory of gravitation, it will follow that the velocity at every point of the real orbit

must be multiplied by a constant factor. But since no alteration of the inclination can change the radius vector at the line of nodes, it follows that at these points the orbital velocities would necessarily be the same however the inclination might vary. And since we have seen that the line of nodes is identical with that given by the theory of gravitation, we conclude that the velocities in the orbits could not differ throughout by a constant ratio. Hence it is evident that  $\cos \gamma + \sin \gamma \cot i = 1$ , or  $\gamma = 0$ , and the inclination is identical with that resulting from the theory of gravitation. It follows therefore that no other conceivable law of attraction could produce the same relative motion in the line of sight as the law of inverse squares. Consequently if observation shall give for every point a relative motion in the line of sight which accords with theory, we may confidently conclude that Newtonian gravitation is the force which retains the stars in their orbits.

§ 7. *On the Theoretical Possibility of Determining the Distances of Star-Clusters and of the Milky Way, and of Investigating the Structure of the Heavens by Actual Measurement\**

The practical problem of measuring the parallaxes of the fixed stars is one of the greatest of modern Astronomy, and has been solved heretofore very imperfectly. The quantity to be deduced is so very small that accidental and systematic errors often wholly obscure the element desired, and render the probable errors of most of our parallaxes painfully large compared to the minute quantities sought. Moreover, the method of relative parallax, which is the only one in general use, aside from its theoretical inaccuracy, is encumbered with many practical difficulties, the chief of which is in finding suitable comparison stars; and hence not a few astronomers have practically abandoned hope of determining the distances of the fixed stars with any considerable degree of precision. None have felt these difficulties more keenly than those astronomers who have attempted investigations requiring exact knowledge of the masses and dimensions of the stellar systems. At the present time the only parallaxes of binaries which lay claim to any considerable precision are those of  $\alpha$  Centauri ( $0''.75$ ),  $\alpha$  Canis Majoris ( $0''.38$ ),  $\gamma$  Ophiuchi ( $0''.162$ ), and  $\eta$  Cassiopeie ( $0''.154$ ). To this list we might perhaps add a few spectroscopic binaries whose parallaxes have been investigated, but even then the number of systems

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\**Astronomische Nachrichten*, No. 3323

would remain very small, and altogether insufficient to support any sound generalization respecting the masses and dimensions of binary stars as a class.

If we consider single, instead of double stars, it will be evident that while a much larger number have been measured for parallax, and in a good many cases reliable values have been derived, yet in the majority of instances the divergence of results obtained by different observers, may fairly be taken to indicate that our knowledge of stellar parallax is still very limited; and owing to the small dimensions of the earth's orbit, very little hope has been entertained of material improvement in time to come.

The method which we have developed in section 5 is full of promise for the case of binary stars. This method is theoretically applicable to any pair where the components have an angular separation of  $0''.1$ , and a single application of the spectrograph at a suitable epoch gives us the absolute dimensions, mass and parallax of the system.

As  $0''.1$  is about the present limit of exact micrometrical or heliometrical measurement, and as this angle would correspond to the parallax of a fixed star at the distance of 36 light-years (eight times the distance of  *$\alpha$  Centauri*) we see that all smaller parallaxes determined by methods heretofore in use must necessarily remain very uncertain. On the other hand the spectroscopic method will apply satisfactorily to much more distant systems—to pairs which have an angular separation of  $0''.1$ , and where an observer by the ordinary method would find that our sun had a parallax of this amount. This is equivalent to using the major semi-axis of the stellar orbit for a base line instead of the mean distance of the earth from the sun; and thus the parallaxes deduced by the spectroscopic method might be as much smaller than  $0''.1$  as the major axis of the stellar orbit is larger than that of the earth, provided of course that the combined mass of the stars is great enough to give a relative motion of the companion in the line of sight which can be measured with the desired precision.

Thus, by the usual method the parallax of  *$\alpha$  Centauri* would be just measurable at the distance of 36 light-years, and would amount to  $0''.1$ , and as the major semi-axis of the orbit would there subtend an angle of  $2''.2$ , the spectroscopic method could be applied at 22 times that distance, or when the system is removed from us by about 800 light-years. Of course we can never hope to measure the distance of a system so remote by the ordinary method, since at the distance of 800 light-years the parallax would amount to only  $0''.0045$ . If the mass and dimensions of the system be larger than those of  *$\alpha$  Centauri*, the spectroscopic method would enable us to measure a parallax correspondingly



smaller. While at present little is known of the magnitude of binary systems, it seems probable that in some cases at least the masses and dimensions will much surpass those of *α Centauri*. It is therefore probable that it will occasionally be possible to determine the distances of systems removed from us by several thousand light-years.

The present state of Astronomy does not permit us to make a confident assertion with regard to the distances of the clusters or of the Milky Way, but it seems exceedingly probable that both are very remote. In each of these species of stellar aggregation there exists a considerable but unknown number of binary stars which can be detected with our present optical means. Thus, BURNHAM has searched for double stars in several of the great northern clusters, such as *Praesepe*, the *Pleiades* and the great clusters in *Perseus*, *Hercules*, &c. (Publications of Lick Obs., vol II pp 211–216), and discovered a number of pairs which promise to be physically connected. He observes that interesting stars are apparently more frequent in wide clusters like the *Pleiades*, *Praesepe*, and the great cluster in *Perseus*, than in the more compact clusters like that in *Hercules*. Yet he has discovered an important pair in this dense globular cluster, and SIR JOHN HERSCHEL has likewise detected double stars of special interest in several of the great clusters of the southern hemisphere. It is not to be doubted that many more such objects will be detected when the clusters generally are critically examined under the powers of our great modern refractors.

When the orbits of these binaries have been found by exact micrometrical measurement, the spectroscopic method will eventually afford the means for determining their immense distances, not by probable assumptions but by exact computation. It is evident therefore that if we are ever to determine the distances of clusters from the earth — and no sound ideas of the nature of these masses of stars can be formed until such determination is made — we must first search the clusters critically for binary stars, and determine their orbits by micrometrical measurement. If, when the orbit is known, it shall appear that the binary has the same proper motion as the adjacent stars of the group, there will be a strong presumption that the system forms a part of the cluster. If the pair be also of about the same magnitude as its neighbors, and of the same color and spectral type, we may conclude with practical certainty that the binary is intimately connected with the mass of stars in which it is projected.

Determination of the parallax of the binary will therefore give the distance of the cluster from the earth, and supply all desired information as to the dimensions of the cluster, the brilliancy of its stars, their mutual distances, &c. If in like manner any group of stars in the Milky Way could be carefully

searched for binary systems, and some intimate connection of a pair with neighboring stars shown to exist, a determination of its orbit and an application of the spectroscopic method would lead to a knowledge of the distance of that part of the Milky Way. By extending the same process to all parts of the Galaxy it will be possible in the course of time to ascertain the nature of that immense aggregation of stars, and throw light upon the construction of the heavens. While the spectroscopic method applies only to binary stars, it is evident that their great abundance and universal distribution in space will some day give a means for determining with precision and certainty the actual structure of the sidereal universe.

We must not expect that the immense possibilities here outlined will be practically realized at once, or even in the near future, yet giant refractors like the 40-inch Yerkes Telescope will give such power for separating close double stars and for supplying a great amount of light for the spectroscopic study of faint objects, that an application of these ideas may not be found impossible in the course of the coming century. If there be spectroscopic or photographic difficulties, the progress of spectroscopic Astronomy during the last thirty years justifies the belief that such obstacles will not continue to be insurmountable. The great philosophic interest attaching to the foregoing method for investigating the structure of the visible universe by exact spectroscopic measurement, instead of by the doubtful processes of gauging employed by HERSCHEL and STRUVE, appears to be a sufficient justification for considering what is at present only a theoretical possibility. The history of Astronomy shows that it is not always the theories that can be realized in a decade or even in a century which in the long run exercise the most important influence on the development of science.

#### § 8. *Historical Sketch of the Different Methods for Determining Orbits of Double Stars*

It is assumed that the law of gravitation governs the motions of double stars, and therefore that the orbits are ellipses with the principal stars in the foci. From the nature of conic sections the centre of the real ellipse will be projected into the centre of the apparent ellipse. But in general the foci of the real ellipse will not fall upon the foci of the apparent ellipse. If, however, a line be drawn from the centre of the apparent ellipse to the principal star and prolonged in either direction until it intersects the curve, the result will define the projection of the real major axis. The diameter of the

ellipse conjugate to this line will be the projection of the minor axis. Thus it is easy to fix the positions of the real major and minor axes as seen in the apparent orbit. Since all parts of the major axis are shortened in the same ratio, the eccentricity of the real orbit may be deduced from the apparent orbit, by dividing the distance from the centre to the principal star by the major semi-axis as seen in projection. The end of this axis which is nearest the principal star will be the periastron, that farthest away, the apastron, the dates corresponding to the passage of the companion through these points will give the epochs of periastron and apastron passage respectively. It is evident that only one diameter of the real ellipse will suffer no shortening, owing to projection, and this is the diameter parallel to the line of nodes. If from points on the apparent ellipse perpendiculars be drawn to this diameter, and then increased in the ratio of  $\cos i$  to 1, we shall get points of the real orbit whose projections give points on the apparent orbit.

The observations of a double star are expressed in polar coordinates,  $\rho$  and  $\theta$ , which give the angular separation of the components in seconds of the arc of a great circle, and the position-angle of the companion with respect to the meridian. The companion is thus referred to the principal star regarded as fixed, and hence the observations give the means of finding only the relative orbit of one star about the other. The absolute orbit of either star about the centre of gravity of the system has a form similar to that of the relative orbit, but the linear dimensions are reduced in the ratio of  $M_2$  or  $M_1$  to  $M_1 + M_2$ , where  $M_1$  and  $M_2$  are the masses of the stars. The absolute orbits of the stars have the same shape, but are reversed in relative position. The centre of gravity of a pair of stars can be determined only by the criterion that the centre of gravity of a system moves uniformly in a right line, and as most of the systems have too little motion to define this point with any considerable degree of precision, owing to the imperfect state of our absolute positions as determined by the meridian circle, it is in general impossible to define the absolute orbits or relative masses of the stars. With few exceptions, therefore, astronomers have contented themselves heretofore with determining the relative orbit of one body about the other.

The first method for determining the orbit of a double star was proposed by SAVARY in 1827 (*Connaissance des Temps*, 1830). This method is closely analogous to those used for planets and comets, in so far as it rests on the treatment of four complete observations for the definition of the seven elements. The problem is solved by elaborate geometrical constructions, such as characterize work in pure mathematics rather than the practical processes which must

be invoked by the working computer. SAVARY's principal equation is based on the difference between the sector and triangle, the area derived from the time being equated with an expression involving the products of the semi-axes and eccentric angles of the apparent ellipse. The method is thus ill adapted to the determination of an orbit from such positions as are furnished by the measures of double stars.

ENCKE recast the method of SAVARY, from the point of view of a practical computer, and deduced formulæ similar to those used by astronomers in their work on planets and comets. Rejecting the equations depending on conjugate diameters, so much employed by the French geometer, he based his formulæ on recognized astronomical processes and developed tables to facilitate their application. As SAVARY had applied his method to  $\xi$  *Ursae Majoris*, ENCKE was led to illustrate his computations on the equally well-known system of 70 *Ophiuchi* (*Berliner Astronomisches Jahrbuch*, 1832).

SIR JOHN HERSCHEL took up the problem about 1830, and sought to improve the processes by a graphical method which enabled him to make use of all the observational material, and to eliminate the grosser errors of the individual observations. He was convinced that in order to obtain orbits of a satisfactory character, it would be necessary to correct the angles by an interpolating curve, one axis representing the time, the other the angle, and that the distances must be rejected altogether, except for the determination of the major axis. He proceeds by successive approximations to deduce normal places for the angles, and by gradual improvement of his graphical results renders them consistent with an ellipse, and finally obtains a satisfactory apparent orbit. The elements are then deduced by formulæ not very different from those employed by SAVARY. The method is illustrated by applications to  $\gamma$  *Virginis*,  $\alpha$  *Geminorum*,  $\sigma$  *Coronae Borealis*,  $\xi$  *Ursae Majoris*, and 70 *Ophiuchi* (*Memoirs, Royal Astronomical Society*, Vol. V).

While the process of interpolation invented by HERSCHEL has been extensively employed, and in some cases is very useful, I am satisfied that in general it is better to plot the observations directly and to make a trial ellipse the interpolating curve. This enables us to use both angles and distances and secures all the advantage of judgement which HERSCHEL considered so essential. It often happens that the length of the radius vector changes with extreme rapidity, and as the areas are constant this will imply very great and unequal changes in the angular motion; when the angular velocity of the radius vector is so variable in different parts of the apparent ellipse the course of the interpolating curve becomes altogether uncertain. Under these conditions it is much

better to use the observations directly. It is also recognized that modern measures of distance should be allowed an equal or nearly equal weight in the determination of orbits.

After SAVARY, ENCKE and HERSCHEL had given such an impetus to the study of sidereal systems, the work was carried forward by MADLER and VILLARCEAU, both of whom published a number of orbits with some minor improvements in the processes of computation.

KLINKERFUES took up the subject about 1856, and in the course of work on several orbits developed very elegant formulæ and more practical methods than any which had been used before. His analytical method is marked by rigor and generality, but in the present state of double-star Astronomy is not so practicable as the graphical method treated in section 10.

THIELE, some years later, devised an elegant graphical method which has many good points, and is much admired by those who are inclined to determine all the elements geometrically. It will be found in the *Astronomische Nachrichten*, Band LII\*.

Among the more recent investigations those of PROFESSOR KOWALSKY are remarkable for their extreme elegance and great generality. This method, depending on the general equation of a conic, is all that can be desired from a mathematical point of view, and as simplified by GLASENAPP has been extensively used by several computers. The original exposition of the method will be found in the *Proceedings of the Imperial University of Kasan* for 1873, the valuable modification introduced by GLASENAPP is given in the *Monthly Notices*, Vol XLIX, p 278.

Other recent investigations which are worthy of special notice include those of SEELIGER (*Inaugural Dissertation* of SCHORR, Munich, 1889), and of ZWIERS (*Astronomische Nachrichten*, No. 3336).

It is singular that nearly all the methods given above have been developed from the point of view of analysis rather than of practical Astronomy. BURNHAM has recently rendered double-star Astronomy a conspicuous service by reviving the method of representing observations first employed by WILLIAM STRUVE (*Mensurae Micrometricae*, last plate). This consists in plotting the points as determined by the micrometer, and in finding from the places thus laid down the apparent ellipse which best satisfies the observations. We have used a modification of this method throughout the present work, and have discussed it in connection with the graphical method of KLINKERFUES, which supplies the process for deriving the elements from the apparent orbit.

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\*It is also explained by PROFESSOR HALL in *The Astronomical Journal*, No 324.

§ 9. *Kowalsky's Method.*

We shall now give an exposition of the elegant method of KOWALSKY, which seems likely to be the one that will ultimately be adopted by astronomers. The general equation of the ellipse with the origin at any point, here taken at the principal star, is

$$F = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, \quad (1)$$

which may be reduced to the form .

$$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + 1 = 0 \quad (2)$$

This equation contains five unknown constants, and hence five values of  $x$  and  $y$  will enable us to determine the constants of the ellipse. Each observation gives one equation by means of the relations

$$x_0 = \rho_0 \cos \theta_0, \quad y_0 = \rho_0 \sin \theta_0,$$

where the  $x$ -axis is directed to the north-point. And hence five observations at different epochs will give a determination of the apparent orbit. In practice it is found that a larger number of observations is desirable, and if the observations are sufficiently good, the best results will generally be obtained by a least-square adjustment of the residuals.

When the apparent ellipse is determined, the problem arises to express the elements of the real orbit in terms of the constants which fix the apparent orbit.

It is evident that projection does not alter the diameter coinciding with the line of nodes, and this enables us to pass from the apparent to the real orbit. The real orbit is evidently the curve determined by the intersection of the orbit-plane with the elliptical cylinder whose right section is the apparent orbit. In the apparent orbit the axis of  $x$  is directed to the north-point, but in passing to the real orbit we shall direct the new axis of  $x$  to the ascending node, while the new axis of  $y$  will be taken in the plane of the real orbit, and the origin retained at the principal star. Calling the new system of coordinates  $x', y', z'$ , it is evident that we shall have

$$\left. \begin{aligned} x &= x' \cos \Omega - y' \sin \Omega \cos i + z' \sin \Omega \sin i \\ y &= x' \sin \Omega + y' \cos \Omega \cos i - z' \cos \Omega \sin i \\ z &= \phantom{x' \sin \Omega +} y' \sin i + z' \cos i \end{aligned} \right\} \quad (3)$$

If we put  $z' = 0$ , we shall have the coordinates of a point in the plane of the real orbit. Thus our expressions are simplified, and become equations for turning the axis of  $x$  through the angle  $\Omega$ , and that of  $y$  through the angle  $\iota$ . If we put

$$x = x' \cos \Omega - y' \sin \Omega \cos \iota, \quad y = x' \sin \Omega + y' \cos \Omega \cos \iota,$$

in (2), we shall obtain the equation of the intersection of the plane  $x'y'$  with the elliptical cylinder, which is the equation of the real ellipse. Thus we have, on omitting the accents,

$$\left. \begin{aligned} & A(x \cos \Omega - y \sin \Omega \cos \iota)^2 \\ & + 2H(x \cos \Omega - y \sin \Omega \cos \iota)(x \sin \Omega + y \cos \Omega \cos \iota) \\ & + B(x \sin \Omega + y \cos \Omega \cos \iota)^2 + 2G(x \cos \Omega - y \sin \Omega \cos \iota) \\ & + 2F(x \sin \Omega + y \cos \Omega \cos \iota) + 1 = 0 \end{aligned} \right\} \quad (4)$$

The equation of the real ellipse referred to its centre is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (5)$$

If we shift the origin to the focus, we must increase  $x$  by  $ae$ , and the equation becomes

$$\frac{(x+ae)^2}{a^2} + \frac{y^2}{b^2} - 1 = 0, \quad (6)$$

when referred to the principal star.

Now suppose  $\lambda$  to be the angle from the node to the periastron, measured in the plane of the real orbit, then if we turn the axis of  $x$  back to the line of nodes, the new coordinates are

$$x \cos \lambda + y \sin \lambda, \quad -x \sin \lambda + y \cos \lambda$$

By means of these values of  $x$  and  $y$ , equation (6) becomes

$$\frac{(x \cos \lambda + y \sin \lambda + ae)^2}{a^2} + \frac{(-x \sin \lambda + y \cos \lambda)^2}{b^2} - 1 = 0, \quad (7)$$

the origin is taken at the focus and the axis of  $x$  is directed to the node

Now this equation is necessarily identical with (4), which also represents the true ellipse referred to the same axes. Hence, when multiplied by a constant factor  $\epsilon$  the coefficients of the variables must equal the corresponding ones in the equation deduced from the apparent orbit, so that (7) and (4) give

$$\epsilon \left( \frac{\cos^2 \lambda}{a^2} + \frac{\sin^2 \lambda}{b^2} \right) = A \cos^2 \Omega + B \sin^2 \Omega + H \sin 2\Omega \quad (8)$$

$$\epsilon \left( \frac{\sin^2 \lambda}{a^2} + \frac{\cos^2 \lambda}{b^2} \right) = (A \sin^2 \Omega + B \cos^2 \Omega - H \sin 2\Omega) \cos^2 \iota \quad (9)$$

$$\epsilon \left( \frac{1}{a^2} - \frac{1}{b^2} \right) \sin 2\lambda = (-A \sin 2\Omega + B \sin 2\Omega + 2H \cos 2\Omega) \cos \iota \quad (10)$$

$$\epsilon \frac{e \cos \lambda}{a} = G \cos \Omega + F \sin \Omega \quad (11)$$

$$\epsilon \frac{e \sin \lambda}{a} = (-G \sin \Omega + F \cos \Omega) \cos \iota \quad (12)$$

$$\epsilon (e^2 - 1) = +1 \quad (13)$$

This last equation gives

$$\epsilon = -\frac{1}{1-e^2} \quad \text{and} \quad \frac{\epsilon e}{a} = -\frac{e}{\rho}$$

Also, since

$$\frac{\epsilon^2 e^2}{a^2} = \frac{e^2}{\rho^2} = \epsilon \left( -\frac{1}{1-e^2} \right) \frac{e^2}{a^2} = \epsilon \left( \frac{1}{1-e^2} \right) \frac{b^2 - a^2}{a^4} = \epsilon \frac{b^2 - a^2}{b^2 a^2} = \epsilon \left( \frac{1}{a^2} - \frac{1}{b^2} \right),$$

we have

$$\frac{e^2}{\rho^2} = \epsilon \left( \frac{1}{a^2} - \frac{1}{b^2} \right)$$

Now (11) and (12) give

$$e \sin \lambda = -\rho (F \cos \Omega - G \sin \Omega) \cos \iota \quad ; \quad e \cos \lambda = -\rho (F \sin \Omega + G \cos \Omega)$$

Multiplying (11) by (12) and reducing, we find

$$\frac{e^2}{\rho^2} \sin 2\lambda = (F^2 \sin 2\Omega - G^2 \sin 2\Omega + 2FG \cos 2\Omega) \cos \iota$$

From (10) we have

$$\frac{e^2}{\rho^2} \sin 2\lambda = (-A \sin 2\Omega + B \sin 2\Omega + 2H \cos 2\Omega) \cos \iota,$$

and hence

$$(F^2 - G^2 + A - B) \sin 2\Omega + 2(FG - H) \cos 2\Omega = 0 \quad (14)$$

If we subtract (9) from (8), we get

$$\frac{e^2}{\rho^2} \cos 2\lambda = \epsilon \left( \frac{\cos^2 \lambda - \sin^2 \lambda}{a^2} - \frac{\cos^2 \lambda - \sin^2 \lambda}{b^2} \right),$$

and the difference of the squares of  $e \cos \lambda$  and  $e \sin \lambda$  gives another value of  $\frac{e^2}{\rho^2} \cos 2\lambda$ . Equating these two values of  $\frac{e^2}{\rho^2} \cos 2\lambda$ , and solving for  $\cos^2 \iota$ , we find

$$\cos^2 \iota = \frac{(F^2 - B) \sin^2 \Omega + (G^2 - A) \cos^2 \Omega + (FG - H) \sin 2\Omega}{(F^2 - B) \cos^2 \Omega + (G^2 - A) \sin^2 \Omega - (FG - H) \sin 2\Omega} \quad (15)$$



The forms of the numerator and denominator show that if we put  $\cos^2 \iota = \frac{P}{Q}$ , and hence  $\tan^2 \iota = \frac{Q-P}{P} = \frac{Q+P}{P} - 2$ , we shall get

$$\tan^2 \iota = \frac{F^2 + G^2 - (A+B)}{P} - 2$$

The first member of (9) gives

$$\epsilon \left( \frac{\sin^2 \lambda}{a^2} + \frac{\cos^2 \lambda}{b^2} \right) = \frac{e^2}{\mu^2} \sin^2 \lambda - \frac{1}{\mu^2},$$

and therefore we obtain

$$\frac{e^2}{\mu^2} \sin^2 \lambda - \frac{1}{\mu^2} = (A \sin^2 \Omega + B \cos^2 \Omega - H \sin 2\Omega) \cos^2 \iota$$

By squaring (12) we find

$$\frac{e^2}{\mu^2} \sin^2 \lambda = (F^2 \cos^2 \Omega + G^2 \sin^2 \Omega - FG \sin 2\Omega) \cos^2 \iota$$

Therefore we have

$$\frac{1}{\mu^2} = [(F^2 - B) \cos^2 \Omega + (G^2 - A) \sin^2 \Omega - (FG - H) \sin 2\Omega] \cos^2 \iota \quad (16)$$

Comparing this with (15), we find  $\frac{1}{\mu^2} = P$ , and hence

$$\frac{2}{\mu^2} + \frac{\tan^2 \iota}{\mu^2} = F^2 + G^2 - (A+B) \quad (17)$$

Now since

$$\frac{1}{\mu^2} = P = (F^2 - B) \sin^2 \Omega + (G^2 - A) \cos^2 \Omega + (FG - H) \sin 2\Omega,$$

we easily find

$$\frac{2}{\mu^2} = F^2 + G^2 - (A+B) - (F^2 - B) \cos 2\Omega + (G^2 - A) \cos 2\Omega + 2(FG - H) \sin 2\Omega \quad (18)$$

Hence (17) gives

$$\frac{\tan^2 \iota}{\mu^2} = (F^2 - G^2 + A - B) \cos 2\Omega - 2(FG - H) \sin 2\Omega \quad (19)$$

If we multiply this equation by  $\sin 2\Omega$ , and (14) by  $\cos 2\Omega$ , and subtract the last result from the first, we get

$$\frac{\tan^2 \iota}{\mu^2} \sin 2\Omega = -2(FG - H)$$

If we use  $\cos 2\Omega$  and  $\sin 2\Omega$ , and add the products, we have

$$\frac{\tan^2 i}{\rho^2} \cos 2\Omega = F^2 - G^2 + A - B$$

Therefore we finally obtain the following set of equations:

$$\left. \begin{aligned} \frac{\tan^2 i}{\rho^2} \sin 2\Omega &= -2(FG - H), \\ \frac{\tan^2 i}{\rho^2} \cos 2\Omega &= F^2 - G^2 + A - B, \\ \frac{2}{\rho^2} + \frac{\tan^2 i}{\rho^2} &= F^2 + G^2 - (A + B), \\ e \sin \lambda &= -\rho(F \cos \Omega - G \sin \Omega) \cos i, \\ e \cos \lambda &= -\rho(G \sin \Omega + F \cos \Omega), \\ a &= \frac{\rho}{1 - e^2} \end{aligned} \right\} (20)$$

These formulae enable us to find  $\Omega, i, \rho, \lambda, e, a$ ; we may then find  $v$  at any epoch by the formula

$$\tan(v + \lambda) = \frac{\tan(\theta - \Omega)}{\cos i}, \text{ and } E \text{ by } \tan \frac{1}{2} E = \sqrt{\frac{1-e}{1+e}} \tan \frac{1}{2} v$$

We find  $M$  by KEPLER'S equation

$$M = E - e'' \sin E.$$

And since  $M_2 - M_1 = n(t_2 - t_1)$ , we see that

$$n = \frac{M_2 - M_1}{t_2 - t_1},$$

and

$$P = \frac{360^\circ (t_2 - t_1)}{M_2 - M_1}, \quad T = \frac{n - M}{n} \quad (21)$$

PROFESSOR GLASENAPP has proposed a simple method for cases in which good drawings of the apparent orbits have been made, but it is not desired to adjust the results by the method of Least Squares, owing to the uncertainty of the data furnished by observation. In the present state of double-star Astronomy this method is very practicable, and can be advantageously employed in the determination of orbits

In the equation (2)

$$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + 1 = 0,$$

we put  $y = 0$ , and then find the roots of

$$Ax^2 + 2Gx + 1 = 0 \quad (22)$$

This may be written

$$x^2 + \frac{2G}{A}x + \frac{1}{A} = 0, \quad \text{or} \quad (x-x_1)(x-x_2) = x^2 - (x_1+x_2)x + x_1x_2 = 0,$$

where  $x_1$  and  $x_2$  are the roots of the equation, or the abscissae of the points of the orbit on the  $x$ -axis.

Hence, by the theory of equations, we have

$$A = \frac{1}{x_1x_2}$$

Also

$$\frac{2G}{A} = -(x_1+x_2), \quad \text{or} \quad G = -\frac{A(x_1+x_2)}{2} = -\frac{(x_1+x_2)}{2x_1x_2}$$

In like manner, putting  $x=0$ , we find

$$By^2 + 2Fy + 1 = 0, \quad \text{or} \quad B = \frac{1}{y_1y_2}, \quad F = -\frac{y_1+y_2}{2y_1y_2}$$

Hence when the coordinates of the intersections of the orbit with the axes of  $x$  and  $y$  are known directly from the apparent orbit, we have the four constants  $A, B, F, G$

And the other constant is given by

$$H = -\frac{Ax^2 + By^2 + 2Gx + 2Fy + 1}{2xy}$$

In finding  $H$  we must take a point  $(x, y)$  such that the product  $xy$  has a large value. It may be desirable to take the mean of several values of  $H$

When all the constants  $A, B, F, G, H$ , have been derived, we find the elements by equations (20) and (21)

## § 10 *Graphical Method of Klinkerfues*

Suppose  $\alpha$  and  $\beta$  to denote the lengths of the real major and minor semi-axes when projected on the plane tangent to the celestial sphere, and  $A$  and  $B$  to be their position-angles. Then we readily find

$$\left. \begin{aligned} \alpha^2 \cos^2(A-\Omega) + \alpha^2 \sin^2(A-\Omega) \sec^2 i &= \alpha^2 \\ \beta^2 \cos^2(B-\Omega) + \beta^2 \sin^2(B-\Omega) \sec^2 i &= \beta^2 \end{aligned} \right\} \quad (1)$$

But it is evident that the sum of these equations is the square of the chord between the vertices of the major and minor axes, and the square of the same chord is given by

$$\{\alpha \cos(A-\Omega) - \beta \cos(B-\Omega)\}^2 + \{\alpha \sin(A-\Omega) - \beta \sin(B-\Omega)\}^2 \sec^2 i = a^2 + b^2$$

Therefore we have

$$\cos(A-\Omega) \cos(B-\Omega) + \sin(A-\Omega) \sin(B-\Omega) \sec^2 i = 0, \quad (2)$$

and hence

$$\cos^2 i = \tan(A-\Omega) \tan(\Omega-B) \quad (3)$$

This equation determines the inclination when the node is known, as the angles  $A$  and  $B$  are taken directly from the apparent orbit

If we divide the second of equations (1) by the first, we get

$$\frac{b^2 \alpha^2}{a^2 \beta^2} = \frac{\cos^2(B-\Omega) + \sin^2(B-\Omega) \sec^2 i}{\cos^2(A-\Omega) + \sin^2(A-\Omega) \sec^2 i},$$

and on substituting for  $\sec^2 i$  its value, we find

$$\frac{b^2 \alpha^2}{a^2 \beta^2} = - \frac{\sin 2(B-\Omega)}{\sin 2(A-\Omega)} \quad (4)$$

In this equation  $\alpha$  and  $\beta$  are given directly by the apparent orbit, and as  $e$  is known, we have also the ratio  $\frac{b^2}{a^2} = 1 - e^2$ . Therefore the only unknown quantity is  $2\Omega$ , which we may determine in the following manner. Since the left member of (4) is the square of a real quantity, the right member must be essentially positive, and we may put

$$\tan \zeta = \frac{b\alpha}{a\beta} = \sqrt{\frac{\sin 2(B-\Omega)}{\sin 2(A-\Omega)}}, \quad (5)$$

and since

$$\sec 2\zeta = \frac{\sin 2(A-\Omega) + \sin 2(B-\Omega)}{\sin 2(A-\Omega) - \sin 2(B-\Omega)} = \tan(A+B-2\Omega) \cot(A-B),$$

we get

$$\tan(A+B-2\Omega) = \sec 2\zeta \tan(A-B) \quad (6)$$

The angle  $\zeta$  is known from its tangent, and hence we easily find  $\Omega$

In (3) it is to be observed that  $\cos^2 i$  is necessarily positive and smaller than unity, and hence we have to choose between two values of  $\Omega$  differing by  $180^\circ$ . As it is thus impossible to distinguish between the ascending and descending node, we may arbitrarily take the ascending node between  $0^\circ$  and  $180^\circ$ , and find  $i$  by means of (3)

$$\cos^2 i = \tan(A-\Omega) \tan(\Omega-B)$$

The angular distance from the node to the periastron is denoted by  $\pi - \Omega = \lambda$ , and is given by the equation

$$\tan(A - \Omega) = \cos i \tan \lambda,$$

or by using (3) we obtain\*

$$\tan^2 \lambda = \frac{\tan(A - \Omega)}{\tan(\Omega - B)} \quad (7)$$

If  $u$  denote the argument of the latitude, we have

$$u = v + \lambda = v + \pi - \Omega, \quad \text{and} \quad \tan u = \sec i \tan(\theta - \Omega),$$

where  $\theta$  is the observed position-angle at the given epoch. The latitude  $l$  is given by  $\sin l = \sin i \sin u$ .

From the apparent radius vector  $\rho$ , we may find the corresponding true radius vector by

$$r = \rho \sec l$$

The major semi-axis is then found by the polar equation

$$a = \frac{r(1 + e \cos v)}{1 - e^2} \quad (8)$$

If we take the apastron as the point in question,  $l$  will be given by

$$\sin l = \sin i \sin \lambda,$$

and since  $\rho$  is taken directly from the diagram of the apparent orbit, we easily find  $r$ . Then, since  $v = 180^\circ$ , we have

$$a = \frac{\rho \sec l}{1 + e} \quad (9)$$

To find the time of revolution we take two observations which are widely separated in time, and find the intervening change in the mean anomaly, or we may find from the diagram the part of the area swept over during this interval compared to the whole area of the apparent ellipse. If  $\theta_1$  and  $\theta_2$  be the two angles of position, and  $u_1$  and  $u_2$  the corresponding arguments of the latitude, we shall have

$$\begin{aligned} \tan u_1 &= \sec i \tan(\theta_1 - \Omega), \\ \tan u_2 &= \sec i \tan(\theta_2 - \Omega), \end{aligned}$$

and then

$$v_1 = u_1 - \lambda, \quad v_2 = u_2 - \lambda,$$

whence the mean anomalies are easily found. Instead of computing the change of the mean anomaly, it is generally preferable to measure up the area swept

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\*  $A - \Omega$  and  $\lambda$  must be in the same or in opposite quadrants. Throughout this work  $\lambda$  is taken in the direction of the motion.

over by the radius vector during the interval, and determine the period by the law of areas

Suppose that  $t_1$  and  $t_2$  be the dates of two widely-separated observations, then the double area swept over by the radius vector will be

$$\int_{t_1}^{t_2} \rho^2 \frac{d\theta}{dt}$$

Putting  $a', b'$  for the major and minor semi-axes of the apparent ellipse, it is evident that the time of revolution will be given by

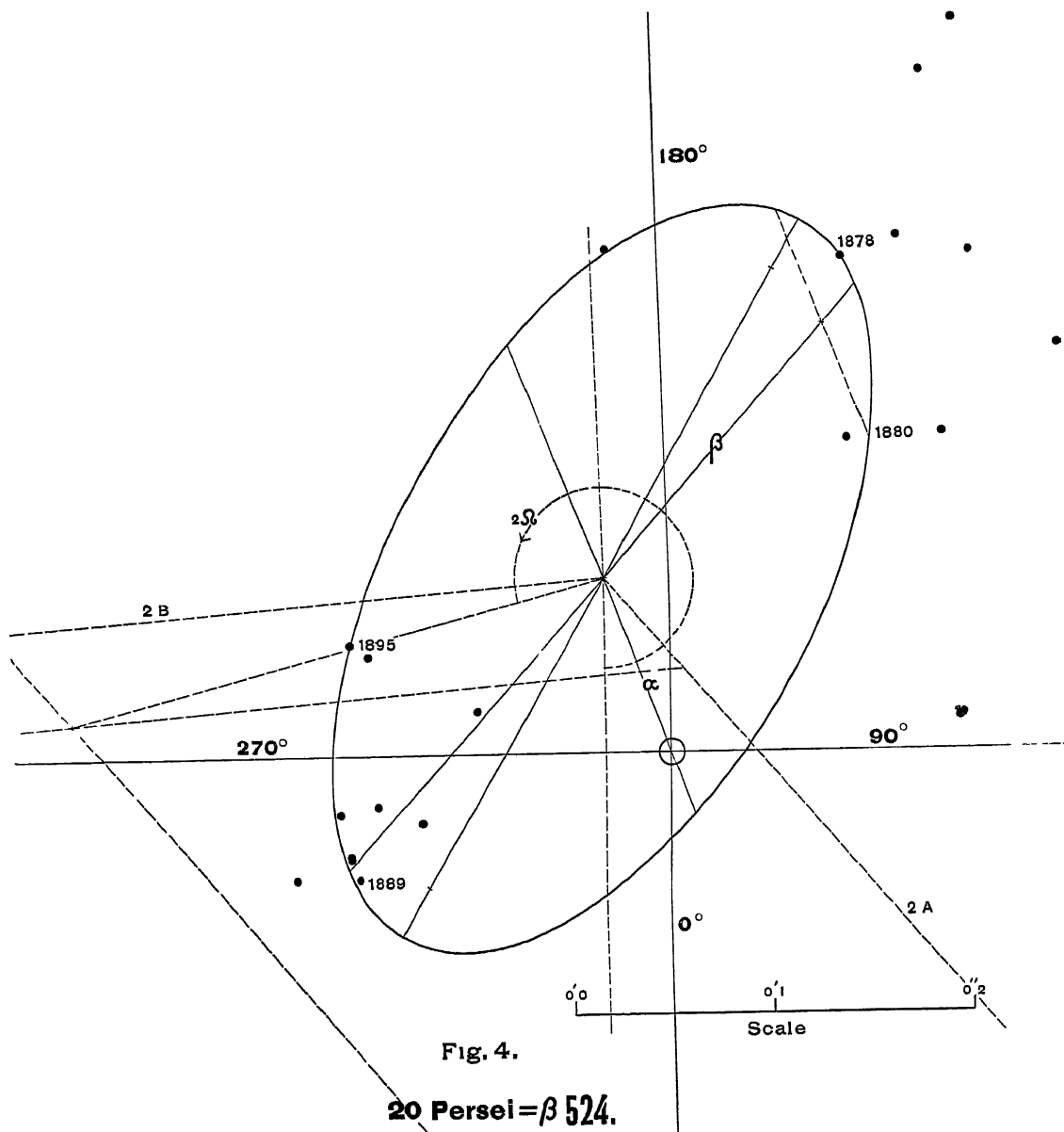
$$P = \frac{2\pi a'b'(t_2 - t_1)}{\int_{t_1}^{t_2} \rho^2 \frac{d\theta}{dt}} \quad (10)$$

In case the period is computed from the change in the mean anomalies, we have

$$n = \frac{M_2 - M_1}{t_2 - t_1}, \quad P = \frac{360^\circ}{n} \quad (11)$$

The periastron passage is given by  $T = t_1 - \frac{M_1}{n}$ , or it may be found from the principle of areas, in the same manner as the period. Thus, since the double areal velocity is known, we simply determine the double area included between a given radius vector and the periastron, and ascertain the intervening time. This interval is to be added to or subtracted from the time of observation, according as the date chosen is before or after the epoch of periastron passage.

To find the node by graphical construction we draw from the centre of the ellipse lines whose position-angles are  $2A$  and  $2B$ ; then, parallels to these at distances related as  $a^2\beta^2$  to  $b^2\alpha^2$ . Connect the intersection of the parallel lines with the centre, and this will give a line whose position-angle is  $2\Omega$ . This construction is easily deduced from (4), and in practice will be found extremely exact. The graphical method is highly practicable, and in the present state of double-star Astronomy is the one which should generally be preferred. The possible inaccuracies of the method are greatly inferior to the uncertainty still attaching to the best orbits. The principal difficulty experienced by computers consists in the finding of a satisfactory apparent orbit.



The apparent orbit of 20 *Persei* =  $\beta$  524 is shown above. We find by the figure  $e = 0.738$ ,

$$\frac{b^2 a^2}{a^2 \beta^2} = 0.194, \quad A = 20^\circ 5', \quad B = 137^\circ 3', \quad \Omega = 142^\circ 2', \quad i = 67^\circ 9',$$

$$\lambda = 103^\circ 1', \quad n = -9^\circ 0', \quad P = 40.0 \text{ years}, \quad a = 0'' 290, \quad T = 1884.10$$

To obtain the apparent orbit it is best to make use of both angles and distances. If the precession has a sensible effect upon the position angles, it is desirable to refer the observations to a common epoch by applying the formula

$$\Delta \theta = n \sin \alpha \sec \delta (t - t_0) \quad (12)$$

where  $n = 20''.04987$ , and  $t_0$  is the date of observation,  $t$  the epoch adopted. We then combine the individual measures of the best observers into suitable annual means, and plot the resulting positions on a convenient scale. The approximate normal places thus defined are subject to two conditions.

(1) That the areas swept over by the radius vector shall be proportional to the times,

(2) That the apparent ellipse which satisfies the law of areas shall conform also to the observed distances.

The ellipse which satisfies these conditions must be found by trial. Fine planimeter measurement renders the approximation comparatively rapid, and when a satisfactory ellipse has been obtained we derive the elements and compare the computed with the observed places.

We first determine  $e$ , then compute the ratio  $\frac{b^2 a^3}{a^2 b^3}$ , and find the node by graphical construction, it is then easy to find  $i, \lambda, P, T$ , and  $a$ , as explained in the foregoing method. If further refinement of the elements be desired, recourse must be had to differential formulae.

It is to be remarked, however, that the assumption of constant areal velocity is equivalent to postulating the absence of unseen bodies or other disturbing influences, and as this is not yet fully established, the orbits which best represent the angular motion are not necessarily correct, as may be seen in the case of *70 Ophiuchi*. If it is necessary to violate the distances in a conspicuous manner in order to preserve the law of the areas, the result must be looked upon with suspicion. In the present state of double-star Astronomy most of our orbits must be regarded as tentative, but when they shall finally be improved there is no doubt that, if the motion is really undisturbed, both angles and distances will be well represented.

If it is desired to compute  $\rho$  and  $\theta$  from the elements, we may employ the formulae

$$\tan(\theta - \Omega) = \tan(\lambda + v) \cos i \quad , \quad \rho = a(1 - e \cos E) \frac{\cos(\lambda + v)}{\cos(\theta - \Omega)}.$$

The element  $\lambda$  is counted from the node between  $0^\circ$  and  $180^\circ$ , in the direction of the motion, in case of retrograde motion the formula for  $\theta$  becomes

$$\tan(\Omega - \theta) = \tan(\lambda + v) \cos i$$





The elements required for this purpose are the following.

Eccentricity,	$e = 0.700 \pm 0.02$
Major semi-axis,	$a = 0''.6549$
Node,	$\Omega = 95^\circ 5$
Inclination,	$i = 77^\circ 72$
Node to periastron,	$\lambda = 75^\circ 28$

We lay down on suitable drawing paper two lines which intersect each other at right angles, and thus mark the four quadrants of position-angle. The intersection of these lines will be the centre of the real orbit and also the centre of the apparent orbit. The line of nodes is then drawn through the centre, having a position-angle of  $95^\circ 5$ . In like manner we lay down the line whose position-angle is  $\Omega + \lambda = 170^\circ 78$ , and this will be the major axis of the real ellipse.

We now adopt a convenient scale, which will give a length on the drawing paper of 10 or 12 inches for the major axis.

With close stars  $0''.1$  may represent one or two inches of the scale, so that the work can be done with the highest degree of accuracy. From the centre the length of the major semi-axis ( $0''.6549$ ) is laid down on the line just drawn, and the distance of the foci of the ellipse from the centre will be  $ae$  ( $0''.6549 \times 0.70$ ). The ellipse is then drawn in the usual manner.

We now lay off points on the line of nodes at equal distances from the centre of the ellipse, and through these points draw lines  $aa'$ ,  $bb'$ ,  $cc'$ ,  $dd'$  etc, perpendicular to the line of nodes. The lengths of these lines on either side are found in seconds of arc by the scale used, and then multiplied by the cosine of the inclination ( $\cos 77^\circ 72 = 0.214$ ), the resulting values are marked on the corresponding lines at  $a'$ ,  $b'$ ,  $c'$ ,  $d'$ ,  $e'$ ,  $f'$ , etc, on both sides of the line of nodes.

The points thus determined will lie on the arc of the true ellipse as seen from the Earth, and when we pass the curve through them, we have the apparent orbit of the double star.

To find the position of the star in the apparent ellipse, we multiply the distance of the focus of the real ellipse from the line of nodes by the cosine of the inclination, and thus find the point  $s'$ , which will be the position of the central star in the projected orbit. A line  $Os'P'$ , drawn from the centre through this point to intersect the arc of the apparent ellipse, gives the position-angle of the real major axis, and the position of the real periastron.

Having thus obtained the position of the central star in the apparent orbit, it only remains to draw through the principal star lines parallel to those inter-

secting at the centre and marking the four quadrants, which may now be erased. In the figure the lines which mark the four quadrants are somewhat heavier than the rest, so that they are easily recognized.

Thus a very simple process of projection enables us to trace the outline of the apparent orbit of any star when the required elements are given, and from the observed positions it is possible to see at a glance whether the apparent orbit represents the observations satisfactorily. It only remains to add that in the case of retrograde motion, the angle  $\lambda$  (which should always be counted in the direction of motion, while the ascending node should be taken between  $0^\circ$  and  $180^\circ$ ) must for purposes of graphical representation be taken as negative, and the position-angle of the major axis of the real ellipse becomes  $\Omega - \lambda$ , whereas for direct motion the angle is  $\Omega + \lambda$ , as in the case of 9 *Argûs*.

### § 11. *Formulae for the Improvement of Elements*

The foregoing graphical method, when judiciously applied, will give elements having all the accuracy which can be desired in the present state of double-star Astronomy. But as some improvement of a very refined character will ultimately be possible, we shall present the differential formulae which may be employed to effect these slight variations of the elements.

The formulae for finding the position-angle  $\theta$  from the elements are

$$\begin{aligned} M &= n(t - T) = E - e^s \sin E, \\ \tan \frac{1}{2} v &= \sqrt{\frac{1+e}{1-e}} \tan \frac{1}{2} E, \\ \tan(v + \lambda) \cos i &= \tan(\theta - \Omega) \end{aligned}$$

Since  $\theta$  is a function of the six elements,  $\Omega$ ,  $i$ ,  $\lambda$ ,  $e$ ,  $T$ ,  $n$ , we have

$$d\theta = \frac{\partial F(\theta)}{\partial \Omega} d\Omega + \frac{\partial F(\theta)}{\partial i} di + \frac{\partial F(\theta)}{\partial \lambda} d\lambda + \frac{\partial F(\theta)}{\partial e} de + \frac{\partial F(\theta)}{\partial T} dT + \frac{\partial F(\theta)}{\partial n} dn$$

When the variations of the elements are finite, but small, we have the approximate formula,

$$\theta_1 - \theta_0 = \Delta\theta = A\Delta\Omega + B\Delta i + C\Delta\lambda + D\Delta e + G\Delta T + H\Delta n,$$

where  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $G$  and  $H$ , denote the partial differential coefficients

From the equations which enable us to compute  $\theta$  we obtain these coef-

ficients by partial differentiation with respect to the several elements. Thus we find

$$\begin{aligned} A &= +1, \\ B &= -\cos^2(\theta - \Omega) \tan(v + \lambda) \sin i, \\ C &= \sec^2(v + \lambda) \cos^2(\theta - \Omega) \cos v, \\ D &= \left( \frac{2 \tan \frac{1}{2} E}{(1-e)\sqrt{1-e^2}} + \sqrt{\frac{1+e}{1-e}} \frac{\sec^2 \frac{1}{2} E \sin E}{1-e \cos E} \right) \cos^2 \frac{1}{2} v \quad C, \\ G &= -\sqrt{\frac{1+e}{1-e}} \frac{\sec^2 \frac{1}{2} E \cos^2 \frac{1}{2} v}{1-e \cos E} \quad C, \\ H &= \frac{(t-T)}{n} \quad G \end{aligned}$$

The formulae usually employed by astronomers for effecting the differential corrections of the elements thus take the form

$$\begin{aligned} A_1 \Delta \Omega + B_1 \Delta i + C_1 \Delta \lambda + D_1 \Delta e + G_1 \Delta T + H_1 \Delta n - \Delta \theta_1 &= 0, \\ A_2 \Delta \Omega + B_2 \Delta i + C_2 \Delta \lambda + D_2 \Delta e + G_2 \Delta T + H_2 \Delta n - \Delta \theta_2 &= 0, \end{aligned}$$

$$A_v \Delta \Omega + B_v \Delta i + C_v \Delta \lambda + D_v \Delta e + G_v \Delta T + H_v \Delta n - \Delta \theta_v = 0$$

There are six quantities to be deduced from this system of equations, a solution by the method of Least Squares will generally ensure the best results. In the above form of the equations it is tacitly assumed that the residuals in angle represent absolute displacements of the companion in space, regardless of its distance from the central star, which is evidently inexact. The importance of a given error in angle increases in proportion to the length of the radius vector, and as the distance of the companion is generally very unequal in different parts of the apparent orbit, the formulae should be so modified as to render the absolute displacements of the observed positions a minimum. This improvement can be effected as follows. We shall assume that the major axis can be best determined from the apparent orbit, which serves as an interpolating curve analogous to that recommended by SIR JOHN HERSCHEL, and hence this element need not be regarded as variable. It is, therefore, required to compute the slight variations for the other six elements.

Let us suppose that the value of  $\rho$  corresponding to the position-angle  $\theta_0$  is  $\rho_a$ , this value may be computed or measured graphically from the diagram. Let the corrected angle and distance be  $\theta_c$  and  $\rho_c$  respectively. Then it is easy to see that the displacement of a point on the apparent orbit due to the correction of the elements will be given by

$$\Delta s = \sqrt{\left(\frac{\rho_a + \rho_c}{2}\right)^2 (\theta_0 - \theta_c)^2 + (\rho_a - \rho_c)^2},$$

In case the length of the radius vector in the apparent orbit is practically constant, the last term of the radical becomes insensible, and the displacement in space at a given distance is proportional to the displacement in angle. But as many of the orbits are very eccentric and highly inclined, and the radius vector therefore changes rapidly, the best result can be obtained only by the use of the complete residuals expressed above. In computing these values numerically we may express  $(\rho_a - \rho_c)$  in degrees by the formula  $2 \left( \frac{\rho_a - \rho_c}{\rho_a + \rho_c} \right) 57^\circ 3$ , and since  $(\theta_a - \theta_c)$  is already given in degrees, we must express the coefficient as an abstract number in units of the major semi-axis, in order to give the displacements in angle weight proportional to the length of the radius vector.

Since the second term of the resulting expression under the radical sign

$$10^\circ = \sqrt{\left[ \left( \frac{\rho_a + \rho_c}{2a} \right) (\theta_a - \theta_c)^\circ \right]^2 + \left[ \frac{2(\rho_a - \rho_c)}{(\rho_a + \rho_c)} 57^\circ 3 \right]^2}$$

will often be very small, it will frequently be sufficient to use the first term only, or in other words, to assign the residuals in angle weights proportional to the lengths of the radii vectores.

This method of improving the elements will be found very much shorter than that involved in the process of correcting both angles and distances by separate differential formulae, and will lead to the same results without loss of accuracy.

## § 12. A General Method for Facilitating the Solution of Kepler's Equation by Mechanical Means \*

The standard works on planetary motion, such as GAUSS' *Theoria Motus*, OPOLZER'S *Bahnbestimmung*, and WATSON'S *Theoretical Astronomy*, give methods for solving KEPLER'S Equation which are very satisfactory when the eccentricity of the orbit is small, and also when this element is large, as in the case of most of the periodic comets. When the eccentricity is small, an expansion in series, usually by LAGRANGE'S Theorem, enables us to find the eccentric anomaly with the desired facility. The series frequently employed has the form

$$E_0 = M + e'' \sin M + e'' \left( \frac{e}{2} \right) \sin 2M +$$

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\* *Monthly Notices*, June, 1895, also Note in *Monthly Notices* for December, 1895

To the approximate value  $E_0$ , obtained from a few terms of this series, we apply a correction resulting from the expansion by TAYLOR'S Theorem.

$$E = E_0 + \frac{dE_0}{dM_0} dM_0 +$$

The equation of KEPLER gives

$$\frac{dM_0}{dE_0} = 1 - e \cos E_0,$$

and since

$$dM_0 = M - M_0,$$

we find two terms of the series to be

$$E = E_0 + \frac{M - M_0}{1 - e \cos E_0}$$

Successive applications of this formula will readily yield the true value of the eccentric anomaly. But when the eccentricity is considerable the expansion in series fails to converge with the desired rapidity. On the other hand, when the orbits differ but little from parabolas, the solution can readily be found by means of special tables, such as those given by GAUSS, WATSON and OPPOLZER.

It is very remarkable that among the many solutions of KEPLER'S Equation discovered by mathematicians there is not one, so far as I am aware, which has come into general use among astronomers that is applicable to ellipses of all possible eccentricities.

The method to which I desire to direct attention is a modification of the graphical method originally invented by J. J. WATERSTON (*Monthly Notices*, 1849-50, p 169), and subsequently rediscovered by DUBOIS (*Astronomische Nachrichten*, no 1404). The method was afterwards discussed by KLINKERFUES in his *Theoretische Astronomie*, p 17, but so far as I am aware\* it never came into practical use until employed in the investigations embodied in this work.

Suppose we construct, on a convenient scale, a semi-circumference of the curve of sines,  $y = \sin x$ . In practice it is desirable to use millimetre paper, and a convenient scale is obtained by taking one degree of the arc as five millimetres, so that the scale may easily be read to  $0^\circ.1$ . The origin of the arc is taken at the origin of coordinates, and as the scale along the axis of abscissae extends from  $0^\circ$  to  $180^\circ$ , it will have a length of 90 centimetres.

In the figure let  $OM$  represent the mean anomaly, and suppose from  $M$

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\* *Monthly Notices*, December, 1895

we draw a right line making an angle  $\psi$  with the axis of abscissae, the angle  $\psi$  being defined by the equation

$$\tan \psi = \frac{1}{e}$$

Let the abscissa of the point  $C$ , determined by the intersection of the right line  $MC$  with the sine curve, be denoted by  $E$ . Then we evidently have

$$OE - ME = OM$$

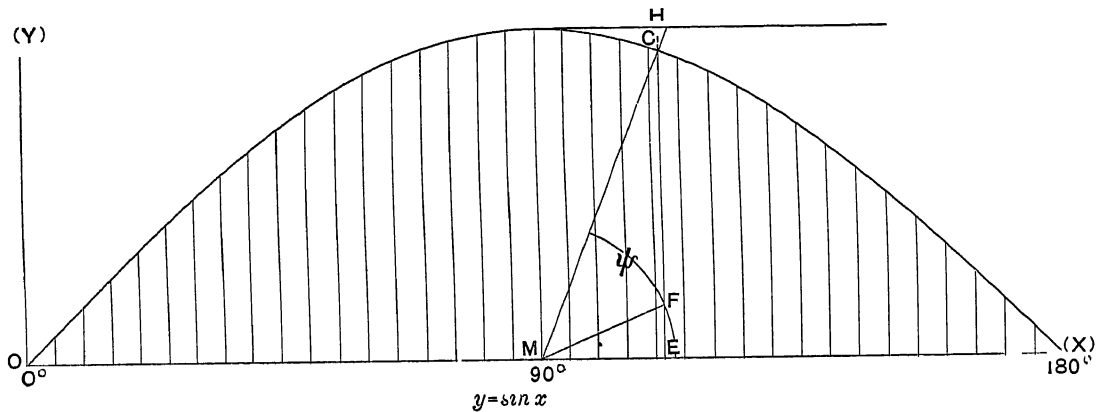


Fig 6

Thus, denoting the arc  $OE$  by  $E$ , and observing that  $e \sin \psi = \cos \psi$ , we find that  $e \sin \psi = ME$ , the radius in the case of  $\sin \psi$  being such that  $\sin \psi$  is always equal to  $\sin E$ .

Hence we get

$$OE - ME = OM,$$

or

$$E - e \sin E = M,$$

which is the Equation of KEPLER.

Therefore we conclude that if for an orbit of given eccentricity we construct a triangle  $CME$  (in practice this may be made of cardboard) and apply the vertex  $M$  of the triangle to the successive mean anomalies, the base coinciding with the  $x$ -axis, the intersection of the hypotenuse with the curve of sines will give at once abscissae which are the corresponding eccentric anomalies. Any actual diagram such as we have described will be subject to slight inaccuracies of construction, owing to the transcendental nature of the sines, and hence we cannot obtain solutions of absolute precision. But it is entirely possible to get approximate solutions exact to  $0^\circ 1$ , and this work can be done with the greatest rapidity. It is merely necessary to slide the base of the

triangle along the  $x$ -axis, placing the vertex  $M$  at the points corresponding to the different values of the mean anomaly, and reading off the corresponding eccentric anomalies.

This triangle device is rendered possible by virtue of the fact that  $\psi$  is constant in  $\tan \psi = \frac{1}{e}$ , and we may observe that in case of elliptic orbits the angle  $\psi$  can vary only from  $45^\circ$  in the case of a parabola to  $90^\circ$  in the case of a circle. This method is therefore directly applicable to ellipses of every possible eccentricity, and the accuracy of the solution is always substantially the same. In the case of parabolic motion, however, the method fails, since when  $\psi = 45^\circ$  the hypotenuse  $MC$  is tangent to the sine curve at the origin. But for  $e < 1$  the hypotenuse  $MC$  intersects the curve  $y = \sin x$ , and the intersection will be well defined except when  $e$  approaches unity and  $M$  is very small. In such cases it is best to use the Special Tables or the Theory of Parabolic Motion. Solutions exact to  $0^\circ 1$  are often sufficient in the present state of double-star observation, and we readily see how great is the practical value of this method in comparing a long series of observations with a given set of elements. One hundred approximate solutions of KEPLER'S Equation, accurate to  $0^\circ 1$ , may be obtained by this method in less than half an hour; while if  $e$  lies between 0.35 and 0.85 probably a skilled computer could not obtain the same results by the ordinary method in less than a day. Thus the time and labor required for this work is much diminished, and it is clear that the chances of large error are correspondingly reduced.

If a curve of sines were engraved on a metallic plate it would be an easy matter to devise a movable protractor which could be set at any angle, such a piece of apparatus would serve for every possible elliptic orbit, and would last for an indefinite time. Considering the immense labor devolving upon astronomers in the computation of the motion of the heavenly bodies, it would seem that such a labor-saving device might be advantageously employed in the offices of the astronomical ephemerides. However, as several astronomers have prepared tables for facilitating the solution of KEPLER'S Equation in the case of orbits which are not very eccentric, such an apparatus would be useful chiefly in work on the more eccentric asteroids, the double stars, and the periodic comets. In dealing with the motions of these bodies the labor saved would be very considerable, and we might hope that the apparatus here suggested would come into actual use. But in case this instrument of precision could not be successfully manufactured, owing to its limited commercial use, it is easy for a working astronomer to construct a curve of sines on millimetre paper.



This can be mounted on a suitable wooden board, and a triangle of cardboard will give the solutions of KEPLER'S Equation for any given orbit

Thus, while the graphical method, originally proposed by WATERSTON, afterwards independently discovered by DUBOIS, and subsequently discussed by KLINKERFUES, was suggested many years ago, it does not appear that it has yet come into general use, and therefore it deserves the careful attention of astronomers. It is worthy of remark that a method of such great practical importance should rest in comparative oblivion during half a century, at a time when astronomers were constantly working on the motions of periodic comets and double stars, but it is probable that neither WATERSTON nor DUBOIS recognized the great generality and high value of the method in practical work. Since writing the paper which I communicated to the Royal Astronomical Society in June, 1895, I have had occasion to make great use of the method in revising the orbits of double stars, and have found it not only the easiest and most rapid process yet invented, but one altogether so satisfactory that we may predict its universal adoption by astronomers. The simplicity and generality of the method and the rapidity and accuracy with which solutions can be obtained, invite the inference that in the nature of the case the method is probably ultimate, and is not likely to be improved upon in any future age.

While this method is of special importance in dealing with the motions of double stars, owing to the wide range of their eccentricities, it will evidently be almost, if not quite, equally important in the case of periodic comets and the asteroids. But in dealing with comets and planets, where we desire very exact solutions of KEPLER'S Equation, it will be necessary to correct the approximate values by the formula

$$\Delta E_0 = \frac{M - M_0}{1 - e \cos E_0},$$

where  $M_0$ ,  $E_0$  are the approximate values of the mean and eccentric anomalies. A second correction will ensure all the accuracy desirable in planetary and cometary ephemerides.\*

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\*Among the other means for solving KEPLER'S Equation we mention especially the tables of ASIRAND (ENGLEMAN, Leipzig), DOBERCK, *A N*, Bd 139, and a graphical method by MR H C PLUMMER, *Monthly Notices*, March, 1896.

## CHAPTER II.

### ON THE ORBITS OF FORTY BINARY STARS.

#### *Introductory Remarks*

THE present chapter is occupied with detailed researches on the motions of the forty stars whose orbits can be best determined at this epoch. The material presented for each star has been collected from all available sources and is very complete. It is highly improbable that any important records have been overlooked, and since we have drawn the material almost wholly from original sources, future investigators will have little need to repeat the labor involved in collecting observations of these stars prior to 1895.

In some cases we have not used all of the available measures, either because the observations appeared to be defective, or because good observations were obtained too late to be incorporated in the discussions, which were not changed unless the elements adopted were found to be inconsistent with the new material. In the main, our choice of observations has been guided by the assumption that it is possible to find an orbit which is consistent with undisturbed elliptical motion. The observations have justified a violation of this principle only in the case of 70 *Ophiuchi*, which presented anomalies too large to be attributed to errors of observation. If the course of time should show that other stars also are perturbed, it will become apparent that we have not always made the best choice of the material now available.

In the determination of these orbits a number of distinguished astronomers have contributed their observations in advance of publication. They have not only sent manuscript copies of valuable measures, but have offered their work with a generosity which merits my most grateful acknowledgement. Among those to whom we return thanks are: M. G. BIGOURDAN, National Observatory, Paris; PROFS. G. C. COMSTOCK and A. S. FLINT, Washburn Observatory, Madison; PROF. S. DE GLASENAPP, Director of the Observatory, Imperial University, St. Petersburg; PROF. G. W. HOUGH, Director of the Dearborn Observatory, Evanston, Ill.; PROF. V. KNORRE, Royal Observatory, Berlin; T. LEWIS, Esq., Royal Observatory, Greenwich, M. W. MAW, Esq., Private

Observatory, London, PROF G. V SCHIAPARELLI, Director of the Royal Observatory, Milan, PROF W SCHUR, Director of the Royal Observatory, Göttingen, JOHN TEBBUTT, Esq, Private Observatory, Windsor, N S Wales, DR. H. C WILSON, Goodsell Observatory, Northfield, Minn

I have also had the constant cooperation of PROFESSORS BURNHAM and BARNARD, who have made valuable suggestions in addition to contributing important observations, some of which were secured expressly for this work. In the investigation of the individual orbits my friends MR. GEO K LAWTON, MR ERIC DOOLITTLE, and MR F R MOULTON have at different times rendered valuable assistance in the execution of a large part of the computations. Without such assistance, uniformly characterized by both zeal and enthusiasm, it would have been impossible to have completed the determination of so many orbits in so short a time. To these gentlemen I acknowledge my deep and lasting obligations. Besides aiding me in the preparation of Chapter I, MR MOULTON has assisted in arranging the manuscript for the printer, and in reading the proofs, and thus not only expedited the work but also ensured greater accuracy than otherwise would have been possible.

While no effort has been spared to ensure exactness in the computations and in the drawings, it can scarcely be hoped that in dealing with so great a mass of material all errors have been avoided. There is reason, however, to believe that such errors as may exist in the work will have no appreciable effect upon the final results.

A number of the orbits embodied in this Chapter have been published in the *Astronomical Journal*, the *Astronomische Nachrichten*, and the *Monthly Notices* of the Royal Astronomical Society, references to these sources will be found in the appropriate places.

### *Abbreviations of the Names of Observers*

A C = Alvan Clark	Bw = Brunnow	Du = Durham Observers
A G C = Alvan G Clark	Cal = Callandean	Ek = Encke
Adh = Adolph	Cin = Cincinnati Observers	El = Elley
Au = Auwers	Col = Collins	En = Englemann
$\beta$ = Burnham	Com = Comstock	Fer = Ferrari
Bar = Barnard	Cop = Copeland	Fl = Flammarion
Be = Bessel	Da = Dawes	Flt = Flint
Bh = Bruhns	Dav = Davidson	Flt = Fletcher
Big = Bigourdan	Dem = Dembowski	Fo = Foerster
Bo = Bond	Dk = Doberck	Fi = Fianz
Bo = Boigen	Du = Duner	Ga = Galle

Gia = Giacomelli	Ma = Main	Sec = Secchi
Gl = Gledhill	Mä = Mädler	See = T J J See
Glas = Glasenapp	Mac = Maclear	Sel = Sellois
Go = Goldeny	Maw = M W Maw	Sh = Schur
H <sub>1</sub> = W Heischel	M <sub>1</sub> = Miller	Sl = Selander
H <sub>2</sub> = J F W Heischel	Mit = Mitchell	Sm = Smith
H <sub>1</sub> = Hind	Ml = Moulton	So = South
Hl = Hall	New = Newcomb	Si = Seale
Ho = Hough	No = Nobile	St = O Stone
Hol = Holden	Pei = Pence	T = Tebbutt
Hv = Harvard Observers	Pei = Periotin	Tai = Tarrant
Ja = Jacob	Pet = Peters	Tj = Tietjen
Jed = Jedrzejewicz	Ph = Philpot	Vo = Vogel
Jo = Jones	Pl = Plummei	Wdo = Waldo
Ka = Kaiser	Po = Powell	Wh = Wichman
Kn = Knott	Pi = Pitchett	Ws = J M Wilson
Knr = Knoie	Rad = Radcliffe Observers	H C W = H C Wilson
Ku = Kustner	Rus = Russell	W & S = Wilson & Seabroke
Ley = Leyton Observers	Σ = W Struve	Well = Wellmann
Lin = Lindstedt	H Σ = H Struve	Winn = Winnecke
Lov = Lovett	O Σ = O Struve	Wlk = Winlock
Ls = Lewis	Sch = Schiaparelli	W <sub>1</sub> = Wootesley
Lu = Luther	Sel = Schlüter	Y = Young
Lv = Leavenworth	Sea = Seabroke	

## Σ 3062.

$\alpha = 0^h 1^m$  ,  $\delta = +57^\circ 53'$   
 69, yellowish , 75, bluish white

*Discovered by Sir William Herschel August 25, 1782*

## OBSERVATIONS

<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1782 65	319 4	—	1	Herschel	1842 80	207 3	0 87	1	Madler
1783 05	319 1	—	1	Heischel	1843 58	208 7	0 92	3	Madler
1823 81	36 7	1 25 ±	1	Struve	1843 80	210 0	0 94	1	Dawes
1831 71	85 7	0 82	2	Struve	1844 49	213 7	0 85	5	Madler
1833 71	108 6	0 56	3	Struve	1846 42	220 3	0 97	2	O Struve
1835 66	132 6	0 41	5	Struve	1847 53	225 1	1 12	5	Madler
1836 61	146 4	0 47	5	Struve	1848 22	229 7	1 14	2	O Struve
1840 32	186 5	0 65	4	O Struve	1848 87	228 8	1 16	1	Dawes
1840 78	186 9	0 8 ±	3-2	Dawes	1849 19	232 5	1 09	3	O Struve
1841 58	193 6	0 89	7	Madler	1850 04	233 9	1 17	3	O Struve
1841 86	193 4	0 95	2	Dawes	1850 71	232 3	1 31	3	Madler
					1850 93	235 2	—	1	Dawes

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1851 16	235 7	1 35	2	O Struve	1871 57	283 8	1 39	7	Dembowski
1851 18	236 9	1 16	8	Madler	1871 60	284 0	1 6	1	Gledhill
1851 75	234 5	1 27	2	Madler	1872 63	285 7	1 47	6	Dembowski
1852 49	238 4	1 23	3	O Struve	1872 80	286 3	1 45	1	W & S
1854 11	243 5	1 48	4	O Struve	1873 63	287 6	1 45	9	Dembowski
1854 32	244 3	1 28	3	Dawes	1873 80	297 8	0 91	1	Leyton Obs'
1854 99	249 9	Sep	6	Dembowski	1873 82	287 8	1 45	1	W & S
1855 05	242 7	1 38	3	O Struve	1873 84	288 0	1 55	2	Gledhill
1855 80	249 4	1 3	8	Dembowski	1874 64	289 8	1 40	6	Dembowski
1855 91	247 9	1 33	3	Morton	1874 72	299 1	1 08	1	Leyton Obs
1856 57	245 5	1 41	1	Winnecke	1874 86	291 2	1 37	1	W & S
1856 62	250 6	1 2	4	Dembowski	1874 91	291 1	1 35	2	Gledhill
1856 66	247 8	1 40	2	O Struve	1875 67	292 2	1 47	6	Dembowski
1856 80	248 8	1 43	1	Madler	1875 69	292 9	1 49	5	Dunér
1857 37	250 4	1 50	3	O Struve	1876 74	293 3	1 61	1	O Struve
1857 60	253 4	1 25	3	Secchi	1876 67	294 5	1 46	5	Dembowski
1857 71	252 2	1 2	4	Dembowski	1876 87	294 5	1 60	3-2	Doberck
1858 54	252 4	1 2	2	Dembowski	1876 93	298 8 <sup>2</sup>	1 44	1	W & S
1859 16	255 3	1 46	3	O Struve	1876 99	294 5	1 46	5-4	Plummei
1861 79	265 2	1 21	2	Madler	1877 61	295 8	1 46	4	Dembowski
1862 18	261 7	1 54	2	O Struve	1877 74	297 3	1 49	4	Doberck
1862 79	263 6	1 46	11	Dembowski	1878 60	299 1	1 51	4	Dembowski
1862 83	266 1	1 29	2	Madler	1878 90	302 3	1 39	5	Doberck
1863 80	266 0	1 43	9	Dembowski	1879 45	301 9	1 50	8	Hall
1863 86	265 6	1 40	1	Dawes	1879 77	303 2	1 33	5	Doberck
1864 73	268 7	1 40	7	Dembowski	1880 60	304 5	1 50	6	Burnham
1865 70	271 2	1 35	6	Dembowski	1880 88	304 3	1 55	4	Doberck
1865 71	269 9	1 43	3	Knott	1881 14	301 0	1 44	3-2	Jedrzejewicz
1865 71	271 9	1 14	2-3	Leyton Obs	1881 60	307 8	1 60	3	Burnham
1866 20	270 4	1 47	2	O Struve	1881 81	306 5	1 97	2-1	Bigourdan
1866 64	270 3	1 46	3	Leyton Obs	1881 83	305 5	1 40	4	Hall
1866 72	275 5	1 13	3	Harvard	1882 11	304 9	1 29	7	Jedrzejewicz
1866 74	273 4	1 44	5	Dembowski	1882 70	312 3	1 62	1	O Struve
1866 97	270 0	1 34	1	Secchi	1882 82	308 1	1 52	4-3	Doberck
1867 74	275 2	1 41	7	Dembowski	1883 60	309 8	1 69	9	Englemann
1868 67	277 5	1 38	4	Dembowski	1883 94	312 8	1 44	3	Hall
1868 75	268 3	1 66	3-1	Leyton Obs	1884 47	311 7	1 26	2	Seabroke
1868 98	276 5	1 59	2	O Struve	1885 80	316 1	1 46	5	Hall
1869 75	279 9	1 48	6	Dembowski	1886 20	315 2	1 43	3-2	Seabroke
1870 18	279 2	1 48	2	O Struve	1886 92	314 6	1 46	5	Hall
1870 44	281 0	1 5	1	Gledhill	1887 06	315 5	1 36	6-3	Schiaparelli
1870 64	280 6	1 63	-	Leyton Obs	1887 10	310 7	1 50	3	Tarrant
1870 67	282 2	1 43	7	Dembowski					

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1888 09	317 7	1 40	1	Schiaparelli	1892 94	323 7	1 62	1	Jones
1888 94	319 4	1 36	4	Hall	1892 99	328 5	1 47	2	Schiaparelli
1888 96	319 5	1 46	6	Schiaparelli	1893 83	327 8	1 58	2	Comstock
1889 57	321 1	1 45	3	Burnham	1893 96	330 9	1 45	2	Schiaparelli
1889 86	323 0	1 45	4	Hall	1894 28	330 6	1 70	3-2	Bigourdan
1889 94	320 5	1 38	1	Seabroke	1894 64	331 99	1 86	1	Glaserapp
1890 76	321 8	1 61	1	Bigourdan	1895 10	151 2	1 58	1	Davidson
1890 79	325 2	1 34	5	Hall	1895 14	330 3	1 61	7-6	Bigourdan
1890 93	323 5	1 52	1	Schiaparelli	1895 15	327 4	1 16	3	Hough
1891 48	322 4	1 5±	1	See	1895 18	331 9	1 46	2-1	Comstock
1891 95	327 3	1 47	2	Schiaparelli	1895 73	334 3	1 53	4	See
1892 71	329 1	1 47	3	Comstock	1895 74	334 5	1 40	2	Moulton
1892 86	325 6	1 52	2	Collins					

When HERSCHTEL discovered this pair he measured the angle and repeated his observation the following year, without finding any sensible change.\* Beginning with 1823, STRUVE followed the star for ten years; and from the measures thus secured he discovered that the system is a binary in rapid orbital motion. Since STRUVE's time the star has been carefully measured by many of the best observers, so that there is abundant material upon which to base an orbit which seems likely to be substantially correct.

Having collected all the published observations of Σ 3062 from original sources, I have formed for each year a mean position which is the arithmetical mean of the mean results obtained severally by the best observers. In accordance with the experience of STRUVE, OTTO STRUVE, DEMBOWSKI, and BURNHAM these yearly means may be held to furnish the most trustworthy basis for the elements of an orbit. The following is a table of the orbits hitherto published for this star.

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
94 765	1837 414	0 4496	1 255	15 04	35 53	135 46	Mädler, 1840	Doip Obs IX, 180
146 83	1834 01	0 57536	0 9982	77 35	38 6	42 2	Mädler, 1847	Die Fixst-Syst
105 61	1836 60	0 4151	1 446	47 6	46 3	93 87	von Fuss, 1867	Mel Acad St Petersburg
112 644	1835 196	0 50090	1 310	32 2	29 97	97 52	Schur, 1867	A N 1636 [1867, p 128
104 115	1834 88	0 4612	1 27	38 6	32 2	92 1	Doberck, 1877	A N 2156
102 913	1835 508	0 4472	1 270	39 15	32 2	92 1	Doberck, 1879	A N 2277

By the method of KLINKERFUES we find the following elements.

$$\begin{aligned}
 P &= 104.61 \text{ years} & \Omega &= 47^\circ 15' \\
 T &= 1836.26 & i &= 43^\circ 85' \\
 e &= 0.450 & \lambda &= 90^\circ 90' \\
 a &= 1'' 3712 & n &= +3^\circ 441355
 \end{aligned}$$

\* *Astronomische Nachrichten*, 3292

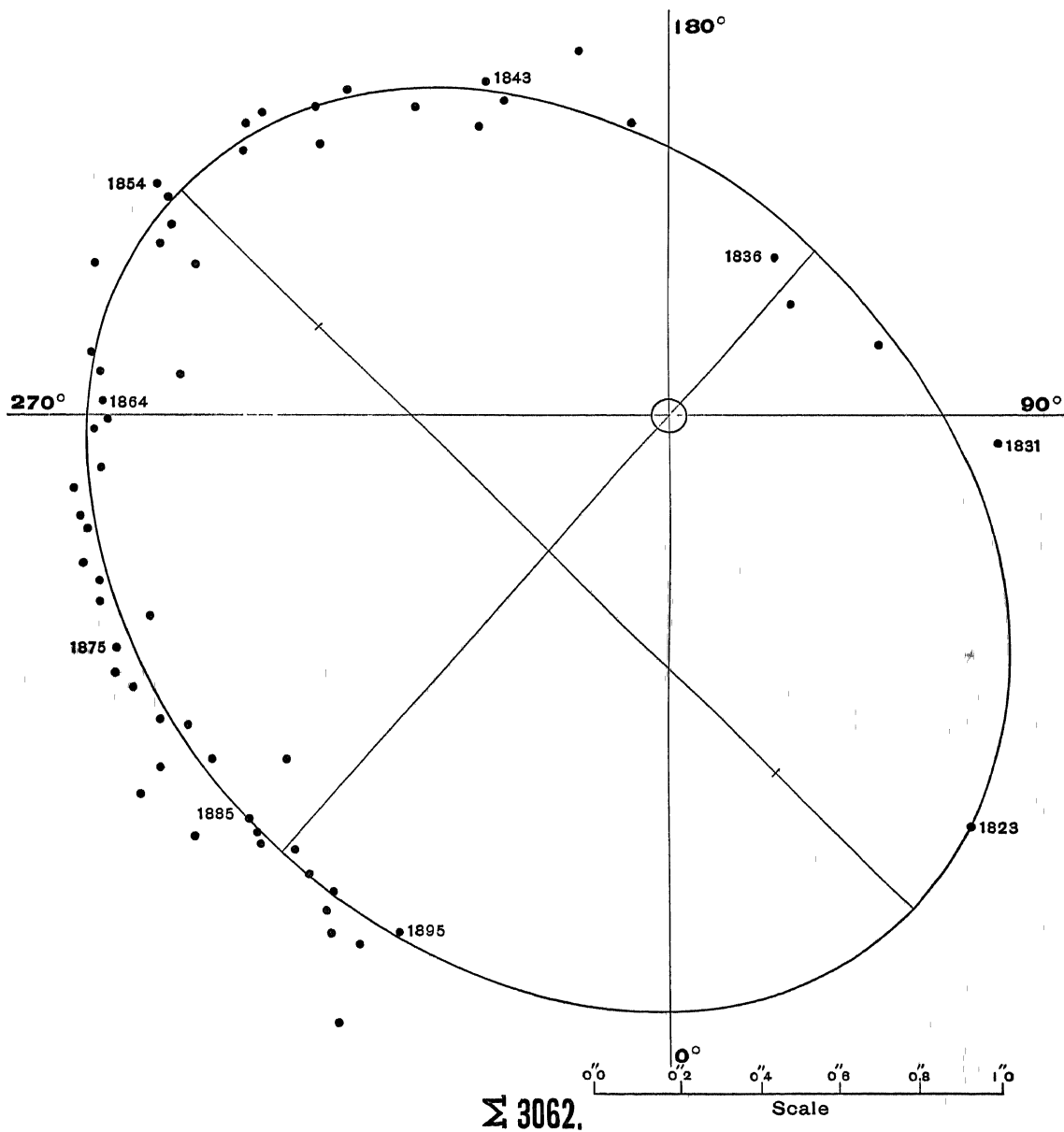
## Apparent orbit

Length of major axis	= 2" 526
Length of minor axis	= 1" 984
Angle of major axis	= 45° 7
Angle of periastron	= 138° 4
Distance of star from centre	= 0" 446

It will be seen that these elements are very similar to those derived by VON FUSS in 1867. The following comparison of the computed and observed places shows that the above elements are highly satisfactory, and that the true elements of this remarkable binary will hardly differ sensibly from the values here obtained.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_0$	$\theta_c$	$\rho_0$	$\rho_c$	$\theta_0 - \theta_c$	$\rho_0 - \rho_c$	$n$	Observers
1782 65	319 4	315 7	"	"	+3 7	"	2	Herschel
1823 81	36 7	45 3	1 25 ±	1 16	-8 6	+0 09	1	Struve
1831 71	85 7	85 1	0 82	0 72	+0 6	+0 10	2	Struve
1833 73	108 6	105 3	0 56	0 61	+3 3	-0 05	3	Struve
1835 66	132 6	130 5	0 41	0 55	+2 1	-0 14	5	Struve
1836 61	146 4	143 8	0 47	0 55	+2 6	-0 08	5	Struve
1840 55	186 7	188 8	0 72	0 71	-2 1	+0 01	7-6	OΣ 4, Dawes 3 2
1841 72	193 5	197 6	0 92	0 79	-4 1	+0 13	9	Madler 7, Dawes 2
1842 80	207 3	204 7	0 87	0 86	+2 6	+0 01	1	Madler
1843 69	209 3	209 5	0 93	0 91	-0 2	+0 02	4	Madler 3, Dawes 1
1844 49	213 7	213 6	0 85	0 96	+0 1	-0 11	5	Madler
1846 42	220 3	222 2	0 97	1 07	-1 9	-0 10	2	O Struve
1847 53	225 1	226 1	1 12	1 11	-1 0	+0 01	5	Madler
1848 54	229 2	229 7	1 15	1 16	-0 5	-0 01	3	OΣ 2, Dawes 1
1849 19	232 5	231 9	1 09	1 18	+0 6	-0 09	3	O Struve
1850 56	233 8	236 1	1 24	1 23	-2 3	+0 01	7-6	OΣ 3, Madler 3, Dawes 1-0
1851 36	235 7	238 3	1 26	1 25	-2 6	+0 01	12	OΣ 2, Madler 8, Madler 2
1852 49	238 4	241 6	1 23	1 29	-3 2	-0 06	3	O Struve
1854 47	245 9	246 7	1 38	1 33	-0 8	+0 05	13-7	OΣ 4, Dawes 3, Dembowski 6-0
1855 58	246 6	249 4	1 34	1 35	-2 8	-0 01	14	OΣ 3, Dembowski 8, Mo 3
1856 69	249 1	251 5	1 31	1 37	-2 4	-0 06	7	Dembowski 4, OΣ 2, Madler 1
1857 56	251 6	254 0	1 32	1 38	-2 4	-0 06	10	OΣ 3, Seabrooke 3, Dembowski 4
1858 54	252 4	256 3	1 2	1 39	-3 9	-0 19	2	Dembowski
1859 16	255 3	257 3	1 46	1 40	-2 0	+0 06	3	O Struve
1861 79	265 2	263 4	1 21	1 42	+1 8	-0 21	2	Madler
1862 60	263 8	265 2	1 43	1 43	-1 4	0 00	15	OΣ 2, Dembowski 11, Madler 2
1863 83	265 8	267 7	1 41	1 43	-1 9	-0 02	10	Dembowski 9, Dawes 1
1864 73	268 7	269 7	1 40	1 43	-1 0	-0 03	7	Dembowski
1865 70	270 5	271 8	1 39	1 44	-1 3	-0 05	9	Dembowski 6, Knott 3
1866 60	271 3	273 6	1 42	1 44	-2 3	-0 02	8	OΣ 2, Dembowski 5, Sea 1
1867 74	275 2	276 1	1 41	1 44	-0 9	-0 03	7	Dembowski
1868 82	277 0	278 2	1 48	1 44	-1 2	+0 04	6	Dembowski 4, OΣ 2
1869 75	279 9	280 6	1 48	1 44	-0 7	+0 04	6	Dembowski
1870 43	280 8	281 5	1 47	1 44	-0 7	+0 03	10	OΣ 2, Gledhill 1, Dembowski 7
1871 58	283 9	283 8	1 49	1 45	+0 1	+0 04	8	Dembowski 7, Gledhill 1
1872 71	286 0	286 1	1 46	1 44	-0 1	+0 02	7	Dembowski 6, W & S 1
1873 76	287 8	288 3	1 48	1 44	-0 5	+0 04	12	Dembowski 9, W & S 1, Gl 2
1874 80	290 7	290 4	1 37	1 44	+0 3	-0 07	9	Dembowski 6, W & S 1, Gl 2
1875 68	292 5	292 2	1 48	1 44	+0 3	+0 04	11	Dembowski 6, Duné 5







$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
	$^{\circ}$	$^{\circ}$	$''$	$''$	$^{\circ}$	$''$		
1876 84	294 5	294 6	1 51	1 44	-0 1	+0 07	13-11	Dembowski 5, Dk 3-2, Pl 5-4
1877 68	296 5	296 2	1 48	1 44	+0 3	+0 04	8	Dembowski 4, Doberck 4
1878 75	300 7	298 4	1 45	1 44	+2 3	+0 01	9	Dembowski 4, Doberck 5
1879 61	302 5	300 2	1 41	1 44	+2 3	-0 03	13	Hall 8, Doberck 5
1880 74	304 4	302 5	1 52	1 43	+1 9	+0 09	10	$\beta$ 6, Doberck 4
1881 59	305 2	304 3	1 60	1 43	+0 9	+0 17	12-10	Jed 3-2, $\beta$ 3, Big 2-1, Hall 1
1882 46	306 5	306 1	1 41	1 43	+0 4	-0 02	11-10	Jed 7, Doberck 4-3
1883 77	311 3	307 7	1 56	1 43	+3 6	+0 13	12	Englemann 9, Hall 3
1884 47	311 7	310 2	1 26	1 43	+1 5	-0 17	2	Seabroke
1885 80	316 1	312 9	1 45	1 43	+3 2	+0 02	5	Hall
1886 56	314 9	314 4	1 44	1 43	+0 5	+0 01	8-7	Seabroke 3-2, Hall 5
1887 08	313 1	315 4	1 43	1 43	-2 3	0 00	9-6	Schiaparelli 6-3, Tarrant 3
1888 66	318 9	317 5	1 41	1 43	+1 4	-0 02	11	Sch 1, Hall 4, Sch 6
1889 79	321 5	320 9	1 43	1 44	+0 6	-0 01	8	$\beta$ 3, Hall 4, Seabroke 1
1890 86	324 3	323 1	1 43	1 44	+1 2	-0 01	6	Hall 5, Schiaparelli 1
1891 71	324 9	323 8	1 48	1 44	+1 1	+0 04	3	Sec 1, Schiaparelli 2
1892 89	326 7	327 2	1 52	1 44	-0 5	+0 08	8	Com 3, Col 2, Jo 1, Sch 2
1893 90	329 3	329 2	1 51	1 44	+0 1	+0 07	4	Comstock 2, Schiaparelli 2
1891 16	331 3	330 3	1 70	1 45	+1 0	+0 35	1-2	(Hlasenapp) 1-0, Bigoudan 3-2
1895 30	332 3	332 1	1 44	1 45	+0 2	-0 01	16-14	Big 7-6, Ho 0-3, Com 2-1, Sec 4

EPIHEMERIS

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
	$^{\circ}$	$''$		$^{\circ}$	$''$
1896 50	331 8	1 45	1902 50	346 8	1 46
1897 50	336 8	1 45	1903 50	348 8	1 46
1898 50	338 8	1 45	1904 50	350 8	1 46
1899 50	340 8	1 45	1905 50	352 8	1 46
1900 50	342 8	1 46	1906 50	354 8	1 46
1901 50	344 8	1 46			

It will be seen that there are occasional systematic errors both in the angles and in the distances, and in some cases these deviations appear to be rather more extensive than we should expect in the work of the best observers; but the star has some peculiar difficulties, especially as regards the distance, and on the whole the measures are fairly accordant for so close an object

This star deserves the careful attention of observers, as the next 20 years will give the material which will make the orbit exact to a very high degree. It may be pointed out that the system has a considerable proper motion in space, in  $\alpha + 0'' 346$ , in  $\delta + 0'' 020$ , and therefore the chances are that it has a sensible parallax. If the parallax could be determined it would give us the absolute dimensions of the system and the combined mass of the components — two elements of the highest interest in the study of the stellar systems.

$\eta$  CASSIOPEAE =  $\Sigma$  60.

$\alpha = 0^h 42^m 9$  ,  $\delta = +57^\circ 18'$   
4, yellow , 7, purple

*Discovered by Sir William Herschel, August 17, 1779*

## OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1779 81	70 $\pm$	11 09	1	Herschel	1850 19	106 8	7 96	15-14	Madler
1780 52	—	11 46	1	Herschel	1850 61	105 5	8 32	2	Johnson
1782 45	62 1	—	1	Herschel	1850 72	106 5	8 01	6-7	Madler
1803 11	70 8	—	1	Herschel	1850 84	105 6	8 16	5	Jacob
1814 10	78 5	9 70	1	Bessel	1851 45	106 6	8 17	7-6	Fletcher
1820 16	81 1	10 68	5	Struve	1851 76	107 7	7 72	3	Madler
1827 21	85 6	10 2	1	Struve	1851 84	108 0	8 04	3	O Struve
1830 75	86 2	10 07	5	Bessel	1851 89	106 9	8 12	4	Miller
1831 75	88 7	9 69	1	Herschel	1851 89	106 4	8 04	3	Jacob
1832 05	87 6	9 78	5	Struve	1852 61	108 5	7 65	7-8	Madler
1832 87	88 7	9 74	2	Dawes	1853 39	108 4	7 57	5	Madler
1834 76	89 6	9 80	1	Bessel	1853 51	109 2	7 98	7	Jacob
1835 26	91 2	9 52	3	Struve	1853 90	110 1	7 52	3	Madler
1836 46	91 1	10 83	2-1	Madler	1853 92	109 4	—	6	Powell
1836 74	92 1	9 39	4	Struve	1854 00	109 6	7 91	1	Dawes
1840 14	95 8	8 98	37-29 obs	Kaiser	1854 56	112 0	7 97	4	O Struve
1841 34	98 1	9 21	3	O Struve	1854 80	110 6	7 60	2	Madler
1841 57	98 3	9 50	4	Madler	1854 91	111 9	7 80	7	Dembowski
1841 80	95 7	9 33	1	Dawes	1854 94	111 5	—	6	Powell
1842 41	98 3	8 76	2-1	Madler	1854 95	110 0	8 12	2	Morton
1842 65	96 4	9 09	7	Schluter	1855 24	110 9	7 95	3	Winnecke
1843 07	98 4	8 97	3	Schluter	1855 52	111 0	7 60	4-3	Madler
1844 56	100 1	8 48	6-5	Madler	1855 79	110 2	7 89	2	Secchi
1845 44	101 1	8 44	8	Madler	1855 93	112 5	7 63	9-4	Powell
1845 86	97 2	8 85	1	Jacob	1855 94	113 2	7 57	4	Dembowski
1846 41	100 5	8 89	2	Jacob	1855 96	112 4	7 80	3	Morton
1846 66	102 5	8 57	12	Madler	1856 07	112 4	7 57	4	Jacob
1846 72	101 5	8 71	2	Jacob	1856 51	112 9	7 22	2-1	Madler
1847 34	102 7	8 28	6-7	Madler	1856 55	117 3	8 34	3	Luther
1847 40	101 8	8 48	5	O Struve	1856 86	114 6	7 33	4	Dembowski
1848 12	102 5	8 60	2-1	Jacob	1857 06	112 9	7 49	3	Jacob
1849 66	105 0	8 26	4	O Struve	1857 22	114 1	7 57	2	O Struve
					1857 23	114 5	7 09	5	Madler
					1857 87	115 8	7 14	4	Dembowski
					1858 06	115 1	7 42	3	Jacob
					1858 19	115 9	7 12	4	Madler
					1858 62	115 8	7 24	3	Dembowski

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1859 27	115.7	6.96	2-1	Madley	1872 01	—	6.0 $\pm$	1	Seabrooke
1859 72	116.6	7.02	6-4	Powell	1872 18	140.8	5.94	2	O Struve
1859 94	117.0	7.08	2	Morton	1872 50	140.5	6.02	7	Dunér
1860 68	119.8	7.17	2	O Struve	1872 63	139.1	5.97	6	Dembowski
1860 97	118.3	6.99	7-6	Powell	1872 65	137.8	6.10	4	Knott
1861 58	119.8	7.37	5	Anwers	1872 77	144.0	5.94	1	Main
1861 70	117.9	7.08	5	Madley	1872 86	124.4	6.32	—	Leyton Obs
1861 82	118.2	6.44	6	Main	1873 06	142.3	—	1	W & S
1861 95	120.6	6.7	3-2	Powell	1873 53	144.6	5.68	3	O Struve
1862 71	120.6	6.85	8	Madley	1873 66	140.7	5.97	7	Dembowski
1862 86	121.3	7.00	12	Dembowski	1873 68	143.7	6.03	2	Gledhill
1862 88	120.4	7.15		Leyton Obs	1873 83	144.7	6.33	1	W & S
1863 80	123.4	6.87	9	Dembowski	1873 86	141.2	5.66	1	Leyton Obs
1864 00	123.1	6.65	4-3	Powell	1873 98	143.6	—	6	Nobile
1864 80	125.0	6.76	9	Dembowski	1874 22	144.9	5.82	1	Dunér
1865 59	125.5	6.52	6	Englemann	1874 63	143.1	5.83	7	Dembowski
1865 62	126.4	6.67	8	Dembowski	1874 90	146.0	5.8	1	W & S
1865 69	125.7	6.75	3	Knott	1875 15	148.6	5.58	2	O Struve
1865 76	123.9	6.43	2-1	Leyton Obs	1875 51	146.7	5.77	10	Dunér
1866 22	132.6	6.44	2	O Struve	1875 66	146.5	5.67	7	Dembowski
1866 63	124.7	6.38	3	Leyton Obs	1875 78	146.1	5.78	1	Main
1866 65	123.9	6.66	1	Seale	1875 94	147.7	—	2	Doberck
1866 72	128.5	6.58	7	Dembowski	1876 61	149.3	5.59	7	Dembowski
1866.84	126.0	—	1	Winlock	1876 79	149.1	5.48	6	Plummer
1866.86	127.7	6.79	4	Secchi	1876 86	149.3	4.72	1	Leyton Obs
1867 15	130.1	6.55	1	Searle	1877 19	152.8	5.44	1	O Struve
1867 65	130.0	6.31	1	Main	1877 69	151.5	5.48	6	Dembowski
1867 74	130.1	6.48	7	Dembowski	1877 76	150.4	5.77	5	Doberck
1868 37	131.8	6.38	5	Dunér	1878 19	154.6	5.25	2	O Struve
1868 53	132.9	6.43	3	O Struve	1878 58	153.7	5.42	5	Dembowski
1868 67	132.1	6.33	4	Dembowski	1878 83	153.9	5.51	1	Goldney
1868 90	124.3	6.21	1	Leyton Obs	1878 90	155.1	5.28	5	Doberck
1869 67	132.4	6.12	1	Main	1879 20	154.7	5.16	2	O Struve
1869 72	124.8	6.58	1	Leyton Obs	1879 01	156.8	5.35	7	Hall
1869 75	134.0	6.20	6	Dembowski	1879 80	158.3	5.21	3	Doberck
1869 93	135.2	6.16	4	Dunér	1879 96	161.9	5.60	5	Franz
1870 07	133.4	6.39	5-4	Powell	1880 14	159.9	5.32	7	Jedrzejewicz
1870 18	136.2	6.28	2	O Struve	1880 60	161.1	5.26	5	Doberck
1870 67	135.3	6.16	7	Dembowski	1881 10	164.1	5.32	2-1	Doberck
1870 72	135.8	6.09	3	Gledhill	1881 14	162.8	5.10	3-2	Jedrzejewicz
1871 10	137.4	5.90	2-1	Powell	1881 16	162.0	5.26	3	O Struve
1871 65	137.6	6.08	6	Dembowski	1881 72	161.4	5.18	2	Pritchett
1871 70	138.0	6.03	2	Gledhill	1881 90	163.1	5.30	4	Hall
1871 93	140.9	—	1	W & S					

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1882 15	165 5	5 08	3	Jedizejewicz	1890 79	188 4	5 07	5	Hall
1882 70	166 8	5 28	1	O Struve	1891 48	191 7	5 02	5-1	See
1882 76	166 3	5 11	6-5	Doberck	1891 74	191 8	4 79	4-3	Maw
1882 87	165 7	5 15	6	Englemann	1892 77	194 1	4 92	3	Comstock
1883 94	168 8	5 12	3	Hall	1892 85	197 3	4 90	2	Collins
1885 23	172 8	5 27	1	Seabroke	1892 95	197 4	4 75	1	Jones
1885 81	173 4	5 06	5	Hall	1893 84	196 0	4 88	1	Comstock
1886 07	176 3	4 92	5	Englemann	1893 97	198 2	5 12	1	Lovett
1886 20	176 6	4 78	3-2	Seabroke	1894 05	201 6	4 89	1	Comstock
1886 95	175 3	4 99	5	Hall	1894 1	200 2	4 96	1	Maw
1886 97	178 6	4 71	7	Tarrant	1895 16	204 8	4 97	3	Hough
1887 35	180 6	4 6	1	Smith	1895 17	203 8	5 01	3	Comstock
1888 48	181 3	4 69	2	Seabroke	1895 29	203 4	4 84	3	See
1888 54	183 9	4 83	5	Maw	1895 73	204 3	4 78	2	See
1888 97	183 2	4 88	4	Hall	1895 73	205 9	4 74	2	Moulton
1889 10	185 9	4 64	3	Seabroke					
1889 86	185 4	4 98	4	Hall					

At the date of discovery SIR WILLIAM HERSCHEL found the distance\* of the component to be  $11''.09$ , and estimated the angle at  $70^\circ$ . At the epoch 1780 52 he found the distance  $11''.46$ , but made no measure of the angle of position until 1782 45, when it proved to be  $62^\circ 07'$ . HERSCHEL observed the angle to be  $70^\circ 8'$ , in 1803, but made no measure of the distance. The earliest observation of both angle and distance is a rough measure by BESSEL, in 1814; and although his angle is nearly correct, it is evident from the subsequent work of STRUVE that the distance is much too small. Since the time of STRUVE  $\eta$  Cassiopeae has been followed by nearly all of the best observers, so that we have good material upon which to base an investigation of the orbit.

Although the observations of  $\eta$  Cassiopeae do not suffice to fix all the elements so well as might be desired, yet it appears that the range of uncertainty is comparatively unimportant, except in the case of the periodic time, which may possibly differ several years from the value here derived. Some of the orbits found for  $\eta$  Cassiopeae by previous computers are indicated in the following Table of Elements.

$P$	$T$	$e$	$\alpha$	$\Omega$	$i$	$\lambda$	Authority	Source
181 <sup>yr</sup>	1896 0	0.77083	10 335	25.55	57 98	243 65	Powell	M N, vol XXI, p 66
176.37	1924 78	0.6268	10 68	50 80	68.5	245 9	Dunér	Mes Micro, p 166
222.435	1909.24	0.5763	9 83	39 95	53 83	223 33	Doberck	A N 2091
195.235	1901 25	0.6244	8 639	33 33	48 3	229 45	Grüber	A N 2111
167.4	1904 0	0.622	8 702	41 02	52 09	233 1	Coit	M N, vol XLII, p 359
208.1	1908.9	0.500	8 45	47 1	47 6	214 2	Lewis	M N, vol LV, p 20
190.50	1906 12	0.547	8 2047	43 0	46 08	222 02	See	A J 343

\*Astronomical Journal, 343; and Astronomical Journal, 355

We find the following elements for this celebrated binary

$P = 195\ 76$ years	$\Omega = 46^\circ 1$
$T = 1907\ 84$	$i = 45^\circ 95$
$e = 0\ 5142$	$\lambda = 217^\circ 87$
$a = 8''\ 2128$	$n = +1^\circ 83899$

#### Apparent orbit

Length of major axis	= $15''\ 80$
Length of minor axis	= $10''\ 24$
Angle of major axis	= $55^\circ 8$
Angle of periastron	= $254^\circ 5$
Distance of star from centre	= $3''\ 80$

The table of computed and observed places shows that these elements are highly satisfactory. But the rapid orbital motion near periastron will make it possible to effect a slight improvement in about ten years.

The parallax of the system recently determined by DR HERMANN S. DAVIS of Columbia College seems to be entitled to great weight; and yet the value is so large that with these elements the mass is only 0.166 that of the sun. The distance of the system is 464540 times the distance of the earth from the sun, and the semi-major axis of the orbit is 18.54 astronomical units. This mass is very small for the size of the system, and if the parallax of  $0''.43$  be confirmed, say, by Heliometer measures, our ideas of the nature of the stellar systems will have to be considerably modified. The parallax of  $0''.154$  found by OTTO STRUVE in 1856, from measures with the micrometer, gives a distance for the system of 1339400 astronomical units. The semi-major axis comes out 53.33 times the distance of the earth from the sun, and the combined mass proves to be 3.96.

The companion is at present near the line of nodes, and its relative motion in the line of sight is near its maximum value. The brightness and width of this pair is such as to justify an application of the spectroscopic method for determining parallax developed in § 5, Chapter I.

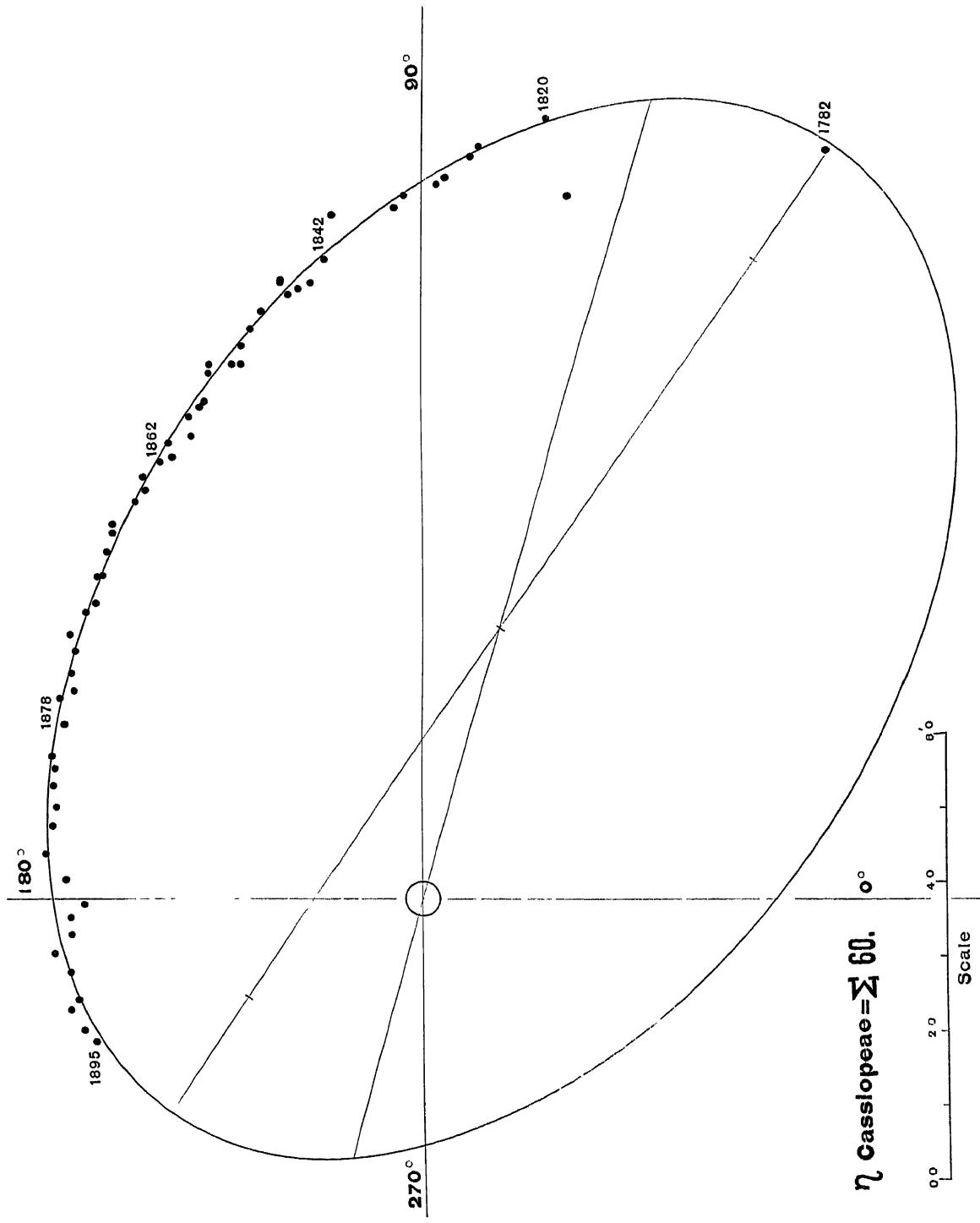
In this connection we may point out the great importance of the determination of the parallaxes of *double* rather than of *single* stars. The parallaxes of single stars are of comparatively little interest, since they give us only the distance and hence the velocity perpendicular to the line of vision, and the radiation compared to that of the sun. On the other hand, the parallaxes of double stars whose orbits are known give us, besides these data, the absolute dimensions of the orbits and the combined masses of the components — two elements of the highest importance in the study of the systems of the universe.  $\eta$  Cassiopeae is remarkable for the great angular distance of the components,

and for the rapid proper motion of the system. Both of these circumstances support the belief that the star is comparatively near to us in space, and render it certain that the parallax is sensible.

In 1881 Mr. LUDWIG STRUVE discussed the relative motion of the components about the common center of gravity of the system, and from his investigation it follows that  $\frac{M_2}{M_1} = 0.268$ , or the masses of the two stars, according to OTTO STRUVE's parallax, are respectively 2.90 and 1.06 times the combined mass of the sun and earth. The companion is therefore more massive than the sun and moves in an ellipse nearly twice the size of the orbit of *Neptune*, but the eccentricity is so large that in periastron the companion would come considerably within the orbit of the outer planet, while at apastron it would recede to more than three times that distance.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1779.81	$70^\circ \pm$	$57.2^\circ$	11.09	11.33	$+12.8 \pm$	$-0.24$	1	Herschel
1780.52	—	57.6	11.46	11.36	—	$+0.10$	1	Herschel
1782.45	62.1	58.7	—	11.42	$+3.1$	—	1	Herschel
1803.11	70.8	70.3	—	11.11	$+0.5$	—	1	Herschel
1814.10	78.5	76.7	9.70	11.00	$+1.8$	$-1.30$	1	Bessel
1820.16	81.1	80.5	10.68	10.67	$+0.6$	$+0.01$	5	Struve
1827.21	85.6	85.1	10.2	10.21	$+0.2$	$-0.01$	1	Struve
1830.75	86.2	87.9	10.07	9.91	$-1.7$	$+0.13$	5	Bessel
1831.75	88.7	88.6	9.69	9.87	$+0.1$	$-0.18$	1	Herschel
1832.46	88.1	89.1	9.76	9.82	$-1.0$	$-0.06$	7	$\Sigma$ 5, Dawes 2
1835.26	91.2	91.1	9.52	9.58	$-0.2$	$-0.06$	3	Struve
1836.74	92.1	92.6	9.39	9.44	$-0.5$	$-0.05$	1	Struve
1841.57	97.4	96.9	9.35	9.02	$+0.5$	$+0.33$	8	$O\Sigma$ 3, Madler 1, Dawes 1
1842.41	98.3	97.8	8.76	8.91	$+0.5$	$-0.15$	2-1	Madler
1844.56	100.1	99.7	8.48	8.73	$+0.4$	$-0.25$	6-5	Madler
1845.65	99.2	100.7	8.64	8.62	$-1.5$	$+0.02$	9	Madler 8, Jacob 1
1846.60	101.5	101.7	8.72	8.51	$-0.2$	$+0.21$	16	Madler 12, Jacob 1
1847.37	102.3	102.5	8.38	8.44	$-0.2$	$-0.06$	11-12	Madler 6-7, $O\Sigma$ 5
1848.12	102.5	103.4	8.60	8.37	$-0.9$	$+0.23$	2-1	Jacob
1849.66	105.0	105.0	8.26	8.25	$\pm 0.0$	$+0.01$	4	O Struve
1850.87	106.4	106.4	8.04	8.12	$\pm 0.0$	$-0.08$	26	Madler 21, Jacob 5
1851.80	107.8	107.5	7.88	8.00	$+0.3$	$-0.12$	6	Madler 3, $O\Sigma$ 3
1852.61	108.5	108.5	7.65	7.91	$\pm 0.0$	$-0.25$	7-8	Madler
1853.68	109.3	109.8	7.69	7.81	$-0.5$	$-0.12$	21-15	Ma 8, Ja 7, Po 6-0
1854.76	111.5	111.2	7.79	7.69	$+0.3$	$+0.10$	13	$O\Sigma$ 4, Ma 2, Dem 7 [Mo 3]
1855.81	111.9	112.5	7.70	7.59	$-0.6$	$+0.11$	22-16	Ma 4-3, Sec 2, Po 9-1, Dem 1,
1856.45	113.4	113.8	7.37	7.48	$-0.4$	$-0.11$	10-9	Ja 4, Ma 2-1, Dem 1
1857.34	114.1	114.8	7.32	7.40	$-0.7$	$-0.08$	14	Ja 3, $O\Sigma$ 2, Ma 5, Dem 1
1858.29	115.6	116.4	7.26	7.30	$-0.8$	$-0.04$	10	Ja 3, Ma 1, Dem 3
1859.60	116.4	118.3	7.02	7.14	$-1.9$	$-0.12$	10-7	Ma 2-1, Po 6-4, Mo 2
1860.68	119.8	119.4	7.17	7.09	$+0.4$	$+0.08$	2	O Struve
1861.82	119.2	121.4	6.89	6.95	$-2.2$	$-0.06$	8-7	Madler 5, Powell 3-2
1862.78	120.9	122.9	6.92	6.87	$-2.0$	$+0.05$	20	Madler 8, Dembowski 12
1863.80	123.4	124.7	6.87	6.75	$-1.3$	$+0.12$	9	Dembowski







$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1864 40	124 1	125 8	6 70	6 68	- 1 7	+0 02	13-12	Powell 4-3, Dembowski 9
1865 63	125 9	127 8	6 65	6 57	- 1 9	+0 08	17	En 6, Dem 8, Kn 3
1866 60	129 6	229 4	6 61	6 49	+ 0 2	+0 12	13	O $\Delta$ 2, Dem 7, Sec 1
1867 44	130 3	131 2	6 51	6 39	- 0 9	+0 12	8	Searle 1, Dembowski 7
1868 52	132 3	132 9	6 37	6 31	- 0 6	+0 06	12	Du 5, O $\Delta$ 3, Dem 4
1869 84	134 6	135 6	6 18	6 17	- 1 0	+0 01	10	Dembowski 6, Dimer 4
1870 41	135 2	136 6	6 23	6 13	- 1 4	+0 10	17-16	Po 5-4, O $\Delta$ 2, Dem 7, Gl 3
1871 48	137 3	138 6	6 00	6 05	- 1 3	-0 05	10-9	Po 2-1, Dem 6, Gl 2
1872 49	139 5	140 7	6 01	5 96	- 1 2	+0 05	19	O $\Delta$ 2, Du 7, Dem 6, Kn 1
1873 62	143 3	143 3	6 00	5 86	$\pm$ 0 0	+0 11	19-13	W & S 2-1, O $\Delta$ 3, Dem 7, Gl 2
1874 58	144 7	145 5	5 82	5 79	- 0 8	+0 03	9	Du 1, Dem 7, W & S 1 [No 6 0
1875 54	147 4	147 6	5 67	5 72	- 0 2	-0 05	21-19	O $\Delta$ 2, Du 10, Dem 7, Dk 2-0
1876 70	149 2	150 2	5 53	5 64	- 1 0	-0 11	13	Dembowski 7, Plummer 6
1877 73	151 0	152 6	5 62	5 57	- 1 6	+0 05	11	Dembowski 6, Doberck 5
1878 77	154 2	155 1	5 40	5 51	- 0 9	-0 11	11	Dem 5, Gold 1, Dk 5
1879 59	159 0	157 4	5 39	5 44	+ 1 6	-0 05	15	Hall 7, Doberck 3, Franz 5
1880 37	160 5	159 2	5 29	5 41	+ 1 3	-0 12	12	Jedrzewicz 7, Doberck 5
1881 46	162 8	162 1	5 22	5 37	+ 0 7	-0 15	11-9	Dk 1, Jed 3-2, Pl 2, Hl 1
1882 59	165 8	165 3	5 11	5 30	+ 0 5	-0 19	15-14	Jed 3, Dk 6-5, En 6
1883 94	168 8	168 9	5 12	5 21	- 0 1	-0 12	3	Hall
1885 52	173 1	172 8	5 16	5 17	+ 0 3	-0 01	6	Seabroke 1, Hall 5
1886 55	176 7	174 9	4 85	5 12	+ 1 8	-0 27	20-19	En 5, Sea 3-2, Hl 5, Tau 7
1887 35	180 6	178 4	4 6	5 08	+ 2 2	-0 28	1	Smith
1888 66	182 8	182 1	4 80	5 03	+ 0 7	-0 23	11	Seabroke 2, Maw 5, Hall 1
1889 48	185 6	181 6	4 81	5 00	+ 1 0	-0 19	7	Seabroke 3, Hall 1
1890 79	188 4	188 5	5 07	4 95	- 0 1	+0 12	5	Hall
1891 61	191 7	191 2	4 90	4 92	+ 0 5	-0 02	9-7	See 5-4, Maw 1-3
1892 86	196 3	195 0	4 82	4 87	+ 1 3	-0 05	6	Com 3, Col 2, Jo 1
1893 90	197 1	198 5	5 00	4 84	- 1 4	+0 16	2	Comstock 1, Lovett 1
1894 07	200 9	199 0	4 92	4 83	+ 1 9	+0 09	2	Comstock 1, Maw 1
1895 29	203 4	202 9	4 84	4 79	+ 0 5	+0 05	3	See

EPHEMERIS

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 50	207 6	4 73	1899 50	217 2	4 55
1897 50	210 1	4 68	1900 50	221 1	4 16
1898 50	213 7	4 62			

$\gamma$  ANDROMEDAE BC = OY 38.

$\alpha = 1^h 57^m 8$  ,  $\delta = +41^\circ 51'$   
5 5, bluish , 7, bluish

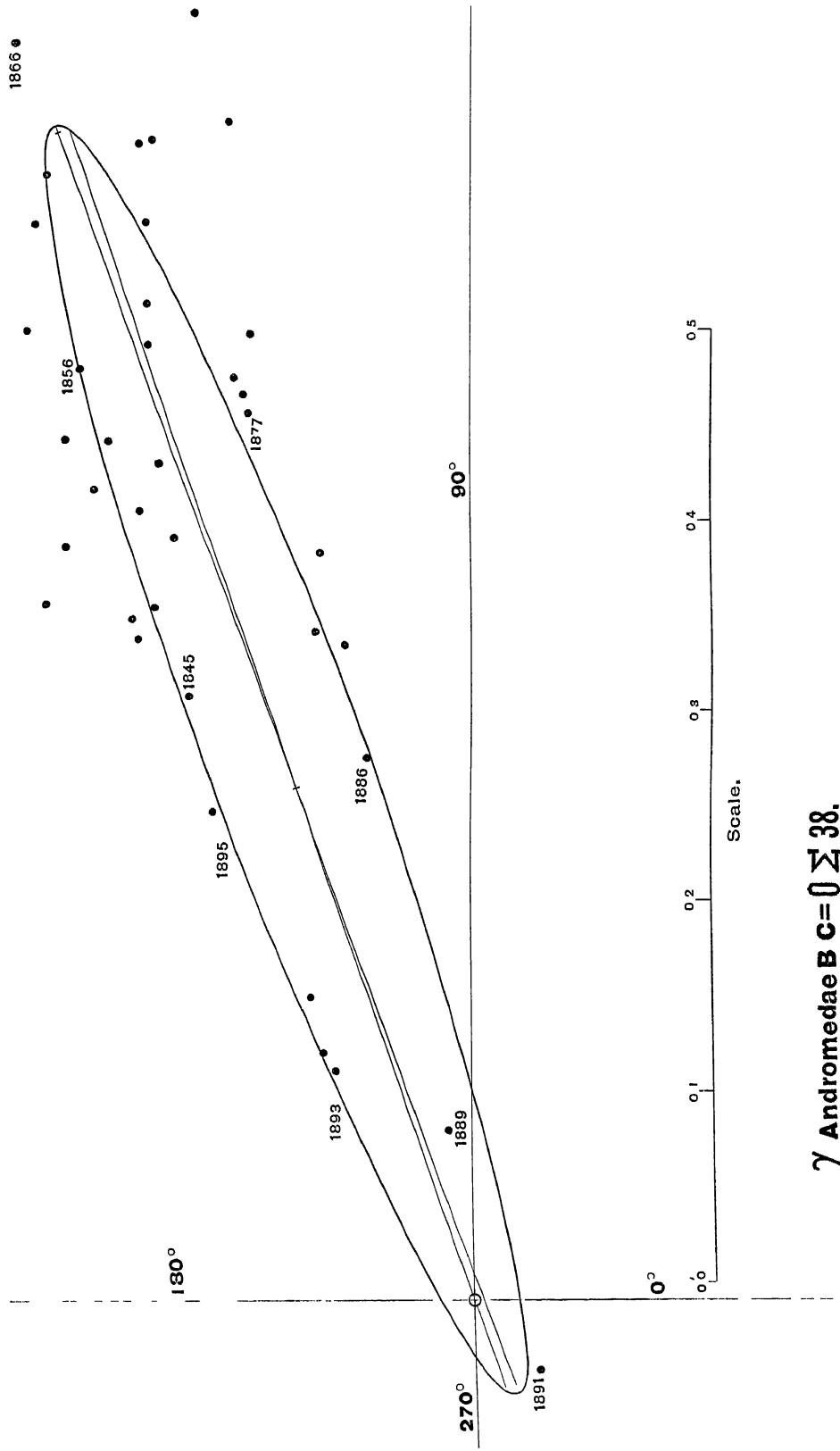
Discovered by Otto Struve in 1842

OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1843 00	119 7	0 45 $\pm$	2	Dawes	1846 64	111 3	0 43	7-3	Mitchel
1843 19	119 8	0 35	2-1	Madler	1847 13	117 9	0 52	5	O Struve
1843 55	125 5	0 48	3	O Struve	1847 82	111 3	0 6 $\pm$	4	Dawes
1845 15	116 9	0 39	1	Madler	1849 69	114 9	0 47	4	O Struve

$t$	$\theta_0$ °	$\rho_0$ "	$n$	Observers	$t$	$\theta_0$ °	$\rho_0$ "	$n$	Observers
1851 19	116.6	0.40	4	Madler	1869 81	107.0	0.63	3	O. Struve
1852 21	114.5	0.48	2	Madler	1869 95	105.6	0.5±	13	Dembowski
1852 78	111.3	0.5±	2	Jacob	1871 01	110.6	0.63	15	Duner
1853 23	116.0	0.47	3	Madler	1872 83	101.5	0.63	1-2	Brunnow
1853 79	108.5	0.55±	4	Dawes	1872 92	91.8	0.5±	2-1	W & S
1853 94	106.8	0.4±	4	Jacob	1873 17	105.1	0.63	5	O. Struve
1854 75	112.0	0.61	1	Dawes	1874 00	109.3	0.53	1	Newcomb
1855 02	119.4	—	1	Madler	1874 53	96.3	0.51	2	Gledhill
1855 09	109.8	0.40	1	Secchi	1876 79	105.7	—	1	W & S
1856 12	116.7	0.5±	1	Jacob	1877 05	101.1	0.48	6	Schiaparelli
1856 20	116.5	0.45	1	Madler	1877 71	103.9	—	1	Doberck
1856 21	121.7	0.41	2	Winnecke	1877 94	102.1	0.81	1	Seabroke
1856 84	113.0	0.67	3	O. Struve	1878 21	101.0	0.36	8	Hall
1856 90	109.7	0.47	3	Secchi	1878 65	102.1	0.43	2	Burnham
1857 23	115.4	0.45	3-1	Madler	1880 06	107.9	0.36	1	Burnham
1858 06	114.0	—	2	Jacob	1880 11	106.7	—	2	Seabroke
1858 22	115.4	—	2	Madler	1880 12	91.1	—	8	Jedrzejewicz
1858 99	108.9	0.45	3	Secchi	1882 05	101.0	0.49	6-1	Bigourdan
1859 81	108.7	0.53	1	Dawes	1883 15	93.1	0.29	7	Englemann
1862 55	115.2	0.50	4-2	Madler	1883 16	106.7	—	1	Seabroke
1863 27	108.5	0.45±	8	Dembowski	1883 87	103.1	0.40	2	Perrotin
1863 86	107.7	0.59	1	Dawes	1884 18	113.3	—	3	Seabroke
1863 99	107.6	0.61	—	Romberg	1884 65	117.6	0.35	1	Perrotin
1865 67	107.1	0.59	4	Knott	1886 83	101.0	0.29	1	Newcomb
1865 68	106.9	0.60	1	Dawes	1889 51	98.2	0.09	1	Burnham
1865 76	106.3	0.58	2-1	Leyton Obs	1891 72	312.6	0.05±	3	Burnham
1866 21	110.0	0.70	3	O. Struve	1893 79	121.8	0.11	3	Barnard
1866 74	132.3	—	1	Winlock	1894 56	121.6	0.15	3	Barnard
1866 74	107.2	—	1	Seale	1895 63	118.5	0.18	3	Barnard
1866 74	100.4	—	1	Winlock	1895 72	121.2	0.29	3	See
1866 85	104.2	0.64	1	Leyton Obs	1895 72	115.3	elongated	1	Moulton
1867 79	104.3	0.5±	1	Newcomb					
1868 82	102.0	0.69	6-5	Brunnow					

Since OTTO STRUVE's discovery of this extraordinary binary in 1842 the companion has described nearly an entire revolution, but as the orbit is very eccentric and highly inclined nearly all the observations lie in the narrow region included between position-angle  $120^\circ$  and  $100^\circ$ . Only in recent years has it been possible for observers to prove the reality of orbital motion, some ten years ago the object was found to be getting more and more difficult, and



$\gamma$  Andromedae B C=0  $\Sigma$  38.



hence it became clear that the distance was diminishing. In 1886 NEWCOMB found the distance  $0'' 29$  and the angle  $101^\circ$ ; in small telescopes the star appeared single. When BURNHAM examined the object in 1889 he found it exceedingly difficult even with the 36-inch refractor of the Lick Observatory, and during 1890 the companion was wholly invisible. When the star was examined in 1891 it was found that the companion had changed to the opposite quadrant, the angle being  $312^\circ 6'$  and the distance so excessively small that it was estimated at  $0'' 05 \pm$ . BARNARD's examination of the object in 1893 gave the key to the situation. The companion had swept rapidly round to  $121^\circ 8'$ , thus passing over about  $320^\circ$  of position angle since the measure in 1889. BURNHAM at once undertook an investigation of the orbit, and obtained a very satisfactory set of elements. His paper, in the *Monthly Notices* for December, 1893, contains an illustration of the apparent orbit, and a complete list of measures down to 1893. We have added the measures made since that date, and derived a set of elements very similar to that found by BURNHAM. His elements are:

$$\begin{array}{ll} P = 54.8 \text{ years} & \Omega = 113^\circ 5' \\ T = 1892.1 & i = 78^\circ 9' \\ e = 0.875 & \lambda = 200^\circ 8' \\ a = 0'' 37 \end{array}$$

We find the following elements of  $\gamma$  *Andromedae*:

$$\begin{array}{ll} P \doteq 54.0 \text{ years} & \Omega = 113^\circ 4' \\ T = 1892.1 & i = 77^\circ 85' \\ e = 0.857 & \lambda = 200^\circ 1' \\ a = 0'' 3705 & n = -6^\circ 6667 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 0'' 706 \\ \text{Length of minor axis} & = 0'' 084 \\ \text{Angle of major axis} & = 109^\circ 9' \\ \text{Angle of periastron} & = 289^\circ 0' \\ \text{Distance of star from centre} & = 0'' 298 \end{array}$$

The table of computed and observed places shows a good agreement for an object of this difficulty. The residuals are easily within the limits of the errors of observation. The orbit is remarkable for its great eccentricity and high inclination. Both of these elements are well defined, and the values given above will never be materially altered. Thus the error in the eccentricity can hardly surpass  $\pm 0.02$ , while a variation of one year in the period is to be regarded as improbable. In regard to the shape of the real orbit,  $\gamma$  *Andromedae* takes its place between  $\gamma$  *Virginis* and  $\gamma$  *Centauri*. These three remarkable systems are also similar as regards the relative brightness of their components,

which in each case are nearly equal. Since the companion of  $\gamma$  *Andromedae* is now within the reach of ordinary telescopes the accompanying ephemeris will be useful to astronomers

## COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1843 25	121 6	116 6	0 43	0 34	+ 5 0	+0 09	7-6	Dawes 2, Madler 2-1, O $\Sigma$ 3
1845 15	116 9	115 1	0 39	0 41	+ 1 8	-0 02	4	Madler
1846 64	111 3	114 3	0 43	0 45	- 1 0	-0 02	7-3	Mitchell
1847 47	114 6	113 9	0 56	0 48	+ 0 7	+0 08	9	O $\Sigma$ 5, Dawes 4
1849 69	114 9	113 0	0 47	0 53	+ 1 9	-0 06	4	O $\Sigma$
1851 19	116 6	112 5	0 40	0 56	+ 4 1	-0 16	4	Madler
1852 49	112 9	112 1	0 49	0 58	+ 0 8	-0 09	4	Madler 2, Jacob 2
1853 65	110 4	111 8	0 47 $\pm$	0 59	- 1 4	-0 12	11	Madler 3, Dawes 4, Jacob 4
1854 75	112 0	111 5	0 61	0 60	+ 0 5	+0 01	1	Dawes
1855 05	114 6	111 4	0 4 $\pm$	0 61	+ 3 2	-0 21	2-1	Madler 1-0, Secchi 1
1856 18	118 3	111 1	0 45	0 62	+ 7 2	-0 17	4	Jacob 1, Madler 1, Wynn 2
1856 99	112 7	110 9	0 53	0 63	+ 1 8	-0 10	9-7	O $\Sigma$ 3, Secchi 3, Madler 3-1
1858 42	112 8	110 6	0 45	0 64	+ 1 2	-0 19	7-3	Jacob 2-0, Madler 2-0, Secchi 3
1859 81	108 7	110 2	0 53	0 65	- 1 5	-0 12	1	Dawes
1862 55	115 2	109 6	0 50	0 66	+ 5 6	-0 16	4-2	Madler
1863 71	107 9	109 3	0 55	0 65	- 1 4	-0 10	9	Dem 8, Dawes 1, Romberg
1865 70	106 8	108 9	0 59	0 64	- 2 1	-0 05	7-6	Knott 4, Dawes 1, Leyton 2-1
1866 21	110 0	108 7	0 70	0 64	+ 1 3	+0 06	3	O $\Sigma$
1867 79	104 3	108 3	0 5 $\pm$	0 63	- 4 0	-0 13	1	Newcomb
1868 82	102 0	108 1	0 69	0 62	- 6 1	+0 07	6-5	Brunnow
1869 90	106 0	107 8	0 57	0 61	- 1 8	-0 04	16	O $\Sigma$ 3, Dembowski 13
1871 01	110 6	107 5	0 63	0 60	+ 3 1	+0 03	15	Dunér
1872 83	101 5	107 0	0 63	0 58	- 5 5	+0 05	4-2	Brunnow
1873 17	105 4	106 9	0 63	0 57	- 1 5	+0 06	5	O $\Sigma$
1874 26	102 8	106 5	0 52	0 55	- 3 7	-0 03	3	Newcomb 1, Gledhill 2
1876 79	105 7	105 6	-	0 51	+ 0 1	-	1-0	Wilson and Seabroke
1877 05	104 1	105 5	0 48 $\pm$	0 50	- 1 4	-0 02	6	Schiaparelli
1878 43	101 6	104 9	0 40	0 47	- 3 3	-0 07	10	Hall 8, $\beta$ 2
1880.10	102 9	104 1	0 36	0 43	- 1 2	-0 07	11-1	$\beta$ 1, Seabroke 2-0, Jed 8-0
1882 05	104 0	102 9	0 49	0 38	+ 1 1	+0 11	6-1	Bigoudan
1883 39	100 9	101 9	0 35	0 34	- 1 0	+0 01	10-9	Englemann 7, Sea 1-0, Per 2
1884 41	115 4	100 9	0 35	0 30	+14 5	+0 05	4	Seabroke 3, Perrotin 1
1886 83	101 0	96 8	0 29	0 19	+ 4 2	+0 10	1	Newcomb
1889 51	98 2	79 7	0 09	0 07	+18 5	+0 02	1	Bunham
1891 72	312 6	300 5	0 05 $\pm$	0 05	+12 1	$\pm$ 0 00	3	Bunham
1893 79	121 8	125 6	0 14	0 11	- 3 8	+0 03	3	Barnard 3
1894 56	121 6	121 4	0 15	0 16	+ 0 2	-0 01	3	Barnard
1895 63	118 5	118 8	0 18	0 23	- 0 3	-0 05	5	Barnard
1895 72	118 2	118 6	0 29	0 24	- 0 4	+0 05	4-3	See 3, Moulton 1-0

## EPHEMERIS

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 70	117 2	0 30	1899 70	114 70	0 42
1897 70	116 2	0 35	1900 70	114 4	0 44
1898 70	115 5	0 39			

$\alpha$  CANIS MAJORIS = SIRIUS = A.G.C. 1.

$\alpha = 6^h 40^m 4$  ,  $\delta = -16^\circ 34'$   
1, white , 10, yellow

*Discovered by Alvan G Clark, January 31, 1862*

OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1862 08	85 $\pm$	10 $\pm$	1	Alvan Clark	1868 02	73 2	10 25	2	Searle
1862 19	84 6	10 07	3	Bond	1868 04	72 1	—	1	Peirce
1862 20	85 0	10 09	5	Rutherford	1868 23	70 3	11 25	7	Vogel
1862 23	84 5	10 42	2	Chacornac	1868 24	69 6	11 35	5	Bruhns
1862 28	83 8	(4 92)	1	Lassell	1868 26	71 7	10 95	5	Englemann
1863 15	88 4	7 63	1	Secchi	1869 10	74 7	10 26	7-4	Brünnow
1863 21	82 5	10 15	2	O Struve	1869 15	73 6	11 23	3	Vogel
1863 21	81 3	9 51	6	Rutherford	1869 20	68 7	11 17	1	Dunér
1863 23	84 9	10 00	1	Dawes	1869 20	68 6	11 07	2	Winlock
1863 27	82 8	—	1	Bond	1869 23	69 4	10 93	1	Peirce
1864 14	79 4	10 60	3	Marth	1870 13	68 1	11.16	12-4	Peirce
1864 18	80 1	9 60	1-3	Lassell	1870 17	65 9	11.06	7-5	Winlock
1864 22	78 6	10 70	4-2	Bond	1870 24	65.1	12 06	5	Vogel
1864 22	74 8	10 92	6-3	O. Struve	1871 16	65 9	10 75	3	Secchi
1864 23	84 9	—	1	Dawes	1871 20	70 3	11 19	2-1	Peirce
1864 24	79 7	10 08	1	Winnecke	1871 23	64 1	11 11	2	Dunér
1865 10	76 8	—	3	Lass & Mar	1871 25	60 1	12 10	4-3	Pechule
1865 21	77 6	10 59	2	O Struve	1872 18	59 8	11.05	2	Dunér
1865 22	75 5	9 59	8	Secchi	1872 21	66 6	10.69	3	Börgen
1865 23	77 8	10 77	5-4	Foerster	1872 24	62 4	11.50	1	Newcomb
1865 25	76 9	—	3	Tietjen	1872 24	64 3	11.46	6	Hall
1865 26	76 0	—	—	Bond	1872 26	61 3	—	3	Skinner
1865 26	76 9	(9 0)	1	Englemann	1873 20	65 8	11 12	1	Hall
1866 07	77 2	10 43	2-1	Knott	1873 22	60 8	10 57	1-4	Dunér
1866 21	—	10 74	1	Bruhns	1873 23	70 0	9 80	1	Börgen
1866 21	75 2	10 93	3	O Struve	1873 23	66 3	10 42	1	Bruhns
1866 22	73 9	10 97	2-1	Tietjen	1873 93	65 0	11 29	1	W & S
1866 23	74.1	11 29	3-1	Foerster	1874 16	59 0	11 46	7	Newcomb
1866 23	74.0	10 21	2-3	Hall	1874 19	58 7	10 99	2-1	Holden
1866 23	74 9	10 57	3	Newcomb	1874 23	58 0	11 10	2	Hall
1866 25	78 3	10 34	1	Tuttle	1874 83	57.5	—	1	Burton
1866 26	74 7	10 09	3	Eastmann	1875 19	57.1	10 73	4	Dunér
1866 29	71 3	10 11	3	Secchi	1875.21	56 6	11.41	2	Newcomb
1867 02	74 2	11 15	7-6	Winlock	1875 21	55 9	11 89	5-4	Holden
1867 10	73 8	10 66	6-5	Searle	1875 28	56 4	11 08	4	Hall
1867 22	72 1	10 98	1	O Struve					
1867.24	72 3	—	2	Foerster					
1867.27	74 9	9.92	2-1	Eastmann					



$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1876 03	57 8	11 12	1	Watson	1881 99	43 6	9 38	11	Burnham
1876 05	54 6	11 45	1	Peters	1882 13	43 1	9 30	9	Hough
1876 09	54 9	11 82	6	Holden	1882 13	42 4	9 76	4-3	Bigou dan
1876 14	55 0	11 55	4	Russell	1882 18	42 2	9 95	6	Frisby
1876 22	55 2	11 19	6	Hall	1882 23	42 5	9 67	7	Hall
1877 11	52 8	11 19	4-3	Cincinnati	1882 54	44 0	—	6	Englemann
1877 16	52 8	11 35	4	Holden	1883 10	40 1	9 05	10	Burnham
1877 26	53 4	10 95	5	Hall	1883 10	39 0	9 41	1	Young
1877 97	52 4	10 83	8	Burnham	1883 12	39 7	9 02	11	Hough
1878 07	50 5	11 07	4	Holden	1883 14	41 3	—	4	Wilson
1878 15	51 0	10 71	9	Cincinnati	1883 17	41 4	9 75	7	Frisby
1878 19	54 4	11 24	5	Pritchett	1883 19	39 9	9 10	2-1	Bigou dan
1878 22	53 2	11 4	—	Eastmann	1883 21	39 1	9 26	6	Hall
1878 24	51 7	10 76	5	Hall	1884 05	36 0	9 67	6	Pelletin
1878 70	50 0	10 61	20-14	Cincinnati	1884 17	35 3	8 79	3-1	Bigou dan
1879 05	50 7	10 44	10	Burnham	1884 18	36 7	8 51	11	Hough
1879 12	47 8	11 35	5	Holden	1884 19	36 4	8 39	10	Burnham
1879 15	50 3	10 78	5	Pritchett	1884 23	37 7	8 81	8	Hall
1879 20	50 1	10 55	6	Hall	1884 27	36 3	8 70	5	Young
1879 75	46 5	10 29	1	Cincinnati	1885 11	34 1	8 09	8	Young
1880 00	48 8	10 55	1	Russell	1885 20	32 7	7 96	10	Hough
1880 10	47 1	10 48	4	Holden	1885 27	34 7	8 06	8	Hall
1880 11	48 3	10 00	11	Burnham	1886 05	29 8	7 59	4	Young
1880 17	49 6	9 87	3	Hough	1886 14	28 7	7 21	12	Hough
1880 18	46 7	9 92	6-4	Bigou dan	1886 22	30 6	7 39	6	Hall
1880 22	51 1	—	1	Smith	1887 14	25 4	7 08	4	Young
1880 25	47 8	10 30	8	Hall	1887 19	23 7	6 78	7	Hough
1880 28	48 6	10 38	2	Frisby	1887 23	24 2	6 51	4	Hall
1881 07	46 3	9 77	8	Burnham	1888 24	23 3	5 78	5	Hall
1881 12	43 3	10 83	2	Holden	1889 97	13 9	5 27	5	Burnham
1881 14	44 3	10 62	5-3	Bigou dan	1890 27	359 7	4 19	3	Burnham
1881 17	46 9	10 11	6	Frisby					
1881 18	46 5	9 81	7	Young					
1881 26	45 3	9 60	5	Hough					
1881 26	45 3	10 00	6	Hall					

The discovery of the companion of *Sirius* is one of the justly celebrated events of modern Astronomy. It extended to the regions of the fixed stars the principle of theoretical prediction which has proved so admirable in the solar system, and which in the hands of LEVERRIER and ADAMS had led to the discovery of *Neptune*. BESSEL had occasion to make a careful examination of the proper motions of a considerable number of stars, including *Sirius* and *Procyon*. The two dog stars, instead of moving uniformly on the arcs of

great circles, seemed to trace out irregular sinuous paths across the sky, and a further study of these anomalies convinced BESSEL that the two stars were perturbed by invisible bodies. In 1844 he wrote, in a letter to HUMBOLDT "I adhere to the conviction that *Procyon* and *Sirius* form real binary systems, consisting of a visible and an invisible star. There is no reason to suppose luminosity an essential quality of cosmical bodies. The visibility of countless stars is no argument against the invisibility of countless others."

In 1857 the suggestion of BESSEL was taken up by PETERS, who made an investigation of the observed inequalities, and found the following elements for the orbit described by *Sirius* about the common centre of gravity of the system:

Periastron passage	= 1791 431
Mean yearly motion	= $7^{\circ}$ 1865
Period	= 50 01 years
Eccentricity	= 0 7994

In 1861 the question was again examined by SAFFORD, who transmitted to BRUNNOW an investigation which assigned to the companion a position-angle of  $83^{\circ}.8$  for the epoch 1862.1. A short time afterwards, on Jan. 31, 1862, MR ALVAN G. CLARK was trying the new 18-inch object glass of the Dearborn telescope, and on pointing the instrument on *Sirius* exclaimed: "Why, father it has a companion!" And sure enough the faint but massive disturbing body announced by BESSEL was seen within a few degrees of the place assigned by the theoretical astronomers. It now became a matter of great interest to ascertain from the motion of the new companion whether it was really the disturbing body, a few years showed that it had sensibly the required motion, and left no doubt of the identity of the two objects. In 1864 AUWERS undertook a new determination of the elements based on all the observations, and found:

Periastron passage	= 1793 890
Mean annual motion	= $7^{\circ}$ 28475
Period	= 49 418 years
Eccentricity	= 0 6010

A definitive determination afterwards published gave the following results:

$P$ = 49 399 years	$\Omega$ = $61^{\circ}$ 96
$T$ = 1843 275	$i$ = $47^{\circ}$ 14
$e$ = 0 6148	$\lambda$ = $18^{\circ}$ 91
$a$ = $2''$ 331	

When the micrometrical measures began to accumulate, various computers made new investigations of the orbit. The following table of elements is very

complete The last set credited to DR AUWERS were based on all the observations up to 1892

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
$\overset{\text{yrs}}{49.6}$	1891.8	0.58	8.41	42.4	57.1	—	Colbeir, 1885	Dearborn Report
58.47	1896.47	0.4055	8.58	50.0	55.4	216.3	Gore, 1889	M N, XLIX, no 8
51.22	1890.55	0.945	—	188	—	—	Mann	
49.46	1893.18	0.7512	8.31	10.2	53	—	Mann	
57.02	1894.17	0.538	8.50	40.75	51.43	48.58	Howard	A J 235
49.399	1844.216	0.6292	7.568	37.51	42.43	39.94	Auwers, 1892	A N 3084
51.97	1893.5	0.568	8.31	40.3	50.8	135.4	Burnham, 1893	Pub Lick Obs II, p 239
51.101	1893.759	0.6131	7.77	37.06	44.6	223.61	Zwiers, 1895	A N 3336

During 1890 the distance of the companion became so small that it was lost in the rays of the large star, even when viewed with the 36-inch refractor of the Lick Observatory. As it was evident that no further observations could be made until the object emerged on the other side, BURNHAM collected all the measures with great care and embodied them in his important paper in the *Monthly Notices* for April, 1891.

The orbit which we have given in this work is very similar to that found by BURNHAM, except that the eccentricity is higher and more nearly in accord with the value of this element found by AUWERS. The orbit is based wholly on the micrometrical measures, and the data used in deriving the mean places have been very carefully selected.

We find the following elements of the orbit of *Sirius*

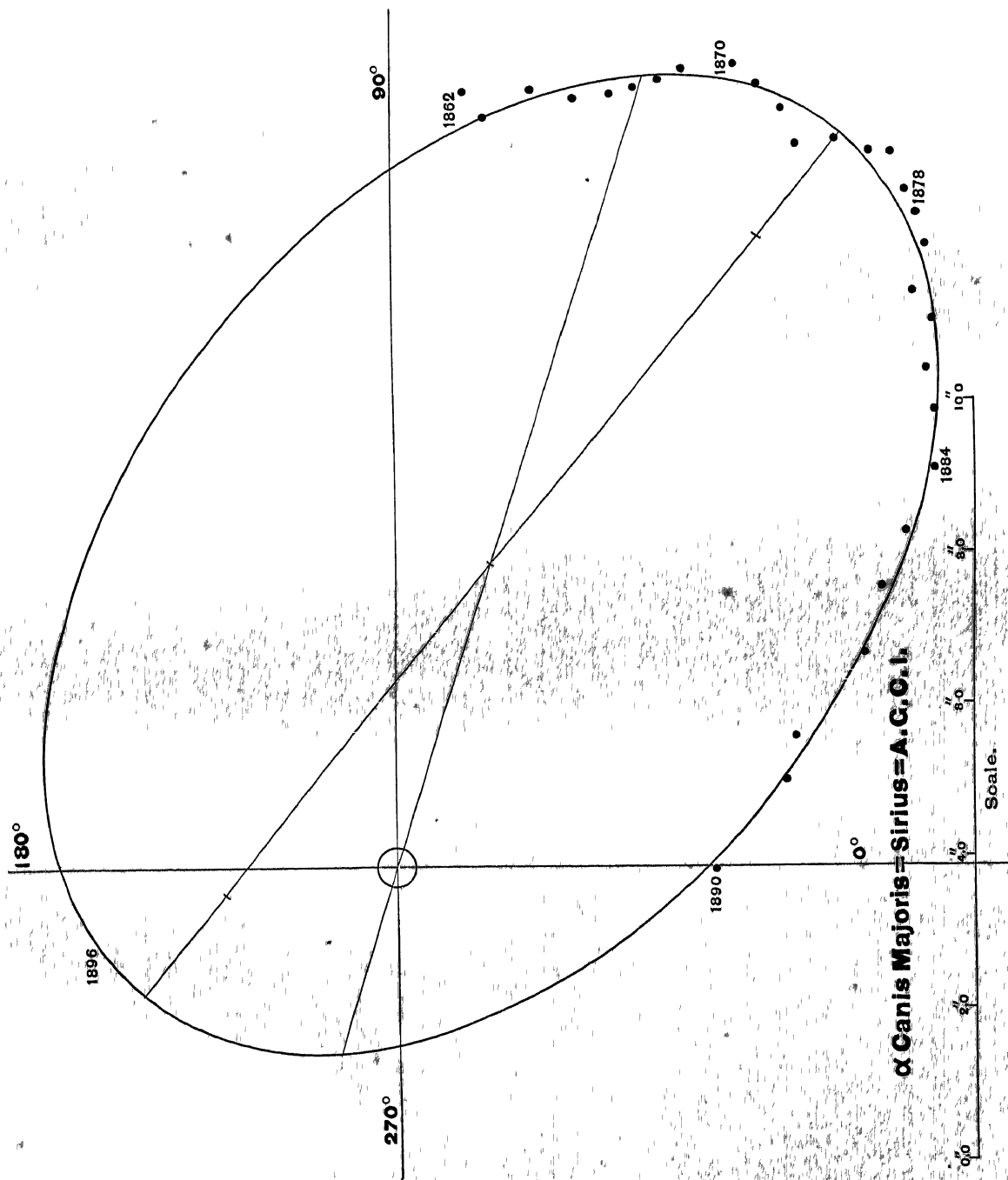
$$\begin{aligned}
 P &= 52.20 \text{ years} & \Omega &= 34^{\circ} 3' \\
 T &= 1893.50 & i &= 46^{\circ} 77' \\
 e &= 0.620 & \lambda &= 131^{\circ} 03' \\
 a &= 8''.0316 & n &= -6^{\circ} 89.655'
 \end{aligned}$$

#### Apparent orbit

$$\begin{aligned}
 \text{Length of major axis} &= 14''.63 \\
 \text{Length of minor axis} &= 9''.50 \\
 \text{Angle of major axis} &= 50^{\circ} 7' \\
 \text{Angle of periastron} &= 252^{\circ} 4' \\
 \text{Distance of star from centre} &= 4''.16
 \end{aligned}$$

#### EPHEMERIS

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896.20	193.9	4.12	1899.20	158.9	4.97
1897.20	180.8	4.44	1900.20	149.5	5.25
1898.20	169.0	4.72			





COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1862 21	84 7	84 1	10 19	9 78	+0 6	+0 41	10	Bond 2, Rutherford 5, Chacornac 2
1863 21	82 9	81 6	9 90	10 03	+1 3	-0 13	10-9	OZ 2, Rutherford 6, Dawes 1, Bond 1-0
1864 20	79 9	79 4	10 36	10 25	+0 5	+0 11	16-12	Mar 3, Las 1-3, Bond 4-2, OZ 6-3, Da 1-0, Winn 1
1865 22	76 8	77 1	10 35	10 48	-0 3	-0 13	22-15	Las 3-0, OZ 2, Sec 8, Fo 5-4, Tj 3, Bd —, En 1-0
1866 22	74 4	75 0	10 52	10 67	-0 6	-0 15	21-20	Kn 2-1, Brh 1, OZ 3, Tj 2-1, Fo 3-1, Hl 2-3, N 3,
1867 14	72 8	73 0	10 68	10 83	-0 2	-0 15	9-13	Wk 0-6, Sr 6-5, OZ 1, Fo 2-0 [Tut 0-1, East 3, Sec 2
1868 16	71 4	71 0	10 90	10 97	+0 4	-0 07	20-19	Searle 2, Peirce 1-0, Vl 7, Bruhns 5, Englemann 5
1869 19	70 1	68 9	11 10	11 09	+1 2	+0 01	7	Vl 3, Dunér 1, Winnecke 2, Peirce 1
1870 18	67 0	67 0	11 42	11 20	$\pm 0 0$	+0 22	19-14	Peirce 12-4, Winnecke 7-5, Vl 0-5
1871 21	65 1	65 1	11 28	11 27	$\pm 0 0$	+0 01	11-9	Secchi 3, Peirce 2-1, Dunér 2, Pech 4-3
1872 23	62 9	63 1	11 17	11 31	-0 2	-0 14	15-13	Dunér 3, Borgen 3, N 1, Hall 6, Doberck 3-0
1873 36	60 8	61 0	10 85	11 32	-0 2	-0 47	1-7	Hall 0-1, Dunér 1-4, Bruhns 0-1, W & S 0-1
1874 19	58 6	59 5	11 18	11 29	-0 9	-0 11	11-10	N 7, Holden 2-1, Hall 2
1875 34	56 3	57 3	11 28	11 22	-1 0	+0 06	16-14	Bur 1-0, Dunér 4, N 2, Holden 5-4, Hall 4
1876 11	54 9	55 7	11 43	11 14	-0 8	+0 29	17-18	Watson 0-1, Peters 1, Holden 6, Rus 4, Hall 6
1877 18	53 0	53 7	11 16	11 02	-0 7	+0 14	13-12	Cin 4-3, Holden 4, Hall 5
1878 14	51 4	51 8	11 00	10 84	-0 4	+0 16	26-31	$\beta$ 8, Holden 4, Cin 9, Pr 0-5, East 0-1, Hall 5
1879 04	49 6	50 0	10 75	10 68	-0 4	+0 07	46-40	Cin 20-14, $\beta$ 10, Holden 5, Pritchett 5, Hall 6
1880 15	47 9	47 5	10 22	10 39	+0 4	-0 17	36-34	Cin 1, Rus 1, Hol 4, $\beta$ 11, Ho 3, Big 6-4, Ill 8, Frs 2
1881 17	45 4	45 2	10 11	10 08	+0 2	+0 03	39-37	$\beta$ 8, Holden 2, Big 5-3, Frs 6, Y 7, Hough 5, Hall 6
1882 20	42 9	42 7	9 60	9 72	+0 2	-0 12	43-36	$\beta$ 11, Hough 9, Big 4-3, Frs 6, Hall 7, Englemann 6
1883 15	40 1	39 8	9 32	9 23	+0 3	+0 09	41-36	$\beta$ 10, Y. 1, Hough 11, Ws 4-0, Frs 7, Big 2-1, Hl 6
1884 18	36 4	37 2	8 81	8 80	-0 8	+0 01	43-41	Ferrotin 6, Big 3-1, Hough 11, $\beta$ 10, Hall 8; Young 5
1885 19	33 2	33 9	8 04	8 24	-0 7	-0 16	26	Young 8, Hough 10, Hall 8
1886 14	29 7	30 4	7 40	7 63	-0 7	-0 23	20	Young 4, Hough 12, Hall 6
1887 19	24 4	25 5	6 79	6 85	-1 1	-0 06	15	Young 4, Hough 7, Hall 4
1888 53	17 9	17 7	5 53	5 75	+0 2	-0 22	4-5	Hall 3, $\beta$ . 1-2
1889 06	12 7	13 6	5 26	5 24	-0 9	+0 02	3	Burnham
1890 27	35 9	0 2	4 19	4 09	-0 5	+0 10	3	Burnham

The comparison of the computed with the observed places shows an extremely satisfactory agreement, and we are led to believe that the elements given above will prove to be near the truth. The differences between these elements and those found by AUWERS are not greater than might be expected from the material used in the two cases. Adopting the foregoing elements and GILL'S parallax of  $0'' 38$ , we find the mass of the system to be 3.473 times that of the sun and earth; the major semi-axis comes out 21.136 astronomical units. Thus the system of *Sirius* is a magnificent one, having 3 47 times the mass of the planetary system, and slightly larger dimensions than the orbit of the planet *Uranus*. The masses, according to AUWERS, are in the ratio 1:2.119; or, in units of the sun's mass, 1.113 and 2.360 respectively. The future observation of this star is a matter of the highest interest. There is some reason to suppose that *Sirius* is very much expanded, more nearly resembling a nebula than the sun; if this inference be true, the action of the companion will raise enormous bodily tides in the mass of *Sirius*. Since the height of the tides varies inversely as the cube of the distance, it will follow that the tidal eleva-

tion at periastron will be about 80 times higher than at apastron. There would thus arise a periodic disturbance in the mass of *Sirius* depending on the revolution of the companion. It seems probable that high tides would increase the radiation of *Sirius*, and hence if it were possible to make photometric measures of absolute accuracy, or of such a character that the brightness could be compared at intervals of 25 years, it might some day be possible to detect the alteration in brightness arising from the tidal action of the companion.

The excessive faintness of this massive body is an extraordinary anomaly which is not easily explained. From the shape of the orbit, however, we may believe that the system has been formed by the usual process, and for some reason the companion has rapidly become obscure. As the companion is apparently still self-luminous, its darkness is not so conspicuous as the excessive brilliancy of *Sirius*. The change in the color of *Sirius* since ancient times is even more remarkable.

### 9 ARGÛS = $\beta$ 101.

$\alpha = 7^h 47^m 1$  ,  $\delta = -13^\circ 38'$   
5 7, yellow , 6 3, yellow

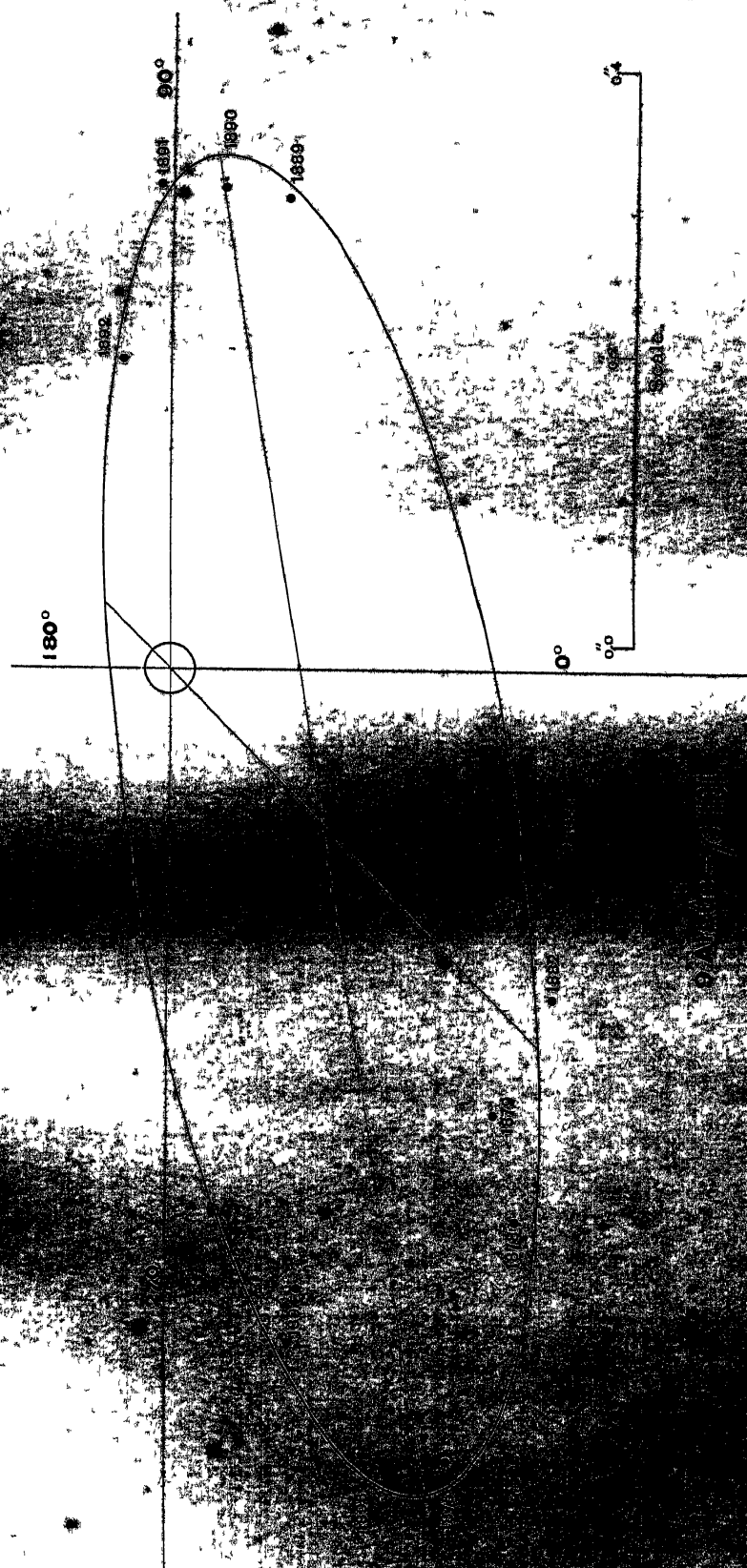
*Discovered by Burnham with his celebrated six-inch Clark Refractor, March 11, 1873*

#### OBSERVATIONS

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1873 19	double	—	1	Burnham	1891 06	91 5	0 34	4	$\beta$ & Sch
1875 24	289 7	0 58	2	Dembowski	1892 05	98 7	0 22	3	Burnham
1878 50	302 2	0 45	4	St & $\beta$	1893 94	282 1	0 44	3	Barnard
1879 68	306 2	0 38	2	Hall	1894 18	282 0	0 42	3	Barnard
1882 21	319 7	0 35	4	Schiaparelli	1894 25	286 6	0 35	3	Comstock
1883 11	336 2	0 30	1	Burnham	1894 85	287 3	0 63	5-4	Barnard
1889 08	76 4	0 34	4	Burnham	1895 21	285 2	0 42	2	Comstock
1890 22	83 8	0 34	6	Burnham	1895 25	285 4	0 59	5	Barnard
					1895 30	283 8	0 58	3	See

The first investigation of the orbit was made by GLASENAPP and published in the *Monthly Notices* for June, 1892. His elements are

$$\begin{array}{ll}
 P = 40.54 \text{ years} & \Omega = 116^\circ 7' \\
 T = 1844.02 & i = 59^\circ 2' \\
 e = 0.090 & \lambda = 251^\circ 3' \\
 a = 0'' 45 & z = +8^\circ 880'
 \end{array}$$







BURNHAM revised this orbit, in May, 1893, and by relying on the distances as well as the angles, arrived at an apparent ellipse of very different character, from which we derived the following elements (*Astronomy and Astrophysics*, June, 1893)

$$\begin{array}{ll} P = 23\,377 \text{ years} & \Omega = 95^\circ 75 \\ T = 1892\,706 & i = 76^\circ 87 \\ e = 0.68 & \lambda = 73^\circ 92 \\ a = 0''\,612 & n = +15^\circ\,3998 \end{array}$$

It did not take long to decide which set of elements was to be preferred.\* BARNARD examined the star with the 36-inch refractor of the Lick Observatory in December, 1893, and found that since 1892.05 the radius vector of the companion had swept over about  $180^\circ$ , so that the small star was in the fourth quadrant. I took occasion recently, while measuring double stars with the 26-inch refractor of the Leander McCormick Observatory of the University of Virginia, to measure 9 *Argûs* on three good nights. The observations confirm those of BARNARD, and show that BURNHAM's apparent orbit is not far from the truth. With the new measures, it seemed worth while to re-investigate the orbit; accordingly, from a consideration of all the observations, I find the following elements of 9 *Argûs*

$$\begin{array}{ll} P = 22\,00 \text{ years} & \Omega = 95^\circ 5 \\ T = 1892\,30 & i = 77^\circ 72 \\ e = 0.70 & \lambda = 75^\circ 28 \\ a = 0''\,6549 & n = +16^\circ\,3636 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 0''\,941 \\ \text{Length of minor axis} & = 0''\,267 \\ \text{Angle of major axis} & = 99^\circ 2 \\ \text{Angle of periastron} & = 134^\circ 5 \\ \text{Distance of star from centre} & = 0''\,152 \end{array}$$

It is confidently believed that these elements will prove to be nearly correct, in spite of the small number of observations upon which they are based.

COMPARISON OF THE COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1875 24	289 7	291 7	0 58	0 58	-2 0	0 00	2	Dembowski
1878 50	302 2	302 5	0 45	0 47	-0 3	-0 02	4	Cincinnati and Burnham
1879 68	306 2	305 4	0 38	0 44	+0 8	-0 06	2	Hall
1882 21	319 7	324 5	0 35	0 31	-4 8	+0 04	4	Schiaparelli
1883 11	336 2	335 7	0 30	0 26	+0 5	+0 04	1	Burnham
1889 08	76 4	73 6	0 34	0 33	+2 8	+0 01	4	Burnham
1890 22	83 8	82 8	0 34	0 36	+1 0	-0 02	6	Burnham
1891 06	91 5	90 1	0 34	0 34	+1 4	0 00	4	Burnham and Schiaparelli
1892 05	98 7	107 0	0 22	0 16	-8 3	+0 06	3	Burnham
1893 94	282 1	276 8	0 44	0 42	+5 3	+0 02	3	Barnard
1895 30	283 8	283 6	0 58	0 57	+0 2	+0 01	3	See

\* *Astronomische Nachrichten*, 3297

It will be seen that the residuals are very small for such a close and difficult star, and it is evident that future observations will not change the present orbit materially, although it is desirable to secure additional exact measures which will improve the elements as much as possible. If adequate attention is given to this object, its orbit will soon be one of the best in the heavens. A short ephemeris is

$t$	$\theta_0$	$\rho_0$	$t$	$\theta_0$	$\rho_0$
1896 3	285 8	0 59	1899 3	295 2	0 55
1897 3	288 8	0 60	1900 3	299 0	0 51
1898 3	291 9	0 59			

As the eccentricity of the orbit is well determined by the rapid motion of the companion round the periastron, the established conspicuous magnitude of this element must be regarded as the most remarkable phenomenon of the system.

For the next few years the star will be relatively easy, and double-star observers should give it particular attention.

### $\zeta$ CANCRI AB = $\Sigma$ 1196.

$\alpha = 8^h 6^m 2$  ,  $\delta = +17^\circ 58'$   
5 5, yellow , 6 2, yellow

*Discovered by Sir William Herschel, November 21, 1781*

#### OBSERVATIONS

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1781 90	363 5	—	1	Herschel	1835 30	28 8	—	1	Madler
1825 27	57.8	1 09	—	South	1835 31	20 2	1 14	3	Struve
1826 22	57 6	1 14	3	Struve	1835 60	15 7	—	3	Madler
1828 80	38 4	1.04	2	Struve	1836 27	15 4	1 20	3	Struve
1831 16	31 8	1 34	5-3	Herschel	1836 31	15 1	—	5	Madler
1831 28	29 8	1 05	6	Struve	1836.68	16 1	—	4	Dawes
1831 30	30 8	1 09	3	Dawes	1840 15	6 1	1 24	35-23 obs	Kaiser
1832 12	27 9	—	8	Herschel	1840 20	4 4	1 19	8	Dawes
1832 12	27 0	—	7	Dawes	1840 29	7 5	1 00	7	O Struve
1832 19	31 3	1 32	5	Bessel	1841.16	0 9	1 18	5	Dawes
1832 28	27 5	1 15	4	Struve	1841 31	1 0	1 05	6-4	Madler
1833 13	26 3	—	9	Herschel	1842 22	356 3	1 18	6	Dawes
1833 21	26 2	1 19	9	Dawes	1842 26	358 9	1 07	6	Madler
1833 27	22 1	1 15	3	Struve	1842 29	359 3	1 29	4	O Struve

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1843 18	355 0	1 12	8	Dawes	1856 07	304 2	1 $\pm$	7	Dembowski
1843 19	356 9	1 06	4	Madler	1856 21	306 3	1 21	4-3	Jacob
1843 30	354 3	1 17	3	O Struve	1856 23	309 4	1 16	2	Morton
1844 28	350 3	1 16	4	O Struve	1856 25	307 2	0 77	2	Secchi
1844 39	354 4	1 02	10	Madler	1856 28	307 5	1 00	2	Madler
1845 25	350 4	1 05	13	Madler	1856 31	307 3	1 01	10-7	Winnecke
1845 31	347 9	0 97	3	O Struve	1856 93	296 5	1 03	3	Dembowski
1845 83	349 4	1 2	1	Jacob	1857 27	298 4	0 98	3	O Struve
1846 27	347 5	1 02	16	Madler	1857 29	304 5	0 96	3-2	Madler
1846 29	344 8	0 95	3	O Struve	1857 29	303 9	0 78	6	Secchi
1846 29	344 4	—	1	Jacob	1857 90	299 7	1 14	3-1	Jacob
1847 18	344 6	1 09	4	Madler	1858 18	294 2	1 $\pm$	7	Dembowski
1847 33	342 2	0 96	5	O Struve	1858 20	297 6	1 05	3	Madler
1848 13	338 5	1 05	1	Dawes	1858 28	295 5	0 98	1	O Struve
1848 24	338 1	1 06	6	Dawes	1859 27	294 9	0 98	8	Madler
1848 25	342 8	1 0	1	W C Bond	1859 30	286 5	0 91	2	O Struve
1848 28	340 0	1 03	7-6	Madler	1860 26	282 9	—	—	Dollen
1848 30	337 7	0 91	5	O Struve	1860 26	283 3	—	—	Wagner
1849 29	334 2	1 11	5	Dawes	1860 26	281 0	0 70	1	Dawes
1849 32	336 1	0 80	4	O Struve	1860 26	284 8	—	—	Schiaparelli
1850 29	332 9	0 94	3	O Struve	1860 27	281 3	0 81	2	O Struve
1850 71	330 0	1 03	1	Madler	1860 28	279 9	—	—	Dollen
1851.18	333 5	1 1 $\pm$	3	Fletcher	1860 28	282 0	—	—	Wagner
1851 21	329 0	1 05	9	Madler	1860 28	283 4	—	—	Schiaparelli
1851 28	327 2	1 02	3	O Struve	1860 28	285 0	—	—	Winnecke
1851 25	327 9	1 01	7	Dawes	1860 30	286 0	1 02	5-4	Madler
1852 16	329 0	1 0 $\pm$	3	Fletcher	1861 14	282 8	—	5	Powell
1852 23	324 4	1 06	3	Dawes	1861 26	282 2	0 97	2	Madler
1852 25	326 9	1 06	6	Madler	1861 27	275 3	0 87	3	O Struve
1852 32	321 7	0 89	2	O Struve	1862 31	267 5	0 74	2	O Struve
1853.20	322 0	1 22	3	Jacob	1862.32	274 4	0 97	4	Madler
1853 24	323 5	1 06	8-7	Madler	1863 13	263.1	0 74	15	Dembowski
1853 30	319 8	0 97	2	O Struve	1863 25	267 3	0 95	—	Leyton Obs
1854 20	315 3	0 98	3	Dawes	1863 25	262.5	0 67	1	Dawes
1854 27	318 6	1 08	10-9	Madler	1863 30?	268 1	0 70	1	Knott
1854 29	320 2	1 02	1	Morton	1864 15	255 0	0 55	10	Dembowski
1854 37	321 9	—	12	Powell	1864 29	253 2	0 71	2	Dawes
1855 10	308 6	1 $\pm$	7	Dembowski	1864 31	350 0 $\pm$	0 60	1	Englemann
1855 19	312 4	1 07	3	Secchi	1864 30	253 3	0 72	2	O Struve
1855 26	310 6	1 06	4	Madler	1865 21	245 7	0 50	12	Dembowski
1855.31	310 3	0 91	3	O Struve	1865 30	243 4	0 63	3-2	Dawes
1855 31	305 9	1 04	7-6	Winnecke	1865 33	245 3	0 64	2	Secchi
					1865 36	241 4	0 61	3	Knott
					1865 30	244 0	0 86	4	Englemann

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1866 19	238 4	0 52	9	Dembowski	1877 17	108 7	0 68	7	Dembowski
1866 27	237 8	0 70	1	O Struve	1877 23	107 9	0 79	7	Schiaparelli
1866 28	234 6	0 40	2	Secchi	1877 23	110 3	0 81	3-6	Plummer
1866 31	233 3	0 78	4	Knott	1877 24	108 1	0 87	3-2	Doberck
1866 37	231 5	0 72	1	Leyton Obs	1877 27	108 0	0 72	3	O Struve
1866 94	228 3	0 66	1	Knott	1877 32	107 3	0 74	1	Pritchett
1867 08	229 7	0 59	3-1	Harvard	1878 16	104 1	1 01	1-2	Doberck
1867 22	224 4	obl	9	Dembowski	1878 18	100 3	0 66	6	Dembowski
1868 20	210 9	0 5	7	Dembowski	1878 26	100 8	0 7	7	Jedrzejewicz
1868 28	214 7	0 72	2	O Struve	1878 29	99 1	0 76	3	O Struve
1869 26	197 6	0 64	1	Peuce	1878 32	102 3	0 81	3	Hall
1869 32	198 4	0 62	2	O Struve	1879 27	93 1	0 87	6	Schiaparelli
1869 37	203 6	0 48	4	Dunér	1879 29	91 8	0 74	3	O Struve
1870 08	188 1	0 64	5-2	Harvard	1880 21	85 2	0 61	5	Hall
1870 15	187 3	0 5	9	Dembowski	1880 22	89 8	0 89 $\pm$	6	Jedrzejewicz
1870 28	186 3	0 66	4	O Struve	1880 24	88 9	—	2	Doberck
1870 30	188 3	0 43	3-4	Dunér	1880 29	85 2	0 73	6	Burnham
1870 56	181 0	0 2	2	Gledhill	1881 24	81 1	0 91 $\pm$	4	Jedrzejewicz
1871 15	175 5	Contatto	7	Dembowski	1881 24	84 9	0 84	5	Doberck
1871 26	175 1	0 2	2	Gledhill	1881 28	86 8	0 88	3	O Struve
1871 29	178 2	0 55	3	Dunér	1881 30	79 0	0 71	3	Hall
1871 30	169 4	—	—	Schamhorst	1881 30	80 2	0 92	6	Schiaparelli
1871 31	171 3	0 59	3	O Struve	1881 31	73 7	0 77	2	Pritchett
1872 11	166 7	0 6	2	Knott	1882 09	75 7	0 74	1	Bigoudan
1872 21	167 5	0 70	3	Wilson	1882 20	73 3	0 79	4	Hall
1872 23	162 8	Contatto	7	Dembowski	1882 22	76 2	1 05	6	Englemann
1872 31	163 0	0 58	3	O Struve	1882 25	75 1	0 98	6	Schiaparelli
1872 33	163 3	0 69	2	Dunér	1882 26	75 0	0 94 $\pm$	4	Jedrzejewicz
1873 19	150 2	0 5	10	Dembowski	1883 24	72 4	1 05	6	Englemann
1873 22	150 9	0 5 $\pm$	4	W & S	1883 29	69 3	1 00	6	Schiaparelli
1873 28	152 0	0 61	3	O Struve	1883 31	66 4	0 82	4	Hall
1873 63	149 3	0 55	2	Gledhill	1884 19	62 7	1 06	3	Perrotin
1874 09	141 6	0 74	7	Dembowski	1884 22	61 9	—	8	Bigoudan
1874 13	140 1	0 45 $\pm$	2	Gledhill	1884 25	63 9	0 98	7	Schiaparelli
1874 18	141 3	0 58	3-2	W & S	1884 26	60 6	0 98	3	O Struve
1874 28	144 5	0 64	3	O Struve	1884 27	64 5	0 88	5	Hall
1874 29	142 8	0 62	2	Dunér	1884 28	67 0	0 94	4	Englemann
1875 14	130 1	0 74	8	Dembowski	1884 38	64 4	—	3	Sea & Smith
1875 26	128 9	0 70	6	Schiaparelli	1885 27	59 0	1 25	2	Seabroke
1875 28	132 4	0 62	3	O Struve	1885 29	58 0	1 04	5	Schiaparelli
1875 29	133 3	0 77	2	W & S	1885 29	59 4	1 05	4	Englemann
1875 33	129 5	0 59	5	Dunér	1886 08	57 2	1 09	4	Tarrant
1876 14	119 4	0 72	6	Dembowski	1886 24	51 4	1 06	2-1	Sea & Smith
1876 26	120 7	—	6	Doberck	1886 28	55 0	1 03	4	Hall
1876 29	119 45	0 66	2	O Struve	1886 29	51 2	0 98	3	Jedrzejewicz
					1886 30	56 3	1 08	5	Englemann

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1887 24	50 4	0 89	4	Hall	1891 22	35 7	1 04	5	Hall
1887 26	48 4	0 97	11	Schiaparelli	1891 24	34 1	1 14	3	Bigourdan
1887 35	46 0	1 21	4-1	Sea & Smith	1892 24	31 6	1 09	3	Maw
1888 25	46 5	1 03	4	Hall	1892 25	31 3	1 26	2-3	Knorre
1888 26	49 2	—	3	Smith	1892 26	30 1	1 11	11	Schiaparelli
1888 27	43 7	1 04	9	Schiaparelli	1892 28	30 4	1 10	6	Bigourdan
1888 33	45 8	1 09	2	O Struve	1892 89	28 7	0 99	3	Jones
1888 36	41 4	1 13	1	Maw	1893 20	27 2	0 98	2	Comstock
1889 17	42 0	1 20	4	Sea & Hodges	1893 22	26 4	1 07	3	Maw
1889 19	40 3	1 05	3	Leavenworth	1893 24	27 6	1 12	13	Schiaparelli
1889 21	40 7	1 08	12	Schiaparelli	1894 15	26 0	1 47	1	Ebell
1889 21	43 4	—	2	Glasenapp	1894 16	23 8	1 24	3	H C Wilson
1889 23	43 6	0 99	5	Hall	1894 23	22 9	0 93	3	Comstock
1889 28	43 7	1 23	2	O Struve	1894 24	23 5	1 08	13	Schiaparelli
1889 29	40 9	1 07	3	Maw	1894 24	25 0	1 05	4	Maw
1890 23	37 2	11 1	9-7	Schiaparelli	1894 39	23 2	1 39	5-4	Bigourdan
1890 26	36 4	0 95	2	Comstock	1895 23	21 9	1 22	2	Lewis
1890 28	36 9	0 99	4	Hall	1895 23	20 9	1 01	3	Comstock
1891 05	32 3	1 04	5-4	Flint	1895 27	17 1	1 09	1	Davidson
1891 21	34 3	1 14	9-10	Schiaparelli	1895 28	22 8	1 13	4	See

The closer components of this ternary (or quarternary) system have been found to revolve rapidly in a period of about sixty years, while the remote component moves much more slowly, and probably will complete its orbit in six or seven centuries. Both stars move retrograde, and the system thus made up is one of great interest to the physical astronomer. From the time of WILLIAM STRUVE the observations are both abundant and exact, and hence the orbit of the close pair can now be determined with a high degree of precision. We shall treat only of the close binary, neglecting the remote companion and the dark body which PROFESSOR SEELIGER supposes to attend it. It is evident that the third component will exercise a considerable disturbing influence upon the close pair, but PROFESSOR SEELIGER has shown that this influence is probably obscured by the large errors incident to the measurement of a system which is never much wider than one second of arc. Assuming that the motion will be sensibly undisturbed, we shall deduce the orbit of the closer pair by the same process which is employed in the case of other binaries. The motion of this system has been investigated by numerous computers; the following list of orbits is fairly complete:

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
58 91	1853 37	0 2346	1 292	1 47	63 3	266 0	Madler, 1840	Donpat obs IX, p 177
58 27	1816 687	0 444	0 892	33 67	24 01	133 01	Madler, 1848	Fixi-Syst I, p 218
42 501	1805 67	0 4743	1 013	10 52	65 65	227 15	Villaiseau 1849	A N 967
58 94	1815 53	0 256	1 030	18 4	48 6	141 9	Winnecke 1855	
58 23	1872 44	0 3023	0 908	150 3	36 24	171 78	Plummer, 1871	M N XXXI, p 195
60 45	1869 9	0 365	0 908	107 5	23 5	85 3	Flam, 1873	Catal d ét doub p 19
62 4	1869 3	0 353	0 908	109 0	20 7	199 0	O Struve, 1871	C R LXXIX, p 1467
59 486	1870 82	0 3318	0 886	358 05	18 52	188 55	Dobereck, 1880	A N 2322 [1881
60 3	1866 0	0 391	0 853	81 55	15 53	109 73	Seeliger, 1881	Wien Akad LXXXIII,
59 11	1868 112	0 3819	0 853	80 18	11 13	109 73	Seeliger, 1888	Akad d Wiss, Munc '88

An examination of all the measures led to the mean places given in the accompanying table; from these we find the following elements

$$\begin{aligned}
 P &= 60.0 \text{ years} & \Omega &= 88^\circ 7' \\
 T &= 1870.40 & i &= 7^\circ 4' \\
 e &= 0.340 & \lambda &= 264^\circ 0' \\
 a &= 0''.8579 & n &= -6''.000
 \end{aligned}$$

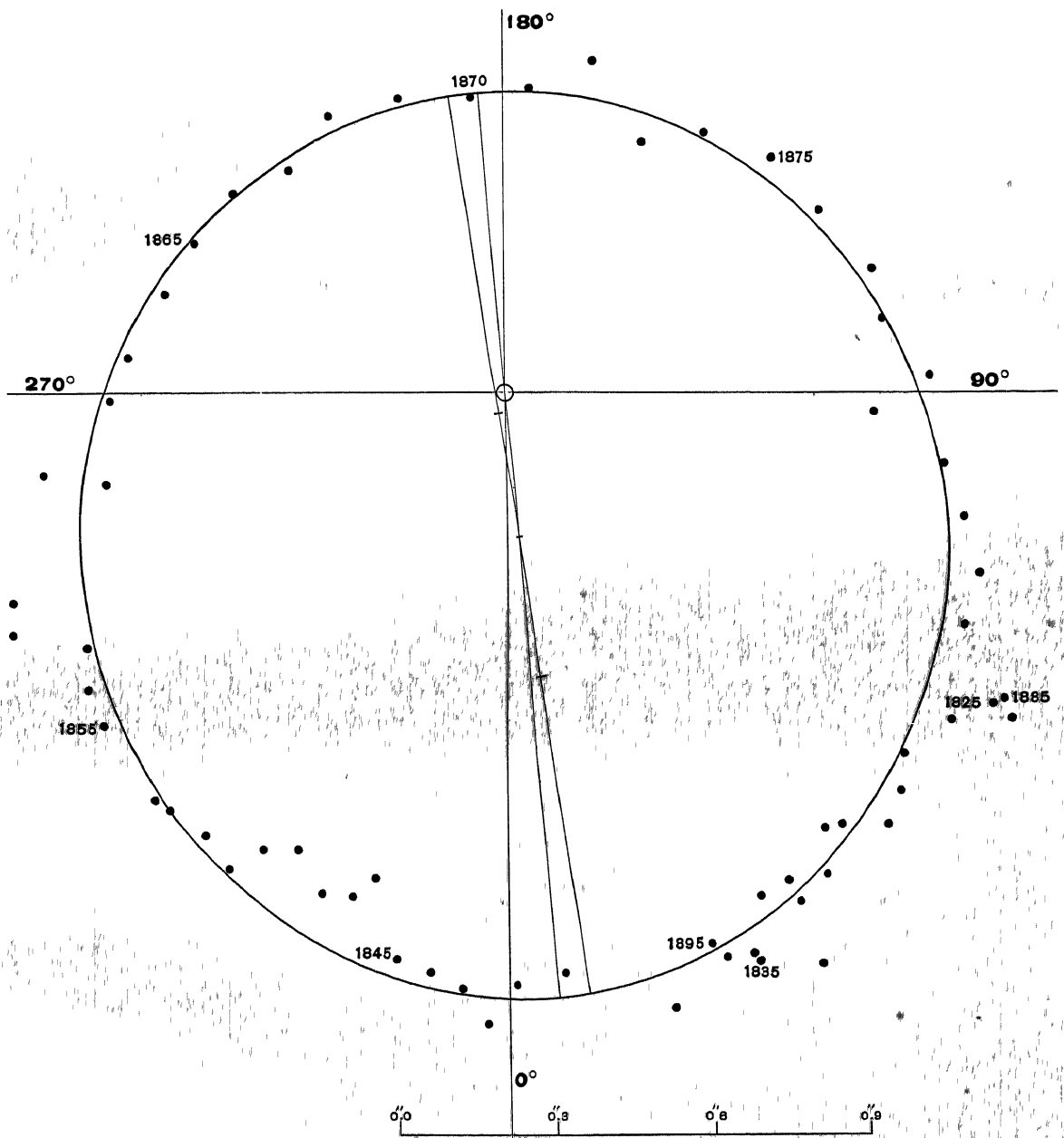
Apparent orbit.

$$\begin{aligned}
 \text{Length of major axis} &= 1''.701 \\
 \text{Length of minor axis} &= 1''.632 \\
 \text{Angle of major axis} &= 8^\circ 8' \\
 \text{Angle of periastron} &= 184^\circ 9' \\
 \text{Distance of star from centre} &= 0''.290
 \end{aligned}$$

The comparison of the computed with the observed places shows a good agreement, and indicates that no radical change in the above elements is to be expected. The period is perhaps uncertain by half a year, while the eccentricity can hardly be varied by more than  $\pm 0.03$ . The motion extends over more than one revolution, and is well represented by the above elements in all parts of the orbit. The apparent ellipse is remarkable for its circularity, and the small inclination renders the motion almost the same in the apparent as in the real orbit. The general interest thus attaching to this system is greatly enhanced by problems arising from the perturbations of the third star and its theoretical companion.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1781 90	363 5	359 6	—	1 14	+3 9	—	1	Herschel
1825 27	57 8	59 0	1 09	0 96	-1 2	+0 13	—	South
1826 22	57 6	55 0	1 14	0 98	+2 6	+0 16	3	Struve
1828 80	38 4	44 1	1 04	1 03	-5 7	+0 01	2	Struve
1831 29	30 3	34 9	1 07	1 07	-4 6	$\pm 0.00$	9	Struve 6, Dawes 3
1832 23	29 4	30 9	1 23	1 09	-1 5	+0 14	9	Bessel 5, Struve 4
1833 24	24 2	28 0	1 17	1 10	-3 8	+0 07	12	Dawes 9, Struve 3



$\zeta$  Cancri AB =  $\Sigma$  1196.





$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
	$^{\circ}$	$^{\circ}$	$''$	$''$	$^{\circ}$	$''$		
1835 40	21 6	21 5	1 14	1 12	+0 1	+0 02	7-3	Madler 1, $\Sigma$ 3, Madler 3
1836 42	15 5	17 4	1 20	1 13	-1 9	+0 07	12-3	$\Sigma$ 3, Madler 5-0, Dawes 4-0
1840 24	6 0	5 2	1 09	1 14	+0 8	-0 05	15	Dawes 8, O $\Sigma$ 7
1841 23	0 9	2 1	1 11	1 14	-1 2	-0 03	11-9	Dawes 5, Madler 6-4
1842 25	358 2	358 8	1 18	1 14	-0 7	+0 04	16	Dawes 6, Madler 6, O $\Sigma$ 4
1843 22	355 4	355 8	1 12	1 13	-0 4	-0 01	15	Dawes 8, Madler 4, O $\Sigma$ 3
1844 33	352 4	352 2	1 09	1 12	+0 2	-0 03	14	O $\Sigma$ 4, Madler 10
1845 57	348 6	348 1	1 08	1 12	+0 5	-0 04	4	O $\Sigma$ 3, Jacob 1
1846 29	344 6	345 7	0 95	1 11	-1 1	-0 16	4-3	O $\Sigma$ 3, Jacob 1-0
1847 31	342 6	342 3	0 99	1 10	+0 3	-0 11	7	Madler 2, O $\Sigma$ 5
1848 24	339 4	339 2	1 01	1 09	+0 2	-0 08	20-19	Dawes 1, Dawes 6, Bond 1, Madler 7-6, O $\Sigma$ 5
1849 31	335 1	335 3	0 95	1 07	-0 2	-0 12	9	Dawes 5, O $\Sigma$ 4
1850 50	331 4	330 9	0 98	1 06	+0 5	-0 08	4	O $\Sigma$ 3, Madler 1
1851 23	329 4	328 3	1 04	1 04	+1 1	$\pm$ 0 00	22	Fletcher 3, Madler 9, O $\Sigma$ 3, Dawes 7
1852 24	325 5	323 7	1 00	1 02	+1 8	-0 02	14	Fletcher 3, Dawes 3, Madler 6, O $\Sigma$ 2
1853 25	321 8	320 3	1 01	1 00	+1 5	+0 01	13-9	Jacob 3-10, Madler 8-7, O $\Sigma$ 2
1854 28	319 0	316 0	1 00	0 98	+3 0	+0 02	26-4	Dawes 3, Madler 10-0, Mo 1, Powell 12-0
1855 23	309 6	311 8	0 98	0 96	-2 2	+0 02	24-17	Dem 7, Secchi 3-0, Madler 4-0, O $\Sigma$ 3, Winnecke 7-6
1856 33	305 5	306 6	0 96	0 93	-1 1	+0 03	30-21	Dem 7, Ja 4-0, Mo 2-0, Sec 2, Ma 2, Winn 10-7
1857 44	301 6	301 0	0 91	0 90	+0 6	+0 01	15-17	O $\Sigma$ 3, Madler 3-2, Secchi 6, Jacob 3-1 [Dem 3
1858 22	295 8	296 7	0 99	0 88	-0 9	+0 11	11-8	Dem 7, Madler 3-0, O $\Sigma$ 1
1859 28	290 7	290 9	0 95	0 85	-0 2	+0 10	10	Madler 8, O $\Sigma$ 2
1860 28	282 8	284 6	0 76	0 82	-1 8	-0 06	8-3	Dawes 1, O $\Sigma$ 2, Madler 5-0
1861 22	280 1	278 6	0 87	0 79	+1 5	+0 08	10-3	Powell 5-0, Madler 2, O $\Sigma$ 3
1862 31	270 9	270 9	0 86	0 75	$\pm$ 0 0	+0 11	6-2	O $\Sigma$ 2, Madler 4
1863 23	264 6	263 4	0 70	0 72	+1 2	-0 02	17	Dembowski 15, Dawes 1, Knott 1
1864 21	253 8	255 0	0 66	0 69	-1 2	-0 03	14	Dembowski 10, Dawes 2, Englemann 1, O $\Sigma$ 2
1865 30	244 0	245 2	0 60	0 65	-1 2	-0 05	24-19	Dembowski 12, Dawes 3-2, Secchi 2, Knott 3, En 4
1866 39	233 9	233 8	0 63	0 62	+0 1	+0 01	18-13	Dem 9, O $\Sigma$ 1, Secchi 2, Knott 4-0, Ley 1-0, Knott 1
1867 15	224 4	225 3	0 59	0 61	-0 9	-0 02	9-1	Harvard 3-1; Dembowski 9-0
1868 24	212 8	212 4	0 61	0 58	+0 4	+0 03	9-7	Dembowski 7, O $\Sigma$ 2-0
1869 32	199 9	199 1	0 58	0 57	+0 8	+0 01	7-6	Peirce 1-0, O $\Sigma$ 2, Dunér 4
1870 27	186 2	186 7	0 56	0 56	-0 5	$\pm$ 0 00	23-21	Harvard 5-2, Dembowski 9, O $\Sigma$ 4, Dunér 3-4, Gl 2
1871 25	175 0	173 7	0 57	0 56	+1 3	+0 01	15-6	Dembowski 7, Gledhill 2-0, Dunér 3, O $\Sigma$ 3
1872 24	164 6	161 3	0 64	0 58	+3 3	+0 06	17-10	Knott 2, Wilson, 3, Dembowski 7-0, O $\Sigma$ 3, Dunér 2
1873 33	150 6	147 8	0 54	0 59	+2 8	-0 05	19	Dembowski 10, W & S 4, O $\Sigma$ 3, Gledhill 2
1874 19	142 1	138 1	0 61	0 62	+3 0	-0 01	17-16	Dembowski 7, Gledhill 2, W & S 3-2, O $\Sigma$ 3, Dunér 2
1875 26	130 8	126 5	0 68	0 65	+4 3	+0 03	24	Dembowski 8, Sch 6, O $\Sigma$ 3, W & S 2, Dunér 5
1876 23	119 8	117 4	0 69	0 68	+2 4	+0 01	13-7	Dembowski 5, Doberck 6-0, O $\Sigma$ 2
1877 24	108 4	108 6	0 74	0 72	-0 2	+0 02	24-26	Dem 7, Sch 7, Plummer 3-6, Dk 3-2, O $\Sigma$ 3, Pr 1
1878 24	101 3	100 4	0 73	0 74	+0 9	-0 01	20-19	Doberck 1-0, Dembowski 6, Jed 7, O $\Sigma$ 3, Hall 3
1879 28	92 4	92 7	0 81	0 78	-0 3	+0 03	9	Schiaparelli 6, O $\Sigma$ 3
1880 24	87 3	86 4	0 75	0 81	+0 9	-0 06	19-17	Hall 5, Jedzejewicz 6, Doberck 2-0, $\beta$ 6
1881 28	80 9	79 9	0 84	0 84	+1 0	$\pm$ 0 00	23	Jed 4, Doberck 5, O $\Sigma$ 3, Hall 3, Sch 6, Pritchett 2
1882 20	75 1	74 4	0 90	0 87	+0 7	+0 03	21	Bigourdan 1, Hall 4, Englemann 6, Sch 6, Jed 4
1883 28	69 3	68 8	0 96	0 90	+0 5	+0 06	16	Englemann 6, Schiaparelli 6, Hall 4
1884 26	63 6	63 8	0 97	0 93	-0 2	+0 04	33-22	Per 3, Big 8-0, Sch 7, O $\Sigma$ 3, Hl 5, En 4, S & S 3-0
1885 28	58 8	59 2	1 11	0 95	+1 7	+0 16	11	Seabroke 2, Schiaparelli 5, Englemann 4
1886 24	54 2	54 8	1 05	0 98	-0 6	+0 07	18-17	Tarrant 4, S & S 2-1, Hall 4, Jed 3, Englemann 5
1887 28	48 3	50 2	1 02	1 00	-1 9	+0 02	19-16	Hall 4, Schiaparelli 11, S & S 4-1
1888 29	45 3	46 4	1 07	1 02	-1 1	+0 05	19-16	Hall 4, Smith 3-0, Schiaparelli 9, O $\Sigma$ 2, Maw 1
1889 22	42 1	42 5	1 10	1 04	-0 4	+0 06	31-29	Sea 4, Leav 3, Hl 5, O $\Sigma$ 2, Maw 3, Sch 12, Gl 2-0
1890 26	36 8	38 5	1 02	1 06	-1 7	-0 04	16-14	Schiaparelli 9-7, Comstock 2, Hall 4
1891 18	34 1	35 2	1 09	1 07	-1 1	+0 02	22	Flint 5-4, Schiaparelli 9-10, Hall 5, Bigourdan 3
1892 38	30 4	30 9	1 11	1 09	-0 5	+0 02	25-26	Maw 3, Knott 2-3, Schiaparelli 11, Bigourdan 6, Jo 3
1893 22	27 1	27 1	1 06	1 10	$\pm$ 0 0	-0 04	18	Comstock 2, Maw 3, Schiaparelli 13
1894 23	24 0	24 6	1 16	1 11	-0 6	+0 05	29-28	Eb 1, H C W 3, Com 3, Sch 13, Maw 4, Big 5-4
1895 25	20 7	21 3	1 11	1 12	-0 6	-0 01	10	Lewis 2, Comstock 3, Davidson 1, See 4

A more critical investigation of these problems will commend itself to the attention of astronomers; the best results will depend upon the reduction of exact observations by the refined methods of analysis. In the present state of micrometrical measurement, a very refined treatment is seriously embarrassed by the errors of observation, but the methods of physical Astronomy ought eventually to enable us to improve the theory of the motion of the system, which is here taken as undisturbed

The following is a short ephemeris for the use of observers.

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 25	18 0	1 13	1899 25	8 4	1 13
1897 25	14 8	1 13	1900 25	5 3	1 14
1898 25	11 6	1 13			

### Σ 3121.

$\alpha = 9^h 12^m 1$  ,  $\delta = +29^\circ 0'$   
7 2, white , 7 5, yellowish

*Discovered by William Struve in 1831*

#### OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$		$t$	$\theta_o$	$\rho_o$	$n$	
1832 31	20 0	0 85	3	Struve	1868 30	27 6	0 81	2	O Struve
1840 31	246 5	0 40 ±	3-1	O Struve	1869 31	26 1	0 88	1	O Struve
1844 28	193 5	0 33	2-1	O Struve	1870 33	206 9	0 65	2	Dunér
1846.29	27 6	0 55	1	O Struve	1870 44	210 4	0 5 ±	1	Gledhill
1847 34	214 2	0 54	1	O Struve	1871 20	212 7	0 5 ±	1	Gledhill
1848.25	33 0	0 53	1	O Struve	1871 27	208 2	0 75	3	Dunér
1849.32	43 3	0 48	1	O Struve	1871 30	35 3	0 79	2	O Struve
1850.30	228 6	0 42	1	O Struve	1871 44	211 0	0 57	5	Dembowski
1851.26	59 7	0 33	1	O Struve	1872 09	209 3	0 68	1	Dunér
1861.29	Double vers le Nord		1	O Struve	1872 31	36 4	0 68	1	O Struve
1861 30	8 9	0 67	1	O Struve	1873 69	214 2	obl	8	Dembowski
1863 11	194 8	0 7	1	Dembowski	1873 70	214 5	0 5 ±	1	Gledhill
1864.30	13 0	0 71	1	O Struve	1874 24	220	< 0 3	2	Dunér
1865.77	206 8	0 80	2	Englemann	1874 28	46 7	0 53	2	O Struve
1867.65	201 3	0 70	5	Dembowski	1875 20	225	0 2 ±	1	Dunér
					1875 29	250 1	obl	1	O Struve
					1875 29	65 2	0 30	4	Schiaparelli
					1875 31	251 9	ovale	2	Dembowski

$t$	$\theta_0$	$\rho_0$	$n$		$t$	$\theta_0$	$\rho_0$	$n$	
1877 25	183 0	oblong	1	O Struve	1885 30	215 8	0 4 ±	3	Schiaparelli
1878 21	185 2	0 25 ±	1	Burnham	1886 33	221 2	0 27	4	Englemann
1879 21	193 0	0 40	2	Burnham	1887 27	250 4	0 22 ±	9	Schiaparelli
1879 33	186 8	0 43	1	O Struve	1888 27	286 3	0 22 ±	7	Schiaparelli
1879 57	200 4	0 43	5	Schiaparelli	1889 30	132 3	0 23 ±	7	Schiaparelli
1880 26	200 3	0 35	3	Hall	1890 29	152 9	0 27 ±	4	Schiaparelli
1880 31	199 8	0 50	1	Burnham	1891 26	163 3	0 35	4	Hall
1881 29	198 0	0 61	1	O Struve	1891 32	166 7	0 33 ±	2	Schiaparelli
1881 34	205 3	0 46	2	Schiaparelli	1892 26	175 3	0 41 ±	7	Schiaparelli
1882 25	194 8	0 31	4	Englemann	1893 25	182 3	0 47	7-2	Schiaparelli
1882 31	205 8	0 45	4	Schiaparelli	1893 25	185 9	0 44	1	Comstock
1882 34	205 2	0 53	1	O Struve	1894 18	185 9	0 49	1	Wilson
1883 22	221 2	0 39	6	Englemann	1894 21	186 6	0 58	3	Bigourdan
1883 28	213 8	0 52	3	Schiaparelli	1894 24	183 3	0 45	3	Comstock
1883 31	215 7	0 45	3	Hall	1894 25	186 3	0 48 ±	5	Schiaparelli
1884 27	218 9	0 42	1	O Struve	1895 23	190 5	0 65	3	Lewis
1884 39	222 7	0 38	4	Schiaparelli	1895 26	8 8	0 50	3	Comstock
1884 61	225 6	0 30	4	Englemann	1895 31	12 6	0 55	2	See

WILLIAM STRUVE rated the magnitudes of the components of this pair at 7.5 and 7.8\* respectively. Recent observations with the 26-inch refractor of the Leander McCormick Observatory of the University of Virginia convince the writer that the brightness of the components has been over-estimated by at least a whole magnitude. The star is close and very faint, and the natural difficulty of the object will doubtless account for the rather large discordances in some of the observations.

As Σ3121 has been observed for many years, and the pair revolves with great rapidity, several orbits have been determined by previous investigators. The following is believed to be a complete list of the elements hitherto published:

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
39 18	1850 0	0 3471	0 696	19 94	52 4	143 3	Fritzsche, 1866	{ Bulletin de l'Acad de St Pétersbourg, t X A N 2156 A N 2808
40 62	1850 0	0 3725	0 715	23 5	54 11	141 6	Fritzsche, 1866	
37 03	1842 78	0 26	0 71	16 0	74 25	149 5	Doberck, 1877	
34 642	1878 52	0 3086	0 6725	24 85	75 43	129 45	Celoria, 1887	

\*Astronomical Journal, 349

From an investigation of all the observations, I find the following elements

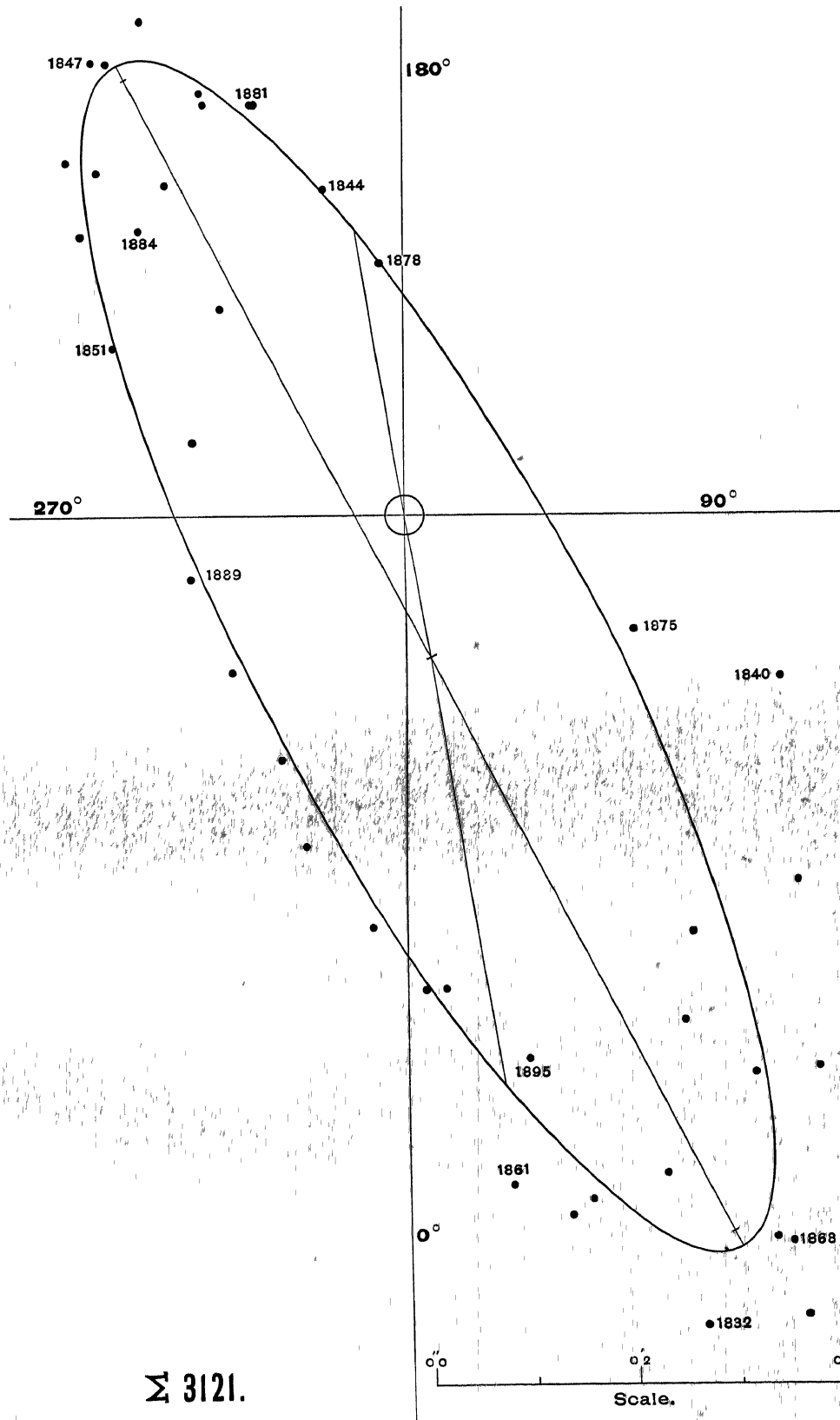
$$\begin{aligned}
 P &= 34.00 \text{ years} & \Omega &= 28^\circ 25' \\
 T &= 1878.30 & i &= 75^\circ 00' \\
 e &= 0.330 & \lambda &= 127^\circ 52' \\
 a &= 0''.6692 & n &= +10^\circ 5883
 \end{aligned}$$

Apparent orbit

$$\begin{aligned}
 \text{Length of major axis} &= 1''.318 \\
 \text{Length of minor axis} &= 0''.349 \\
 \text{Angle of major axis} &= 27^\circ 4' \\
 \text{Angle of periastron} &= 189^\circ 6' \\
 \text{Distance of star from center} &= 0''.142
 \end{aligned}$$

#### COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1832.31	20.0	22.3	0.85	0.79	-2.3	+0.08	3	W. Struve
1840.31	66.5	47.3	0.40 ±	0.35	+19.2	+0.05	3-1	O. Struve
1844.28	193.5	189.8	0.33	0.29	+3.7	+0.04	2-1	O. Struve
1846.29	207.6	205.2	0.55	0.48	+2.4	+0.07	1	O. Struve
1847.34	214.2	210.1	0.54	0.52	+4.1	+0.02	1	O. Struve
1848.25	213.0	214.1	0.53	0.52	-1.1	+0.01	1	O. Struve
1849.32	223.3	218.8	0.48	0.50	+5.5	-0.02	1	O. Struve
1850.30	228.6	223.9	0.42	0.45	+4.7	-0.03	1	O. Struve
1851.26	239.7	230.1	0.33	0.39	+9.6	-0.06	1	O. Struve
1861.26	8.9	9.9	0.67	0.58	-1.0	+0.09	1	O. Struve
1863.11	14.8	14.6	0.7	0.66	+0.2	+0.04	1	Dembowski
1864.30	13.0	18.1	0.71	0.73	-5.1	-0.02	1	O. Struve
1865.77	26.8	21.3	0.80	0.78	+5.5	+0.02	2	Englemann
1867.65	21.3	24.8	0.70	0.79	-3.5	-0.09	5	Dembowski
1868.30	27.6	26.2	0.81	0.79	+1.4	+0.02	2	O. Struve
1869.31	26.1	28.2	0.88	0.76	-2.1	+0.12	1	O. Struve
1870.38	28.6	30.6	0.57	0.71	-2.0	-0.14	3	Dunér, 2, Gledhill 1
1871.30	31.8	32.8	0.65	0.65	-1.0	0.00	11	Gl 1, Du 3, OΣ 2, Dem 5
1872.20	36.4	35.6	0.68	0.58	+0.8	+0.10	1-2	OΣ 1, Dunér 0-1
1873.70	34.3	42.8	0.5 ±	0.42	-8.5	+0.08	9-1	Dembowski 8-0, Gledhill 1
1874.28	46.7	46.7	0.53	0.36	0.0	+0.17	2	O. Struve
1875.27	63.0	63.0	0.25	0.22	0.0	+0.03	8-5	Du 1, OΣ 1, Sch 4, Dem 2
1878.21	185.2	188.4	0.25	0.28	-3.2	-0.03	1	Burnham
1879.57	200.4	200.2	0.43	0.41	+0.2	+0.02	5	Schiaparelli
1880.28	200.0	205.1	0.43	0.48	-5.1	-0.05	4	Hall 3, Burnham 1
1881.34	205.3	210.1	0.46	0.52	-4.8	-0.06	2	Schiaparelli
1882.28	205.8	214.1	0.45	0.52	-8.3	-0.07	4	Schiaparelli
1883.27	221.2	218.3	0.45	0.50	+2.9	-0.05	6-12	En 6, Sch 0-3, Hall 0-3
1884.39	222.7	224.5	0.38	0.44	-1.8	-0.06	4	Schiaparelli
1885.30	215.8	230.5	0.4 ±	0.39	-14.7	+0.01	3	Schiaparelli
1886.33	221.2	239.9	0.27	0.32	-17.7	-0.05	4	Englemann
1887.27	250.4	252.5	0.22	0.27	-2.1	-0.05	9	Schiaparelli
1888.27	286.3	272.6	0.22 ±	0.22	+13.7	0.00	7	Schiaparelli
1889.30	312.3	299.7	0.23 ±	0.21	+12.6	+0.02	7	Schiaparelli
1890.29	332.9	323.5	0.27 ±	0.24	+8.4	+0.03	4	Schiaparelli
1891.29	343.3	340.8	0.34	0.30	+4.2	+0.04	6	Hall 4, Schiaparelli 2
1892.26	355.3	354.0	0.41 ±	0.37	+1.3	+0.04	7	Schiaparelli
1893.25	2.3	359.7	0.47	0.43	+2.6	+0.04	7-2	Schiaparelli
1894.22	5.2	5.0	0.48	0.50	+0.2	-0.02	9	Wilson 1, Comstock 3, Sch 5
1895.29	10.7	9.8	0.53	0.58	+0.9	-0.05	5	See 2, Comstock 3





Some of the observations are vitiated by sensible systematic errors, so that occasionally our best observers differ by so much as  $12^\circ$ ; and in succeeding years the angles are made to retrograde where they ought to be steadily advancing. Under these circumstances the residuals may be considered small, and the elements very satisfactory for so close and difficult a star. In following this star, observers should take every precaution against systematic error, since the orbit is highly inclined, and a small error in angle greatly affects the distance. Good observations are essential for any further improvement of the elements.

EPHEMERIS					
$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 30	13.5	0.64	1899 30	20.7	0.77
1897 30	16.2	0.69	1900 30	22.7	0.79
1898 30	18.5	0.74			

Since the companion is now approaching its maximum distance, the star will be relatively easy for a number of years.

### $\omega$ LEONIS = $\Sigma$ 1356.

$\alpha = 9^h 23^m 1$  ,  $\delta = +9^\circ 30'$   
6, yellow , 7, yellow

*Discovered by Sir William Herschel, February 8, 1782*

OBSERVATIONS									
$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1782 86	110.9	—	1	Herschel	1841 18	354.5	—	1	Dawes
1803 09	130.9	—	2	Herschel	1841 35	194.0	0.3	1	Mädler
1825 21	153.9	0.97	5	Struve	1842 21	249.8	elong ?	1	Mädler
1830 24	146.5	wedge-shaped	1	Herschel	1842 31	302.3	0.3	4	O Struve
1832 25	163.4	0.51	3	Struve	1842 33	einfach		1	Mädler
1833 29	172.8	0.45	3	Struve	1843 30	einfach, rund		3	Mädler
1835 33	178.3	0.3 ±	3-1	Struve	1843 30	316.8	0.37	2	O Struve
1836 28	358.7	0.35 ±	3-2	Struve	1844 29	320.9	0.48	3	O Struve
1836 28	359.8	—	3	O Struve	1844 32	337.0	0.32	4	Mädler
1836 30	171.8	—	1	Mädler	1845 31	321.1	0.44	3	O Struve
1840.29	247.5	0.3	2	O Struve	1846 28	326.9	0.35	11	Mädler
1840 29	255	—	—	Dollen	1846 30	322.9	0.45	2	O Struve
1840 31	250.3	—	—	W Struve	1847 28	337.0	0.37	3	Mädler
					1847 33	328.8	0.53	2	O Struve



$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1848 32	332 1	0 43	4	O Struve	1870 24	44 4	0 25 $\pm$	5-1	Pence
1848 35	346 8	0 38	1	Madler	1870 28	53 6	0 58	2	O Struve
1849 32	331 8	0 43	3	O Struve	1870 30	37 9	0 27 $\pm$	2	Dunér
1850 63	335 8	0 49	3	O Struve	1871 16	52 6	cuneo	3	Dembowski
1851 23	342 6	0 35	9	Madler	1871 30	56 7	0 57	3	O Struve
1852 30	350 0	0 47	4	Madler	1871 31	42 7	0 3 $\pm$	1	Dunér
1852 66	339 1	0 46	3	O Struve	1872 18	66 3	0 48	2	Wilson
1853 18	343 3	0 45 $\pm$	2	Jacob	1872 31	58 8	0 52	2	O Struve
1853 27	346 3	0 35	7-6	Madler	1873 23	56 2	—	2	W & S
1853 96	350 0	0 4 $\pm$	2	Jacob	1873 29	57 0	0 4 $\pm$	1	Gledhill
1854 23	346 2	0 55	2	Dawes	1873 58	62 0	contatto	5	Dembowski
1854 28	348 3	0 53	10	Madler	1873 96	63 6	0 59	3	O Struve
1855 27	obl ?	—	2	Madler	1875 25	64 6	0 46	5	Dembowski
1855 32	348 7	0 47	2	O Struve	1875 26	62 7	0 49	7	Schiaparelli
1855 34	6 2	—	1	Winnecke	1875 31	66 8	0 43	5	Dunér
1856 20	obl ?	—	1	Madler	1875 32	66 4	0 59	3	O Struve
1856 42	1 0	0 36	10-7	Secchi	1876 16	69 4	0 44	2	Dembowski
1857 28	358 1	0 52	1	O Struve	1876 24	52 7	—	3	Doberck
1857 31	obl ?	—	1	Madler	1876 27	73 5	0 55 $\pm$	2	W & S
1857 54	4 3	0 43 $\pm$	3	Jacob	1876 29	65 6	0 57	2	O Struve
1858 28	16 2 ?	—	1	Madler	1877 21	77 2	0 88	1	Copeland
1859 25	16 7	0 35	4-3	Madler	1877 21	71 2	0 54	5-1	Plummer
1859 30	6 7	0 60	2	O Struve	1877 21	73 0	0 51	3-1	Doberck
1860 28	9 2	—	—	Winnecke	1877 27	70 7	0 47	7	Schiaparelli
1860 28	10 2	0 62	2	O Struve	1877 28	71 6	0 54	2	O Struve
1860 33	19 1	0 25	1	Madler	1877 36	76 6	0 41	2	Dembowski
1861 28	11 9	0 56	2	O Struve	1878 11	70 3	0 63	2	Burnham
1862 32	18 6	elong	2	Madler	1878 26	80 3	0 50	1	Doberck
1864 30	29 2	0 52	1	O Struve	1878 28	74 7	0 44	5	Dembowski
1864 89	24	cuneo	4	Dembowski	1878 63	77 7	0 60	3	O Struve
1865 67	23 0	0 50	8	Englemann	1878 95	74 4	0 41	6	Hall
1866 30	32 9	0 3	1	Secchi	1879 31	76 6	0 55	7	Schiaparelli
1867 08	109 4	elong	1	Winlock	1879 78	79 8	0 51	4	Burnham
1867 08	125 7	elong	1	Searle	1880 23	79 7	—	1	Bigourdan
1867 32	29 3	elong	1	Winlock	1880 26	95 2	obl	4	Jedrzejewicz
1867 87	Kreisrund		1	Vogel	1880 26	81 3	0 46	6	Hall
1868 21	15 6	elong	1	Peirce	1881 10	81 0	0 61	2	Bigourdan
1868 63	44 3	0 55	3	O Struve	1881 24	82 3	0 50	5-2	Doberck
1869 13	317 2	elong	1	Peirce	1881 26	98 7	obl	2	Jedrzejewicz
1869 26	36 7	elong	1	Pence	1881 28	83 7	0 68	2	O Struve
					1881 31	84 3	0 48	4	Hall
					1881 33	84 4	0 58	5	Schiaparelli

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1882 12	77 3	—	1	Doberck	1888 21	97 4	0 68	3	Tarant
1882 12	80 5	—	1	Copeland	1888 26	91 6	—	3	Smith
1882 23	80 0	0 56	7	Englemann	1888 27	98 5	0 68	6	Schiaparelli
1882 27	83 3	0 66	3	Doberck	1888 29	98 3	0 66	5	Hall
1882 30	84 1	0 49	4	Hall	1888 33	94 9	0 87	2	O Struve
1882 34	86 7	0 61	2	O Struve	1888 57	95 8	0 71	7	Lv
1882 36	90 0	0 55	4	Schiaparelli	1889 19	94 1	0 70	1	Hodges
1883 24	85 8	0 62	6	Englemann	1889 29	99 8	0 67	5	Hall
1883 31	90 5	0 65	6	Schiaparelli	1889 32	100 2	0 65	9	Schiaparelli
1883 34	90 9	0 62	3	Hall	1890 27	101 8	0 68	2	Comstock
1884 18	90 6	0 55	2	Perrotin	1890 31	101 2	0 64	4	Hall
1884 23	91 4	0 66	4	Englemann	1890 31	101 6	0 68	4	Schiaparelli
1884 26	87 6	0 71	2	O Struve	1891 21	102 1	0 76	2	Bigourdan
1884 30	91 3	0 58	5	Schiaparelli	1891 28	101 2	0 75	5	Hall
1884 32	93 3	0 55	4	Hall	1891 31	103 9	0 66	5	Schiaparelli
1884 34	90 6	—	10	Bigourdan	1892 25	102 4	0 77	3	Maw
1884 39	85 9	1 0 $\pm$	3-2	Sea & Sm	1892 26	104 9	0 72	7	Schiaparelli
1885 27	90 6	0 72	3	Englemann	1892 27	104 5	0 87	5	Lv & Col
1885 17	93 3	—	1	Doberck	1893 25	101 5	0 61	1	Comstock
1885 31	93 7	0 58	4	Schiaparelli	1893 28	105 7	0 70	9	Schiaparelli
1885 31	93 9	0 69	2	Tarant	1894 22	104 5	1 30	1	Bigourdan
1885 35	88 9	1 00 $\pm$	1	Smith	1894 23	106 5	0 67	3	Comstock
1885 72	90 9	0 70	2	Perrotin	1894 25	103 3	0 74	2	H C Wilson
1886 24	90 1	1 19	2-1	Sea & Sm	1894 25	106 7	0 75	8	Schiaparelli
1886 32	92 2	0 73	6	Englemann	1894 88	287 4	0 94	3	Barnard
1887 26	95 0	0 62	9	Schiaparelli	1895 24	106 1	0 67	3	Comstock
1887 30	95 6	0 53	4	Hall	1895 28	106 1	0 83	2	See
1887 37	94 0	—	1	Smith					

At the time of discovery SIR WILLIAM HERSCHEL estimated the position-angle\* to be between  $95^\circ$  and  $100^\circ$ , but later in the year found by measurement that the angle was  $110^\circ.9$ . The pair was soon found to be in slow orbital motion, and in 1804 HERSCHEL concluded that since 1782 the change in angle had amounted to  $+19^\circ 59'$ , and that the distance had sensibly increased. When the star was thus recognized as binary, it naturally claimed the attention of the principal double-star observers, and accordingly since the time of STRUVE, a long list of measures has been secured. But while the closeness of the companion in most parts of the apparent ellipse has made the pair a classic test-object for the dividing power of small telescopes, it has, on the other hand, rendered micrometrical measurement extremely difficult, and some of the observations are therefore far from satisfactory. In spite of the fact that the measures

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\* *Astronomische Nachrichten*, 3311

are sometimes difficult to reconcile, the angles and distances of the best observers, when properly combined, in conjunction with the important principle of the preservation of areas, enable us to fix the apparent ellipse with a relatively high degree of precision, and the resulting elements are found to be incapable of any large variation. The orbit is based chiefly upon the observations of HERSCHEL, STRUVE, O. STRUVE, DAWES, DEMBOWSKI, BURNHAM, HALL, SCHIAPARELLI, and the measures which the writer recently secured at the McCormick Observatory in Virginia. The elements of  $\omega$  Leonis are.

$$\begin{aligned} P &= 116.20 \text{ years} & \Omega &= 146^\circ 70' \\ T &= 1842.10 & i &= 63^\circ 47' \\ e &= 0.537 & \lambda &= 124^\circ 22' \\ a &= 0''.88241 & n &= +3^\circ 09.81 \end{aligned}$$

#### Apparent orbit.

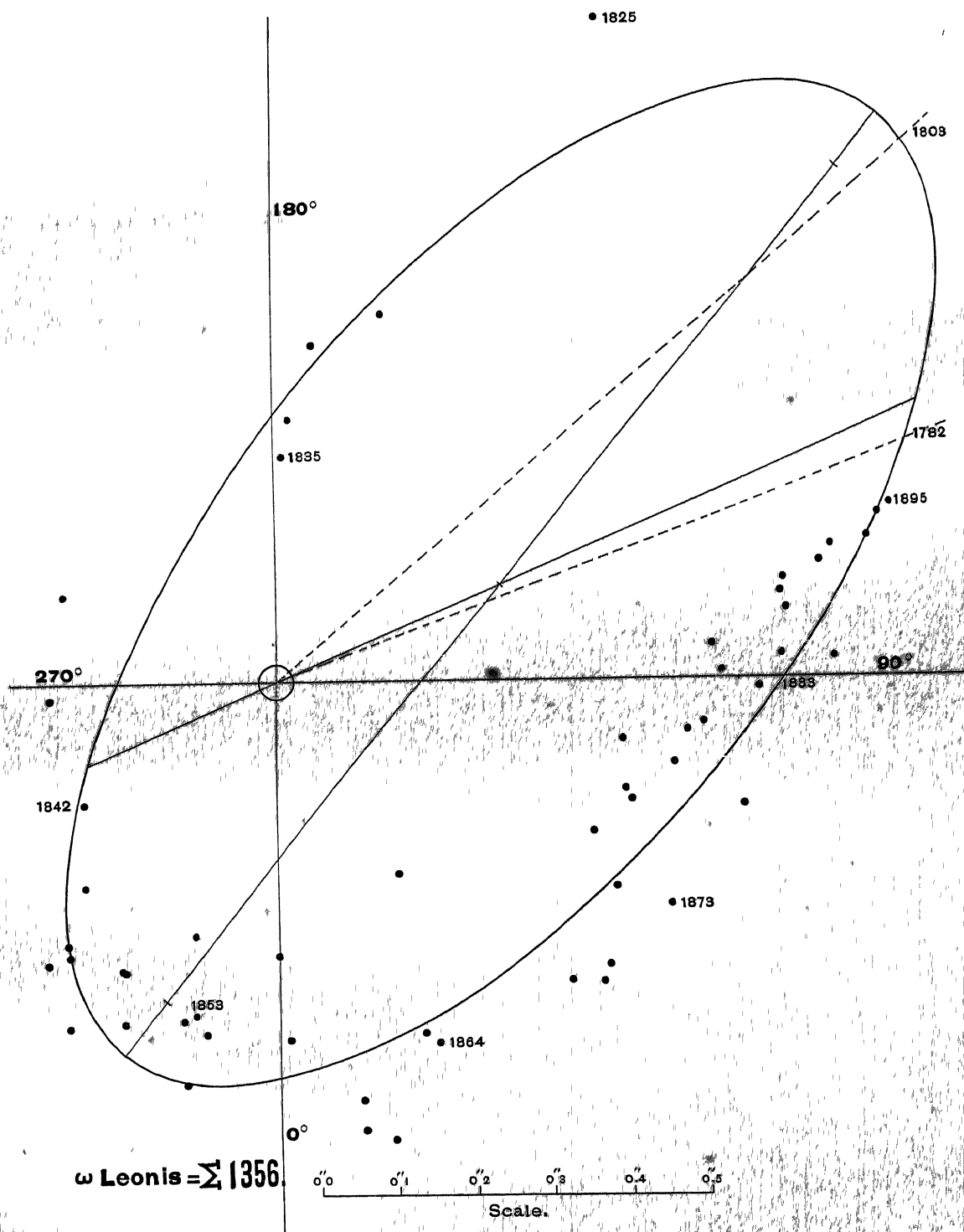
$$\begin{aligned} \text{Length of major axis} &= 1''.576 \\ \text{Length of minor axis} &= 0''.738 \\ \text{Angle of major axis} &= 141^\circ 1' \\ \text{Angle of periastron} &= 293^\circ 4' \\ \text{Distance of star from centre} &= 0''.317 \end{aligned}$$

Several astronomers have previously investigated the orbit of this star; the following table gives the elements hitherto published.

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
82.533	1849.76	0.6434	0.857	135.2	46.57	185.45	Madler, 1841	Donp Obs IX, 198
117.577	1843.408	0.6256	0.8505	159.83	50.61	120.45	Madler, 1846	First Syst I, p 250
133.35	1846.44	0.3605	0.703	111.85	57.23	217.37	Klinkerf 1856	A N 990
227.77	1841.40	0.7225	1.307	169.2	60.22	81.17	Klinkerf 1856	A N 990
142.41	1843.39	0.6286	1.092	162.22	54.42	107.15	Klinkerf 1858	A N 1127
136.4	1844.2	0.62	1.05	160.5	52.4	113.4	Klinkerfues	Theor Astron p 395
107.62	1842.77	0.5028	—	151.57	65.37	122.9	Doberck, 1876	A N 2078
110.82	1841.81	0.536	0.890	148.77	64.08	121.07	Doberck, 1876	A N 2095
114.55	1841.57	0.5510	0.85	149.25	64.08	122.3	Doberck	
115.30	1841.99	0.5379	0.864	147.1	64.15	122.9	Hall, 1892	A J 269
115.87	1842.16	0.533	0.8753	145.9	63.05	125.32	See, 1894	A N 3311

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1782.86	110.9	112.1	—	0.89	-1.2	—	1	Herschel
1803.09	130.9	130.3	—	1.08	+0.6	—	2	Herschel
1825.21	153.9	150.4	0.97	0.81	+3.5	+0.16	5	Struve
1832.25	163.4	164.9	0.51	0.52	-1.5	-0.01	3	Struve
1833.29	172.8	168.8	0.45	0.47	+4.0	-0.02	3	Struve
1835.33	178.3	179.9	0.3±	0.35	-1.6	-0.05	3-1	Struve
1836.28	176.8	187.8	0.35	0.30	-11.0	+0.05	7-2	2. 3-2, OΣ 3-0, Madler 1.0
1840.29	247.5	263.8	0.3	0.21	-16.3	+0.09	2	O. Struve
1841.26	274.2	281.6	0.3	0.24	-7.4	+0.06	2-1	Dawes 1-0, Madler 1
1842.31	302.3	295.8	0.3	0.28	+6.5	+0.02	4	O. Struve



$t$	$\theta_o$	$\theta_e$	$\rho_o$	$\rho_e$	$\theta_o - \theta_e$	$\rho_o - \rho_e$	$n$	Observers
1843 30	316 8	305 2	0 37	0 33	+11 6	+0 04	2	O Struve
1844 31	320 9	312 3	0 48	0 38	+ 8 6	+0 10	3	O Struve
1845 31	321 1	317 9	0 44	0 42	+ 3 2	+0 02	3	O Struve
1846 30	322 9	322 6	0 45	0 45	+ 0 3	0 00	2	O Struve
1847 31	328 8	326 8	0 53	0 48	+ 2 0	+0 05	2	O Struve
1818 32	332 1	330 5	0 43	0 50	+ 1 6	-0 07	4	O Struve
1849 32	331 8	334 0	0 43	0 52	- 2 2	-0 09	3	O Struve
1850 63	335 8	338 2	0 49	0 53	- 2 4	-0 04	3	O Struve
1851 23	342 6	340 1	0 35	0 53	+ 2 5	-0 18	9	Madler
1852 48	344 5	344 1	0 46	0 54	+ 0 4	-0 08	7	Madler 4, O Struve 3
1853 47	346 5	347 0	0 45	0 54	- 0 5	-0 09	11-10	Jacob 2, Madler 7-6, Jacob 2
1854 25	347 2	349 4	0 54	0 54	- 2 2	0 00	12	Dawes 2, Madler 10
1855 32	348 7	353 1	0 47	0 53	- 4 4	-0 06	2	O Struve
1856 42	1 0	356 3	0 36	0 53	+ 4 7	-0 17	10-7	Secchi
1857 41	2 4	359 5	0 47	0 52	+ 2 9	-0 05	4	O Struve 1, Jacob 3
1859 27	11 7	5 6	0 60	0 51	+ 6 1	+0 09	6-5	Madler 4-3, O Struve 2
1860 30	14 6	9 2	0 62	0 50	+ 5 4	+0 12	3	O Struve 2, Madler 1
1861 28	11 9	12 8	0 56	0 50	- 0 9	+0 06	2	O Struve
1864 59	24 0	25 0	0 52	0 48	- 1 0	+0 01	4-1	O Struve 1, Dembowski 1-0
1865 67	23 0	28 1	0 50	0 48	- 5 1	+0 02	8	Englemann
1866 30	32 9	31 7	0 30	0 48	+ 1 2	-0 18	1	Secchi
1868 63	44 3	40 7	0 55	0 48	+ 3 6	+0 07	3	O Struve
1870 28	47 3	47 1	0 68	0 49	+ 0 2	+0 18	9-5	Pence 5-1, O Struve 2, Dunér 2
1871 30	49 7	51 0	0 57	0 49	- 1 3	+0 08	7-4	Dembowski 3-0, O $\Sigma$ 3, Du 1
1872 31	58 8	54 7	0 52	0 50	+ 4 1	+0 02	2	O Struve
1873 62	60 3	59 2	0 52	0 51	+ 1 1	+0 01	11-4	W & S 2-0, Gl 1, Dem 5-0,
1875 27	64 7	61 9	0 46	0 52	- 0 2	-0 06	17	Dem 5, Sch 7; Du 5 [O $\Sigma$ 3
1876 21	71 4	67 7	0 49	0 53	+ 3 7	-0 04	4	Dem 2, W. & S 2 [Cop 0-1
1877 25	72 9	71 3	0 56	0 55	+ 1 6	+0 01	17-12	Pl 5-1, Dk 3-1, Sch 7, Dem 2,
1878 40	74 9	74 8	0 63	0 56	+ 0 1	+0 07	14	S 2; Dk 1; Dem 5; Hall 6
1879 54	78 2	77 7	0 53	0 58	+ 0 5	-0 05	11	Schiaparelli 7, Burnham 4
1880 24	80 2	79 7	0 46	0 59	+ 0 5	-0 13	7-6	Bigourdan 1-0, Hall 6
1881 24	83 0	82 1	0 54	0 60	+ 0 9	-0 06	16-13	Big 2, Dk 5-2, Hl 4, Sch 5
1882 29	84 4	84 7	0 56	0 62	- 0 3	-0 06	18	En 7, Dk 3, Hl 4, Sch 4
1883 30	89 2	87 1	0 63	0 63	+ 2 1	0 00	15	En 6, Sch 6, Hl 3 [Big 10-0
1884 27	91 4	89 2	0 58	0 65	+ 2 2	-0 07	25-15	Per 2, En 4, Sch 5, Hall 4,
1885 37	92 9	90 9	0 66	0 66	+ 2 0	0 00	9-8	Dk 1-0, Sch 4, Tai 2, Per 2
1886 32	92 2	93 3	0 73	0 68	- 1 1	+0 05	6	Englemann
1887 31	94 9	95 2	0 57	0 70	- 0 3	-0 13	14-13	Sch 9, Hall 4; Smith 1-0
1888 25	98 1	96 9	0 67	0 72	+ 1 2	-0 05	14	Tarrant 3, Sch 6, Hall 5
1889 30	100 0	98 6	0 66	0 73	+ 1 4	-0 07	14	Hall 5, Schiaparelli 9
1890 30	101 5	100 3	0 67	0 75	+ 1 2	-0 08	10	Hall 4, Comstock 2, Sch 4
1891 27	102 4	101 8	0 72	0 77	+ 0 6	-0 05	12	Hall 5, Bigourdan 2, Sch 5
1892 26	103 9	103 3	0 79	0 79	+ 0 6	0 00	15	Maw 3, Sch 7, Lv & Col 5
1893 26	103 6	104 8	0 74	0 80	- 1 2	-0 06	10	Comstock 1-0, Schiaparelli 9-5
1894 36	105 6	106 3	0 81	0 82	- 0 7	-0 01	17-13	Big 1-0, Com 3-0, H C W 2,
1895 28	106 1	107 5	0 83	0 84	- 1 4	-0 01	2	See [Sch 8, Bar 3

The elements given above confirm the substantial accuracy of the orbit found by HALL, and represent the observations as a whole remarkably well. The changes which future observations will introduce are likely to be very small.

The following is an ephemeris for the next five years:

EPHEMERIS					
$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 28	108 7	0 85	1899 28	112 4	0 90
1897 28	110 0	0 87	1900 28	113 5	0 91
1898 28	111 2	0 88			

It is to be noted that the distance is steadily increasing, and that for many years the pair will be relatively easy. A number of observers of late years have sensibly underestimated the distance. Owing to the closeness of  $\omega$  *Leonis* and its slow orbital motion, one would naturally think that this brilliant system probably has a small mass, and is comparatively near us in space, for if the mass be large, the slow motion of so close a system would indicate that it is very remote, and the resulting brightness of the components would be very great. The eccentricity of this orbit is so well determined that the value given above can hardly be in error by so much as 0.01, and a correction of half this amount does not seem probable.

### $\varphi$ URSAE MAJORIS = $\sigma$ 208.

$\alpha = 9^h 45^m 3$  ,  $\delta = +54^\circ 33'$   
 5 5, yellowish , 5 5, yellowish

*Discovered by Otto Struve in 1842*

OBSERVATIONS									
$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1842 30	4 2	0 42	1	Mädler	1852 39	16 1	0 32	2	O Struve
1842 35	8 5	0 52	2	O Struve	1852 40	209 8	0 25	4	Mädler
1843 37	5 6	0 48	3	Mädler	1853 40	16 7	0 34	3	O Struve
1843 47	188 5	0 39	1	O Struve	1854 28	25 9	0 4 $\pm$	1	Dawes
1844 26	186 6	0 51	1	O Struve	1854 37	23 3	0 42	1	O Struve
1846 01	193 8	0 45	3-2	Mädler	1857 34	30 6	0 3	1	Secchi
1846 37	9 2	0 42	1	O Struve	1858 41	36 1	0 40	3	O Struve
1847 41	196 8	0 30	2	Mädler	1859 37	43 9	0 33	1	Winnecke
1847 41	12 1	0 36	1	O Struve	1859 39	37 6	0 35	2	O Struve
1848 40	10 4	0 35	2	O Struve	1861 40	55 0	0 44	1	Winnecke
1850 39	15 0	0 33	2	O Struve	1861 41	48 5	0 37	2	O Struve
1851 39	207 2	0 31	4	Mädler	1862 39	46 8	0 38	1	O Struve
1851 40	13 7	0 33	2	O Struve	1864 43	48 5	0 27	1	O Struve

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1866 27	46 5	<0 4	1	Englemann	1882 19	139 0	<0 2	3	Englemann
1866 42	48 2	0 24	1	O Struve	1882 34	342 0?	—	1	O Struve
1869 40	45 0	oblong	2	Dunéi	1887 43	218 9	0 23	4	Schiaparelli
1870 42	81 5	oblong	2	Dunéi	1888 43	220 3	cuneiforme	1	O Struve
1872 41	77 7	0 23	2	O Struve	1889 39	214 0	cent elong	1	O Struve
1873 44	87 5	—	1	Lindemann	1892 13	250 8	0 24	3	Bunham
1873 45	96 6	oblong	3	O Struve	1892 31	60 4	0 29	1	Bigourdan
1873 47	95 4	—	1	H Bruhns	1892 58	single	—	1	Comstock
1875 47	115 1	oblong	2	O Struve	1893 36	339 55	0 30	1	Schiaparelli
1876 42	54 0	elongated?	1	O Struve	1894 25	round	—	1	Comstock
1877 43	single	—	1	O Struve	1894 40	82 7	—	3	Bigourdan
1879 44	single	—	1	O Struve	1895 73	276 2	0 29	3	See

Although this close and rapid binary was discovered by OTTO STRUVE, the first observation was secured by MADLER, whose measures supplement STRUVE's work in a very happy manner, and enable us to fix the original position of the companion with much precision. For a long time these two astronomers alone followed the motion of the system, but in later years it has received occasional attention from several other observers. The stars are nearly equal in magnitude, and hence a few of the recorded angles require a correction of  $180^\circ$ . The arc already described amounts to about  $270^\circ$ , and as this covers the most critical parts of the orbit, most of the elements are defined with the desired precision. The chief difficulty encountered by observers lies in the closeness of the components, which places them beyond the reach of small, and even of moderate-sized, telescopes. The pair is, however, gradually widening out, and in a few years will be much more accessible to measurement.

The following elements of this star have been published by previous computers.

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
115 4 91 9	1877 12 1885 4	0 788 0 45	0 54 0 29	105 3 165 7	57 95 34 7	72 1 19 0	Casey, 1882 Glas, 1892	A N 2417 A N 3119

Using all the available measures, we find the following elements

$$\begin{aligned}
 P &= 97.0 \text{ years} & \Omega &= 160^\circ 3 \\
 T &= 1884.0 & i &= 30^\circ 5 \\
 e &= 0.440 & \lambda &= 15^\circ 9 \\
 a &= 0''.3443 & n &= +3^\circ 7114
 \end{aligned}$$

## Apparent orbit:

Length of major axis	= $0'' 69$
Length of minor axis	= $0'' 53$
Angle of major axis	= $167^\circ 6$
Angle of periastron	= $174^\circ 1$
Distance of star from centre	= $0'' 149$

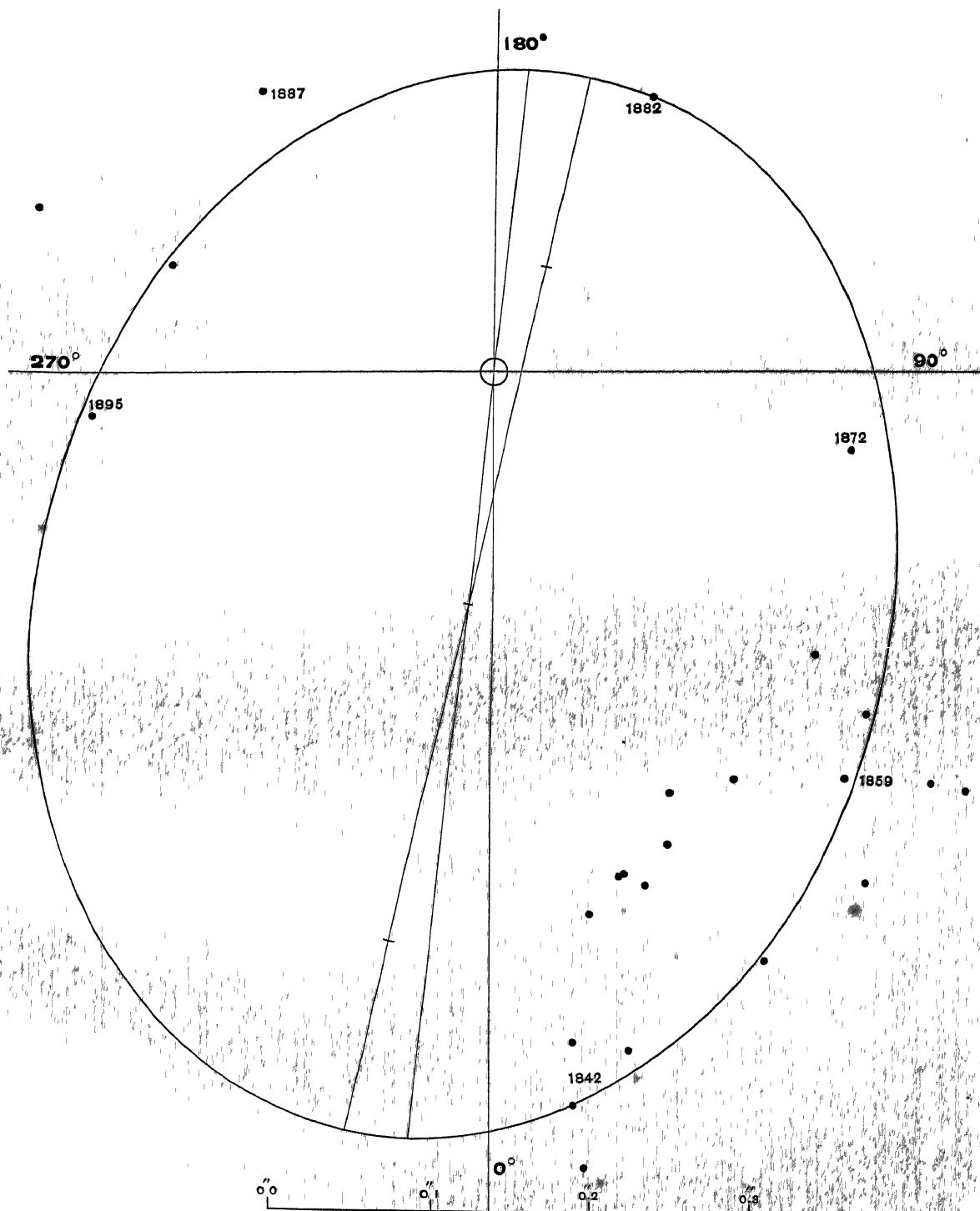
It will be seen that this orbit is essentially similar to that found by GLASENAPP. The table of computed and observed places shows so satisfactory an agreement for this close and difficult object that we may regard these elements as substantially correct, and confidently conclude that such alterations as future observations may render necessary will be of minor importance

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1842 32	6 3	4 0	0 47	0 48	+ 2.3	-0 01	3	Mädler 1, O Struve 2
1843 42	7 0	5 7	0 43	0 47	+ 1 3	-0 04	4	Mädler 3, O Struve 1
1844 26	6 6	7 0	0 51	0 47	- 0 4	+0 04	1	O Struve
1846 19	11 5	10 1	0 44	0 46	+ 1 4	-0 02	4-3	Mädler 3-2; O Struve 1
1847 41	14 4	12 0	0 33	0 45	+ 2 4	-0 12	3	Mädler 2, O Struve 1
1848 40	10 4	13 8	0 35	0 45	- 3 4	-0 10	2	O Struve
1850 39	15 0	17 2	0 33	0 43	- 2 2	-0 10	2	O Struve
1851 40	20 4	19 1	0 32	0 43	+ 1 3	-0 11	6	Mädler 4, O Struve 2
1852 40	22 9	20 9	0 29	0 42	+ 2 0	-0 13	6	O Struve 2, Mädler 4
1853 40	16 7	22 9	0 34	0 41	- 6 2	-0 07	3	O Struve
1854 32	24 6	24 7	0 41	0 41	- 0 1	$\pm 0 00$	2	Dawes 1, O Struve 1
1857 34	30 6	31 3	0 30	0 38	- 0 7	-0 08	1	Secchi
1858 41	36 1	33 9	0 40	0 37	+ 2 2	+0 03	3	O Struve
1859 38	40 8	36 2	0 34	0 36	+ 4 6	-0 02	3	Winnecke 1, O Struve 2
1861 40	48 5	41 8	0 40	0 34	+ 6 7	+0 06	2-3	Winnecke 0-1, O Struve 2
1862 39	46 8	44 6	0 38	0 33	+ 2 2	+0 05	1	O Struve
1864 43	48 5	51 2	0 27	0 31	- 2 7	-0 04	1	O Struve
1866 34	47 4	61 8	0 32	0 29	-14 4	+0 03	2	Englemann 1, O Struve 1
1869 40	45 0	70 0	oblong	0 27	-25 0	—	2	Dunér
1870 42	81 5	75 6	oblong	0 26	+ 5 9	—	2	Dunér
1872 41	77 7	86 4	0 23	0 24	- 8 7	-0 01	2	O Struve
1873 46	96 0	92 4	oblong	0 24	+ 3 6	—	4	O Struve 3, H Bruhns 1
1875 47	115 1	105 1	oblong	0 22	+10 0	—	2	O Struve
1877 43	single	118 9	single	0 21	—	—	1	O Struve
1879 44	single	134 7	single	0 21	—	—	1	O Struve
1882 26	150 5	149 6	0 20	0 20	+ 0 9	$\pm 0 00$	4-3	Englemann 3, O Struve 1-0
1887 43	218 9	206 6	0 23	0 19	+12 3	0 04	4	Schiaparelli
1888 43	220 3	216 1	eune	0 19	+ 4 2	—	1	O Struve
1889 39	214 0	225 2	elong	0 19	-11 2	—	1	O Struve
1892 13	250 8	248 3	0 21	0 21	+ 2 5	$\pm 0 00$	3-2	Burnham
1893 36	249 6	257 1	0 30	0 22	- 7 5	+0 08	1	Schiaparelli
1894 40	262 7	264 0	—	0 23	- 1 3	—	3-0	Bigourdan
1895 73	276 2	271 6	0 25	0 25	+ 4 6	$\pm 0 00$	3-1	See

Some changes will doubtless be required in all the elements, but the two elements of chief interest, the period and the eccentricity, will hardly be varied





by more than five years, and  $\pm 0.03$  respectively. It is desirable to have the theory of this system carefully confirmed, and observers with good telescopes will find it worthy of regular attention. The motion is still tolerably rapid, but is gradually slowing up, as will be seen in the following ephemeris.

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 40	275.2	0.26	1899 40	288.8	0.29
1897 40	280.1	0.27	1900 40	292.7	0.30
1898 40	284.6	0.28			

ξ URSAE MAJORIS =  $\Sigma$ 1523.

$\alpha = 11^h 12^m 9^s$ ,  $\delta = +32^\circ 6'$   
4, yellow, 5, yellowish

*Discovered by Sir William Herschel, May 2, 1780*

OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1781 97	143.8	4.±	—	Herschel	1833 14	189.9	2.06	8-2	Herschel
1802 09	97.5	—	—	Herschel	1833 23	189.8	1.98	4	Dawes
1804 09	92.6	—	—	Herschel	1833.38	188.2	1.69	5	Struve
1819 10	284.5	—	2	Struve	1834 44	184.1	1.87	2	Struve
1820 13	276.3	—	3	Struve	1834 50	182.5	2.17	4-1	Mädler
1821 78	264.7	1.92	3	Struve	1835 27	176.4	1.93	1	Madler
1825 22	244.5	2.44	6-4	South	1835 41	180.2	1.76	5	Struve
1826 20	238.7	1.77	3	Struve	1835 56	175.8	—	4	Mädler
1827 27	228.3	1.71	4	Struve	1836 28	171.4	1.92	1	Dawes
1828 37	224.0	2.01	2	Herschel	1836 28	172.7	1.94	7-2	Mädler
1829 02	219.0	2.00	1	Herschel	1836 44	171.2	1.97	4	Struve
1829 35	213.6	1.67	7	Struve	1837 47	165.3	1.93	3	Struve
1830 18	211.4	—	10±	Herschel	1838 43	160.4	2.26	9	Struve
1830 98	200.9	2.23	10±	Herschel	1839 47	157.9	1.89	—	Galle
1831 08	201.5	1.86	5	Bessel	1840 25	152.2	2.08	40-31 <sub>Obs</sub>	Kaiser
1831 23	201.1	1.93	6-4	Herschel	1840 29	150.8	2.44	6-4	Dawes
1831 34	201.9	1.98	17-4	Dawes	1840 40	153.6	2.28	6	O Struve
1831 44	203.8	1.71	5	Struve	1840 44	—	2.29	—	W Struve
1832 16	198.2	—	5	Herschel	1841 21	148.0	2.40	4-3	Dawes
1832 27	196.7	1.76	10-8	Dawes	1841 29	150.2	2.44	7-6	Mädler
1832 41	195.9	1.75	5	Struve	1841 40	147.5	2.23	6	O Struve
					1842 24	147.0	2.41	4	Madler
					1842 27	144.8	2.44	4	Dawes
					1842 40	147.5	2.34	4	O Struve

<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1843 28	142 2	2 48	7	Dawes	1854 35	116 3	2 90	15	Madler
1843 38	143 7	2 37	1	Madler	1854 36	115 9	2 96	3	Dawes
1843 48	141 9	2 71	9	Schluter	1854 37	115 6	3 46	1	Luthe
1844 34	140 4	2 45	3	O Struve	1854 38	115 9	2 90	4	O Struve
1844 34	141 0	2 60	11-10	Madler	1854 51	116 6	3 06	5	Dembowski
1844 36	141 0	2 47	—	Liapunow	1855 09	116 6	—	12	Powell
1844 36	144 5	2 65	—	Dollen	1855 15	115 6	3 23	7	Dembowski
1845 46	138 1	2 51	2	O Struve	1855 29	114 3	2 96	1	Secchi
1845 82	135 8	3 11	2	Jacob	1855 33	114 1	2 98	1	Winnecke
1846 37	137 2	2 56	4	O Struve	1855 44	115 7	2 87	2	Madler
1847 30	131 6	2 58	1	Dawes	1855 44	115 2	2 85	3	O Struve
1847 38	132 0	2 71	10	Madler	1856 05	114 2	—	6	Powell
1847 41	133 2	2 61	3	O Struve	1856 18	111 9	3 12	3	Jacob
1848 13	129 5	2 70	1	Dawes	1856 26	113 9	3 13	4	Secchi
1848 19	129 3	2 94	3	Dawes	1856 33	114 1	2 99	3	Winnecke
1848 31	129 7	2 71	4	Madler	1856 34	112 3	3 15	7	Dembowski
1848 41	130 0	2 66	5	O Struve	1856 42	112 7	2 98	13	Madler
1848 45	129 1	2 90	2	G <sup>c</sup> Bond	1856 82	110 9	2 99	2	Jacob
1849 30	126 6	3 01	5	Dawes	1857 36	109 7	3 11	2	Secchi
1849 37	127 6	2 78	4	O Struve	1857 43	109 6	2 74	8	Madler
1850 01	127 0	2 65	1	Johnson	1857 46	110 2	2 97	3	O Struve
1850 30	124 2	3 37	2	Jacob	1858 00	108 1	2 90	4	Jacob
1850 39	124 1	2 68	4	O Struve	1858 20	108 1	2 85	2	Morton
1850 85	124 6	2 85	2	Madler	1858 20	108 1	3 10	6	Dembowski
1851 19	123 1	2 83	6-5	Fletcher	1858 39	108 9	2 97	3	O Struve
1851 27	123 3	2 93	6	Madler	1858 43	108 8	2 96	5	Madler
1851 31	122 9	2 98	2	Dawes	1859 39	106 1	2 94	6-3	Madler
1851 41	123 0	2 80	5	O Struve	1859 57	104 9	2 84	5	O Struve
1851 79	122 1	2 91	9	Madler	1860 08	105 2	2 84	2	Morton
1852 13	122 3	2 90	7	Miller	1860 16	104 1	2 99	6-5	Powell
1852 20	119 8	2 92	6	Fletcher	1860 32	105 2	2 88	2-1	Dawes
1852 29	120 9	3 01	1	Jacob	1860 36	102 8	—	—	Oblomievsky
1852 34	120 8	2 73	6	Madler	1860 36	103 6	—	—	Schiaparelli
1852 36	118 2	2 85	2	Morton	1860 36	103 9	—	—	Wagner
1852 38	120 0	—	1	Dawes	1860 39	104 1	3 15	2	Madler
1852 40	120 6	2 76	4	O Struve	1861 14	100 6	3 09	6-2	Powell
1853 19	118 8	3 01	4	Miller	1861 40	101 1	2 70	4	O Struve
1853 20	119 5	3 01	2	Jacob	1861 42	100 8	2 83	8	Madler
1853 20	119 2	—	6	Powell	1861 76	100 4	3 04	5	Auwers
1853 23	118 9	2 98	6	Fletcher	1862 36	100 1	2 95	4	Madler
1853 40	119 0	2 88	4	O Struve	1862 39	99 3	2 62	4	O Struve
1853 45	118 8	2 94	13	Madler	1862 42	100 2	3 20	—	Oblomievsky
1854 12	117 2	3 1	10-1	Powell	1863 20	89 5	2 61	2	Main
					1863 23	96 6	2 55	19	Dembowski
					1863 46	95 7	2 55	2	O Struve

<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1864 31	94 0	2 29	9	Dembowski	1873 28	2 2	0 9	2-1	W & S
1864 38	92 9	2 40	3	Secchi	1873 33	358 9	0 98	10	Dembowski
1864 42	94 2	2 33	3	O Struve	1873 42	358 4	0 88	1	Dunér
1864 46	92 8	2 44	1	Englemann	1873 43	358 4	0 96	5	O Struve
1864 50	93 9	2 42	1	Dawes	1873 78	347 1	0 83	3	Gledhill
1865 12	91 4	2 44	19	Englemann	1874 13	338 4	1 00	3	Gledhill
1865 30	90 1	2 17	10	Dembowski	1874 20	336 2	0 92	2-1	W & S
1865 51	89 9	2 53	4	Secchi	1874 21	337 0	1 48	1	Ferrari
1866 25	92 8	2 72	4-3	Leyton Obs	1874 26	335 5	—	2	Leyton Obs
1866 30	86 5	2 26	3	Secchi	1874 35	333 6	1 02	6	Dembowski
1866 30	86 8	2 05	10	Dembowski	1874 41	338 1	1 03	3	O Struve
1866 39	86 7	2 09	5	Kaiser	1874 45	335 1	0 96	4-5	Dunér
1866 40	85 4	2 12	3	O Struve	1875 27	317 6	1 09	8	Dembowski
1866 45	87 8	2 08	5	Kaiser	1875 31	317 5	1 31	7	Schiaparelli
1866 49	81 1	—	2	Guldén	1875 34	317 2	1 28	4-3	W & S
1866 49	83 6	—	2	Abbe	1875 45	315 8	1 10	4	O Struve
1866 49	87 0	—	2	Foss	1875 45	316 4	1 12	14	Dunér
1867 21	75 5	2 89	1	Winlock	1875 99	311 7	—	1	Doberck
1867 23	82 2	—	1	Leyton Obs	1876 27	306 3	1 75	13-2	Doberck
1867 31	82 2	1 90	8	Dembowski	1876 30	304 8	1 24	7	Dembowski
1867 47	81 0	1 91	2	O Struve	1876 34	334 5	1 65	1	Leyton
1868 14	80 8	1 76	1	Searle	1876 36	305 5	1 45	3	W & S
1868 23	79 1	2 49	2	Leyton Obs	1876 42	303 5	1 35	3	O Struve
1868 30	77 1	1 72	8	Dembowski	1876 46	301 2	1 52	5-4	Plummer
1868 39	77 1	1 77	1	Main	1877 20	297 0	1 57	7-6	Plummer
1868 42	72 6	1 63	4	O Struve	1877 26	294 9	1 42	6	Dembowski
1869 40	68 6	1 34	11	Dunér	1877 26	294 2	1 76	10-9	Doberck
1869 42	69 9	—	—	Krüger	1877 34	293 0	1 52	8	Schiaparelli
1870 18	59 2	1 32	4	O Struve	1877 40	294 6	1 52	3	W & S
1870 24	57 3	1 39	9	Dembowski	1877 43	291 6	1 45	2	O Struve
1870 33	57 2	1 35	2	Gledhill	1877	291 5	1 35	1	Pritchett
1870 35	70 8	—	—	Leyton Obs	1877 41	294 5	2 10	2-1	Hall
1870 43	53 8	1 20	9	Dunér	1878 20	—	2 01	4	Doberck
1871 22	47 7	1 20	8	Dembowski	1878 32	286 8	1 66	6	Dembowski
1871 31	47 7	1 2	2	Gledhill	1878 36	286 3	1 50	3	O Struve
1871 39	66 2	—	—	Leyton Obs	1879 27	284 2	1 82	3	Hall
1871 40	45 7	1 12	2	O Struve	1879 33	280 3	1 79	7	Schiaparelli
1871 47	40 0	1 02	11-10	Dunér	1879 41	278 5	1 74	2	O Struve
1871 48	43 9	1 1	1	Wilson	1880 13	278 2	2 07	6	Franz
1872 05	30 7	1 05	2	Gledhill	1880 27	276 2	1 80	6	Hall
1872 26	23 2	1 09	7-6	W & S	1880 28	274 9	2 05	5	Doberck
1872 33	19 3	1 07	6	Knott	1880 39	273 0	1 90	2	Bigourdan
1872 35	68 0	1 28	1-2	Leyton Obs	1880 48	272 0	1 82	3	Jedrzejewicz
1872 41	17 8	0 97	10	Dembowski	1881 23	270 3	1 84	4	Doberck
1872 46	16 6	0 94	14	Dunér	1881 31	268 0	1 80	2-1	Bigourdan
1872 48	15 4	0 98	8	Ferrari					

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1881 34	269 2	1 84	7	Hall	1889 37	216 9	1 81	3	Maw
1881 35	269 7	1 66	4-3	Bunham	1889 39	218 5	1 64	2	O Struve
1881 36	268 9	1 92	6	Schiaparelli	1889 40	217 4	1 68	5	Tarrant
1882 25	263 5	1 99	6	Hall	1890 27	210 0	1 64	6	Hall
1882 25	259 4	2 00	4-3	Doberck	1890 36	209 7	1 61	7	Schiaparelli
1882 25	262 1	1 99	4	Englemann	1890 40	209 1	1 96	3	Maw
1882 39	261 1	1 93	9	Schiaparelli	1890 42	313 3	1 54	1	Hayn
1882 42	260 4	1 72	3	O Struve	1890 45	209 4	1 87	2	Knorre
1883 32	257 8	2 00	6	Englemann	1891 13	202 6	1 78	1	Bigoudan
1883 38	257 1	1 88	11	Schiaparelli	1891 15	202 1	1 63	1	Flint
1883 40	258 2	1 95	6	Hall	1891 30	200 6	1 59	6	Hall
1883 41	258 1	1 88	3	Jedrzejewicz	1891 31	204 1	1 92	1	Knorre
1884 28	249 2	1 69	3-4	Periotin	1891 41	199 8	1 60	10	Schiaparelli
1884 32	249 0	1 89	7	Hall	1891 47	199 9	1 74	3	Maw
1884 35	247 6	—	14	Bigoudan	1892 32	196 9	1 75	4	Maw
1884 38	249 3	1 82	11	Schiaparelli	1892 35	195 1	1 57	11-10	Schiaparelli
1884 41	249 6	1 92	4	Englemann	1892 36	194 1	1 78	1	Bigoudan
1884 44	249 2	1 56	1	O Struve	1892 39	197 4	1 70	6	Knorre
1885 35	244 7	1 80	5	Hall	1892 45	196 6	1 60	2	Leavenworth
1885 36	245 2	2 12	4	Englemann	1892 46	197 5	1 57	4	Comstock
1885 39	245 4	1 72	10	Schiaparelli	1893 27	188 0	2 05	2	Knorre
1885 41	243 4	1 87	3	Tarrant	1893 33	187 3	1 72	4	Maw
1886 37	237 3	1 63	5	Hall	1893 36	186 4	1 65	7	Schiaparelli
1886 37	237 4	2 06	8	Englemann	1893 37	186 1	1 75	1	Dav Photog
1886 45	237 0	1 80	3	Jedrzejewicz	1894 22	183 2	1 79	3	Comstock
1887 04	226 9	—	1	Glasenapp	1894 30	181 1	2 00	1	Ebell
1887 35	230 3	1 61	5	Hall	1894 32	182 8	1 79	1	H C Wilson
1887 36	230 9	1 65	12	Schiaparelli	1894 34	183 6	1 84	2	Knorre
1888 28	222 2	1 68	6	Hall	1894 35	183 0	1 87	3	Maw
1888 29	222 7	1 63	4	Schiaparelli	1894 47	181 7	1 78	8	Bigoudan
1888 43	226 2	1 61	1	O Struve	1894 56	184 6	1 77	1	Glasenapp
1888 51	222 7	2 20	4	Maw	1895 30	176 5	1 93	3	Comstock
1889 28	218 1	2 09	2-1	Glasenapp	1895 31	176 0	1 78	1	Dav Photog
1889 29	216 5	1 68	5	Hall	1895 32	176 0	1 98	1	Lewis
1889 36	215 9	1 61	9	Schiaparelli	1895 33	176 6	1 95	3	Sec
					1895 46	175 9	1 79	4	Schwarzschild

This celebrated system was first measured by HERSCHEL in 1781. A repetition of the measures in 1802 and 1804 showed\* that the smaller star had a rapid relative motion (*Phil. Trans.* 1804, p 363), and indeed gave indications for the first time that the motion of certain double stars is of an orbital nature.  $\xi$  *Ursae Majoris* thus enjoys the unique distinction of having first aroused interest in observational proof of the universality of the Newtonian law. This

\* *Astronomische Nachrichten*, 3323

star also led SAVARY in 1827 to derive a method for finding the orbit of a double star on gravitational principles, and the first orbit ever computed appeared in the *Connaissance des Temps* for 1830. When SAVARY's method for finding double-star orbits had been successfully applied to ξ *Ursae Majoris*, the subject was taken up by ENCKE and HERSCHEL, who published methods of superior elegance and of greater practical utility, with the result that numerous orbits were soon computed.

The rapid orbital motion of ξ *Ursae Majoris* insured it ample attention, and accordingly since the time of SIR JOHN HERSCHEL and STRUVE, measures have been secured annually by the best observers. The number of orbits computed for this star is very large, the following list is fairly complete:

P	T	e	a	Ω	i	λ	Authority	Source
58 2625	1817 25	0 4164	3 857	95 37	59 67	131 63	Savary, 1828	Conn des Temps, 1830
60 72	1816 73	0 3777	3 278	97 78	56 1	134 37	Herschel, 1832	Mem R A S V, p 209
60 4596	1816 95	0 40368	2 290	95 0	52 27	129 68	Madler, 1836	A N 319
61.464	1816 44	0 4135	2 417	98 87	54 93	130 8	Madler, 1843	A N 486
61 30	1817 102	0 4037	2 295	96 35	50 92	132 47	Madler, 1847	Fixt-Syst I, p 233
61 175	1816 66	0 4116	2 82	96 1	53 87	129.47	Jacob, 1846	Mem R A S XVI, p 322
61 576	1816 86	0 4315	2 439	95 83	52 82	128 57	Villarcceau, 1849	A N. 680
63 14	1816 32	0.3929	2 454	97 3	52 27	132 88	Breen, 1862	M N XXII, p 158
59 88	1816 405	0 3786	2 591	103 6	53 1	135 3	Ball, 1872	Proc R I. A., June, 1872
60 679	1815 008	0 3830	2 587	100 7	56 33	127.15	Knott, 1873	M. N XXXIII, p 101
60 63	1875 50	0 371	2 535	101 0	55 0	216 0	Flam, 1873	Cat des Ét Doub p 65
60 79	1875.29	0 3952	2 549	101 5	56 9	234 3	Dunér, 1876	Meas Micr., p. 196
60 80	1875 26	0 4159	2 580	100 22	56 67	235.0	Pritchard, 1878	Oxford Obs, No. 1
60 50	1814 8	0 410	2.55	280 7	122 9	305 8	Birk, 1879	K Akad Wiss Wien Bd 98

It will be seen that among the more recent orbits there is no wide range of values, and yet the elements are by no means identical. The different results depend upon the observations used and the method of computation employed.

From an investigation of all the observations, I am led to the following elements:

$$\begin{aligned}
 P &= 60\ 00\ \text{years} & \Omega &= 100^{\circ}\ 8 \\
 T &= 1875\ 22 & i &= 55^{\circ}\ 92 \\
 e &= 0\ 397 & \lambda &= 126^{\circ}\ 33 \\
 a &= 2''\ 508 & n &= -6^{\circ}\ 0000
 \end{aligned}$$

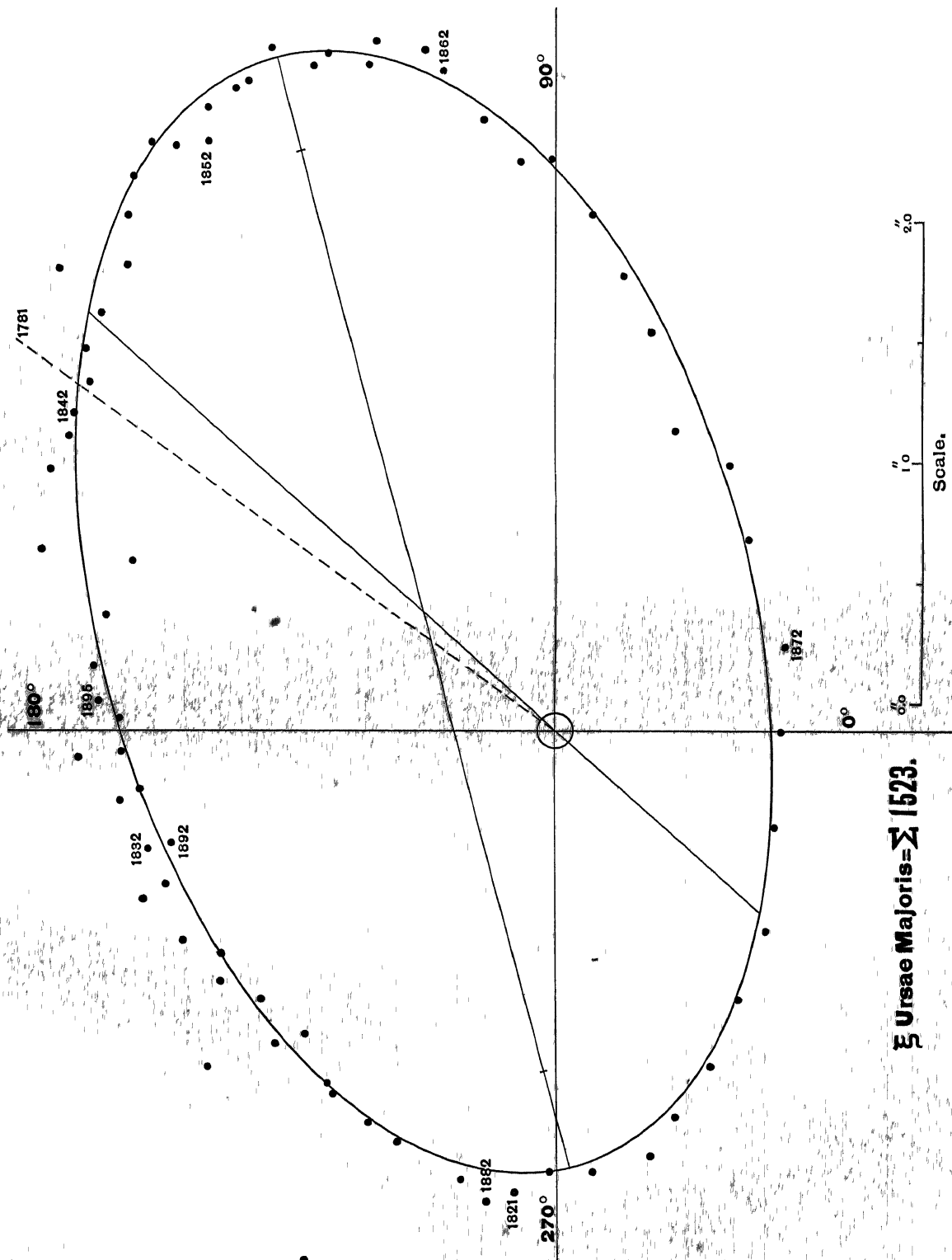
Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 4''\ 76 \\
 \text{Length of minor axis} &= 2''\ 70 \\
 \text{Angle of major axis} &= 104^{\circ}\ 6 \\
 \text{Angle of periastron} &= 318^{\circ}\ 0 \\
 \text{Distance of star from centre} &= 0''\ 75
 \end{aligned}$$

The following table of computed and observed places shows that these elements are extremely satisfactory

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
	$^{\circ}$	$^{\circ}$	$''$	$''$	$^{\circ}$	$''$		
1781 97	143 8	148 4	4 ±	2 34	-4 6	+1 66±	1	Herschel
1802 09	97 5	99 0	—	2 70	-1 5	—	1	Herschel
1804 09	92 6	93 3	—	2 47	-0 7	—	1	Herschel
1819 10	284 5	282 1	—	1 69	+2 4	—	2	Struve
1820 13	276 3	274 0	—	1 79	+2 3	—	3	Struve
1821 78	264 7	264 5	1 92	1 84	+0 2	+0 08	3	Struve
1823 29	258 4	255 8	2 81	1 83	+2 6	+0 98	58-20	Herschel and South
1825 22	244 5	244 5	2 44	1 78	±0 0	+0 66	7-4	South
1826 20	238 7	238 4	1 77	1 75	+0 3	+0 02	3	Struve
1827 27	228 3	231 6	1 71	1 72	-3 3	-0 01	4	Struve
1828 37	224 0	224 3	2 01	1 69	-0 3	+0 32	2	Herschel
1829 35	213 6	217 7	1 67	1 67	-4 1	±0 00	7	Struve
1830 58	206 1	209 3	2 23	1 67	-3 2	+0 56	10±	Herschel
1831 28	202 4	204 5	1 85	1 68	-2 1	+0 17	27-14	Bessel 5, Dawes 17-4, W Struve 5
1832 34	196 3	197 3	1 76	1 69	-1 0	+0 07	15-13	Dawes 10-8, W Struve 5
1833 30	189 0	191 0	1 83	1 72	-2 0	+0 11	9	Dawes 4, W Struve 5
1834 47	183 3	183 7	1 87	1 78	-0 4	+0 09	6-2	W Struve 2, Madler 4-0
1835 34	178 3	178 7	1 84	1 82	-0 4	+0 02	6	Madler 1, W Struve 5
1836 33	171 7	173 1	1 94	1 89	-1 4	+0 05	12-7	Dawes 1, Madler 7-2, W Struve 4
1837 47	165 3	167 2	1 93	1 97	-1 9	-0 04	3	Struve
1838 43	160 4	162 7	2 26	2 05	-2 3	+0 21	9	Struve
1839 47	157 9	157 4	1 89	2 14	+0 5	-0 25	—	Galle
1840 34	152 2	154 5	2 36	2 20	-2 3	+0 16	12-10	Dawes 6-4, O Struve 6
1841 30	148 6	150 2	2 36	2 29	-1 6	+0 07	17-15	Dawes 4-3, Madler 7-6, O Struve 6
1842 30	146 4	147 3	2 40	2 37	-0 9	+0 03	12	Madler 4, Dawes 4, O Struve 4
1843 33	143 0	143 9	2 42	2 45	-0 9	-0 03	11	Dawes 7, Madler 4
1844 34	140 7	140 7	2 52	2 54	±0 0	-0 02	14-13	O Struve 3, Madler 11-10
1845 74	136 9	136 6	2 81	2 65	+0 3	+0 16	4	O Struve 2, Jacob 2
1846 37	137 2	134 9	2 56	2 69	+2 3	-0 13	4	O Struve
1847 36	132 3	132 3	2 63	2 76	±0 0	-0 13	14	Dawes 1, Madler 10, O Struve 3
1848 30	129 5	130 0	2 78	2 82	-0 5	-0 04	15	Dawes 1, Dawes 3, Madler 4, O Struve 5, Bond 2
1849 33	127 1	127 3	2 89	2 87	-0 2	+0 02	9	Dawes 5 O Struve 4
1850 51	124 3	124 8	2 96	2 94	-0 5	+0 02	8	Jacob 2, O Struve 4, Madler 2
1851 39	122 9	122 9	2 89	2 97	±0 0	-0 08	28-27	Flt 6-5, Madler 6, Dawes 2, O Struve 5, Madler 9
1852 30	120 3	120 9	2 84	3 00	-0 6	-0 16	27-26	Miller 7, Flt 6, Jacob 1, Ma 6, Mo 2, Da 1-0, OZ 4
1853 24	119 0	118 9	2 96	3 02	+0 1	-0 06	35-29	Miller 4, Jacob 2, Powell 6-0, Fl 6, OZ 4, Ma 13
1854 34	116 4	116 5	2 98	3 03	-0 1	-0 05	37-28	Powell 10-1, Madler 15, Dawes 3, O Struve 4, Dem 5
1855 33	115 2	114 5	2 98	3 03	+0 7	-0 05	13	Dembowski 7, Sec 1, Madler 2, O Struve 3
1856 45	112 4	112 1	3 07	3 02	+0 3	+0 05	29	Jacob 3, Sec 4, Dembowski 7, Madler 13, Jacob 2
1857 42	109 8	110 0	2 94	3 00	-0 2	-0 06	13	Sec 2, Madler 8, O Struve 3
1858 24	108 4	108 3	2 96	2 97	+0 1	-0 01	20	Jacob 4, Morton 2, Dembowski 6, O Struve 3, Ma 5
1859 48	105 5	105 4	2 87	2 91	+0 1	-0 04	11-8	Madler 6-3, O Struve 5
1860 24	104 6	103 6	2 96	2 86	+1 0	+0 10	12-10	Morton 2, Powell 6-5, Dawes 2-1, Madler 2
1861 32	100 8	101 0	2 87	2 77	-0 2	+0 10	18-14	Powell 6-2, O Struve 4, Madler 8
1862 38	99 7	98 2	2 78	2 67	+1 5	+0 11	8	Madler 4, O Struve 4,
1863 34	96 7	95 6	2 55	2 56	+1 1	-0 01	21	Dembowski 19, O Struve 2
1864 40	93 7	92 2	2 36	2 42	+1 5	-0 06	16	Dembowski 9, Sec 3, O Struve 3, Dawes 1
1865 31	90 5	89 0	2 37	2 27	+1 5	+0 10	33	Englemann 19, Dembowski 10, Sec 4
1866 33	86 2	85 5	2 14	2 13	+0 7	+0 01	16	Sec 3, Dembowski 10, O Struve 3
1867 39	81 6	79 5	1 91	1 89	+2 1	+0 02	11	Dembowski 8, O Struve 2
1868 28	76 8	75 0	1 70	1 73	+1 8	-0 03	13	Searle 1, Dembowski 8, O Struve 4
1869 40	68 6	65 3	1 34	1 45	+3 3	-0 11	11	Dunér



Ursae Majoris =  $\Sigma$  1523.



<i>t</i>	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	<i>n</i>	Observers
1870 19	56 9	57 0	1 32	1 27	-0 1	+0 05	24	O Struve 4, Dembowski 9, Hedhull 3, Dunér 9
1871 35	45 0	40 3	1 13	1 05	+4 7	+0 08	24-23	Dem 8, Gl 2, O Struve 2, Dunér 11-10, Wilson 1
1872 36	20 4	19 5	1 01	0 92	+0 9	+0 09	47-46	Gl 2, W & S 7-6, Kn 6, Dem 10, Du 14, Fer 8
1873 36	359 2	355 4	0 93	0 90	+3 8	+0 03	18-17	W & S 2-1, Dembowski 10, Dunér 1, O Struve 5
1874 29	336 4	334 5	0 99	0 98	+1 9	+0 01	19-18	Gl 3, W & S 2-1, Fer 1-0, Dem 6, OZ 3, Du 4-5
1875 47	316 1	314 4	1 20	1 18	+1 7	+0 02	34-32	Dem 8, Sch 7, W & S 4-3, Dunér 14, Dobereck 1-0
1876 35	304 3	303 5	1 34	1 33	+0 8	+0 01	28-10	Dobereck 13-2, Dem 7, W & S 3, Plummer 5-0
1877 31	294 7	294 0	1 52	1 39	+0 7	+0 13	36-33	Pl 7-6, Dem 6, Dk 10-0, Sch 8, W & S 3, Ill 2-0
1878 32	286 8	286 5	1 66	1 62	+0 3	+0 04	6	Dembowski
1879 30	282 2	279 3	1 80	1 73	+2 9	+0 07	10	Hall 3, Schiaparelli 7
1880 31	274 8	273 1	1 83	1 80	+1 7	+0 03	22-11	Franz 6-0, Hall 6, Dobereck 5-0, Bigourdan 2, Jed 3
1881 32	269 2	267 1	1 82	1 83	+2 1	-0 01	23-21	Dobereck 4, Bigourdan 2-1, Hall 7, β 4-3, Sch 6
1882 28	261 5	261 7	1 97	1 84	-0 2	+0 13	23-19	Hall 6, Dobereck 4-0, Englemann 4, Schiaparelli 9
1883 38	257 8	255 3	1 90	1 82	+2 5	+0 08	26-20	Englemann 6-0, Schiaparelli 11, Hall 6, Jedrzejewicz 3
1884 35	248 9	248 8	1 83	1 80	+0 1	+0 03	39-26	Perrotin 3-4, Hall 7, Bigourdan 14-0, Sch 11, En 4
1885 38	244 5	243 5	1 80	1 77	+1 0	+0 03	18	Hall 5, Schiaparelli 10, Tarrant 3
1886 39	237 2	237 1	1 71	1 74	+0 1	-0 03	16-8	Hall 5, Englemann 8-0, Jedrzejewicz 3
1887 35	230 6	231 5	1 63	1 72	-0 9	-0 09	17	Hall 5, Schiaparelli 12
1888 36	222 5	224 4	1 65	1 69	-1 9	-0 04	14-10	Hall 6, Schiaparelli 4, Maw 4-0
1889 32	216 8	217 9	1 70	1 68	-1 1	+0 02	19-17	Glasenapp 2-0, Hall 5, Schiaparelli 9, Maw 3
1890 37	209 5	210 7	1 77	1 67	-1 2	+0 10	18	Hall 6, Schiaparelli 7, Maw 3, Knorre 2
1891 30	201 5	204 3	1 71	1 68	-2 8	+0 03	22	Big 1, Flint 1, Hall 6, Knorre 1, Sch 10, Maw 3
1892 39	196 3	197 3	1 66	1 69	-1 0	-0 03	28-17	Maw 4, Sch 11-10, Big 1, Knorre 6, Lv 2, Com 4
1893 33	188 0	191 0	1 71	1 72	-3 0	-0 01	14-12	Knorre 2-0, Maw 4, Schiaparelli 7, Davidson 1
1894 38	182 9	184 3	1 81	1 77	-1 4	+0 04	17	Com 3, II C W 1, Knorre 2, Maw 3, Big 8, Glas 1
1895 32	176 2	178 5	1 90	1 83	-2 3	+0 07	5	Davidson 1, Lewis 1, Sec 8

Future observations are likely to produce only very slight alterations in the above values. Thus the period is not likely to be in error by more than one-tenth of a year, and the error in the eccentricity can hardly surpass  $\pm 0.005$ . Indeed the orbit *ξ Ursae Majoris* is practically all that can be desired in the present state of double-star measurement. In order to effect any further improvement of the orbit, astronomers will need to take every precaution against systematic errors, and rough measures by inexperienced observers are unlikely to prove to be of any considerable value.

We remark, however, that continued observation of this star is desirable, because the micrometrical measures of skilled observers will be valuable in throwing light upon the question of the existence of dark bodies or other disturbing influences, and in proving with all possible experimental accuracy that the force which retains the companion in its orbit is directed exactly towards the central star.

*ξ Ursae Majoris*, like *ζ Herculis*, has a large proper motion in space, and this circumstance in connection with the brilliancy of the components, conduces to the belief that the system is comparatively near the earth. Measurement for parallax has never been attempted, but if suitable comparison stars could be found, effort in this direction would be likely to prove successful.

O $\Sigma$ 234.

$\alpha = 11^h 25^m 4$  ,  $\delta = +41^\circ 50'$   
7, yellowish , 7 8, yellowish

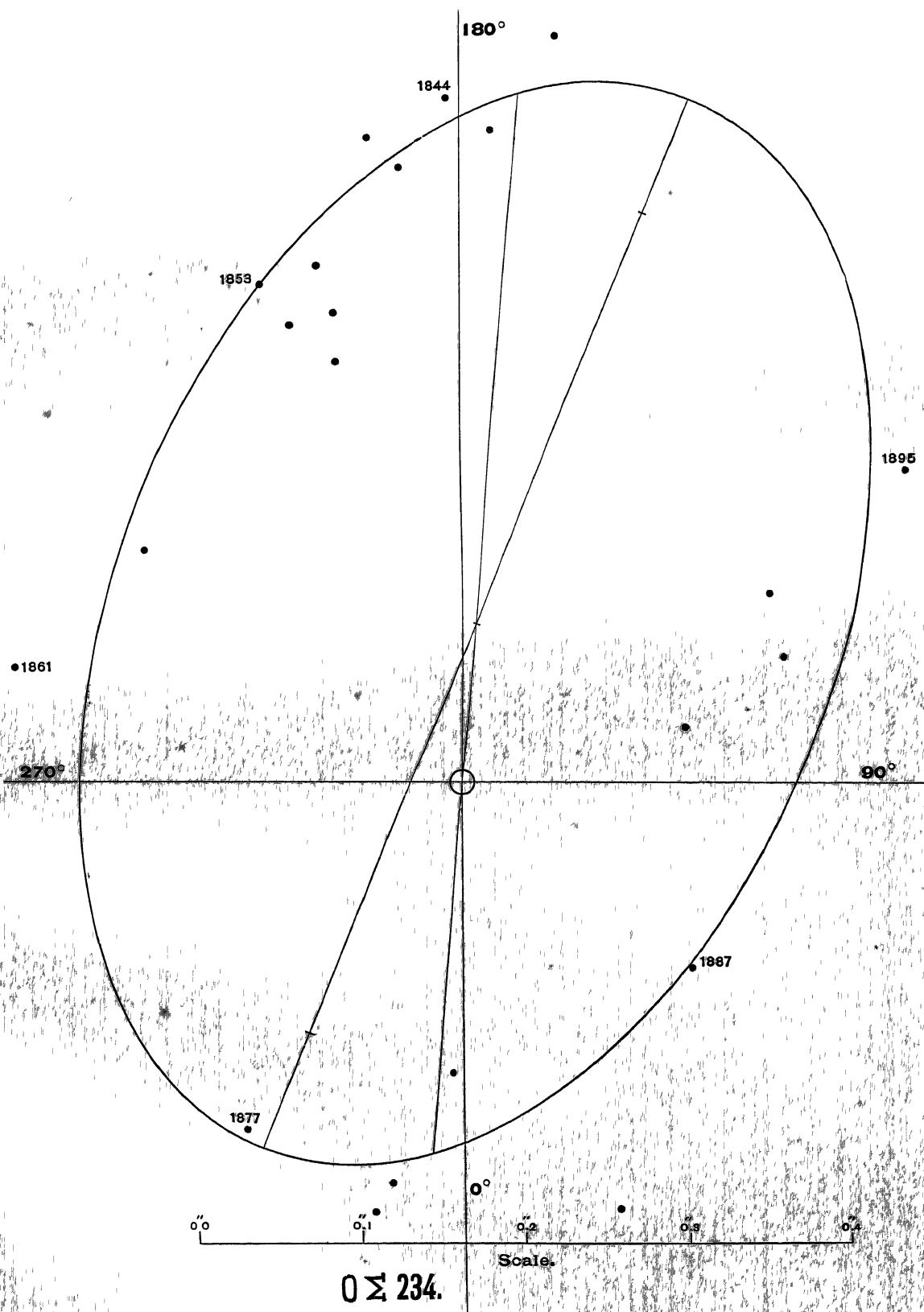
*Discovered by Otto Struve in 1843*

## OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1843 29	182 5	0 42	1	O Struve	1870 46	281 8	cert obl	1	O Struve
1843 33	179 6	0 25	—	Madler	1877 26	127 3	0 25	2	Dembowski
1844 31	172 7	0 46	1	O Struve	1877 32	cuneiforme sous $349^\circ$			1 O Struve
1845 42	194 6	0 30	2	Madler	1878 28	168 4	0 27	2-1	Burnham
1846 37	177 2	0 40	1	O Struve	1880 37	178 4	0 18	1	Burnham
1847 40	187 2	0 25	1	Madler	1882	130	<0 3	3	Englemann
1847 41	183 7	0 38	1	O Struve	1883	350	<0 25	3	Englemann
1848 25	187 9	0 40	1	O Struve	1884 10	20	0 28	1	Englemann
1850 31	195 2	0 33	1	O Struve	1887 42	231 2	0 18	6	Schiaparelli
1851 36	200 4	0 3	1	Madler	1889 39	cuneiforme sous $98^\circ$			1 O Struve
1851 42	199 3	0 30	2	O Struve	1891 23	104 2	0 14	3	Burnham
1852 46	196	0 27	1	O Struve	1892 28	114 2	0 18	3	Burnham
1853 41	201 3	0 33	1	O Struve	1892 39	107 0	0 24	2-1	Bigoudan
1858.36	cert elong in $244^\circ$			1 O Struve	1892 40	293 6	0 22	1	Schiaparelli
1859 40	233	0 24	1	O Struve	1894 29	123 2	0 22 $\pm$	2	Comstock
1861 26	255 0	0 28	2-1	O Struve	1894 84	121 7	0 21	3	Barnard
1862 39	260	oblong	1	O Struve	1895 20	122 2	0 30 $\pm$	1	Comstock
1866 20	single	—	1	Dembowski	1895 75	125 1	0 36	1	See
1866 49	oblong in $283^\circ$		1	O Struve					

Since the discovery of this pair by OTTO STRUVE, the companion has described an arc of  $305^\circ$ . The object is always close and difficult, and hence the measures are by no means so good as could be desired, yet when account is taken of both angles and distances, there is reason to believe that elements based on the observations now available will never be greatly changed. MR GORE is the only computer who has previously investigated the orbit of this pair; using the measures prior to 1886, he found the following elements:

$$\begin{aligned}
 P &= 63.45 \text{ years} & \Omega &= 124^\circ 2' \\
 T &= 1881.15 & i &= 47^\circ 35' \\
 e &= 0.3629 & \lambda &= 71^\circ 97' \\
 a &= 0''.339
 \end{aligned}$$



We find the following orbit of O $\Sigma$ 234:

$$\begin{array}{ll} P = 77.0 \text{ years} & \Omega = 157^\circ 5 \\ T = 1880.10 & i = 50^\circ 6 \\ e = 0.302 & \lambda = 206^\circ 6 \\ a = 0''.3467 & n = +4^\circ 6754 \end{array}$$

Apparent orbit.

$$\begin{array}{ll} \text{Length of major axis} & = 0''.695 \\ \text{Length of minor axis} & = 0''.437 \\ \text{Angle of major axis} & = 158^\circ 0 \\ \text{Angle of periastron} & = 355^\circ 2 \\ \text{Distance of star from center} & = 0''.098 \end{array}$$

The accompanying table shows that these elements are very satisfactory; the period is perhaps uncertain by five years, and the eccentricity by perhaps  $\pm 0.04$ . Larger variations in these elements are not to be anticipated. It is probably worth noting that BURNHAM'S distance in 1891 is sensibly smaller than the computed distance, although the angle agrees perfectly. By this we are not to infer that he under-measured the distance with the great Refractor of the Lick Observatory, but that all small distances with a great Telescope appear diminished in comparison with their magnitude in a small instrument—a phenomenon due mainly to the diminution of the spurious discs under the superior separating power of great Telescopes. The computer must therefore take account of the inequality of the distances due to the different power of the Telescopes employed; but as most of the observations of O $\Sigma$ 234 were made with instruments of about 15-inch aperture, I preferred to make the scale of the major axis such, that on the whole the computed would agree with the observed distances.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1843.31	181.0	178.1	0.42	0.41	+2.9	+0.01	2-1	O $\Sigma$ 1, Mädler 1-0
1844.31	172.7	180.1	0.46	0.41	-7.4	+0.05	1	O Struve
1845.42	194.6	182.3	0.30	0.40	+12.3	-0.10	2	Mädler
1846.37	177.2	184.2	0.40	0.39	-7.0	+0.01	1	O Struve
1847.40	185.4	186.6	0.38	0.38	-1.2	$\pm 0.00$	2-1	Mädler 1-0, O $\Sigma$ 1
1848.25	187.9	188.5	0.40	0.38	-0.6	+0.02	1	O Struve
1850.31	195.2	193.7	0.33	0.36	+1.5	-0.03	1	O Struve
1851.39	199.8	196.6	0.30	0.35	+3.2	-0.05	3	Mädler 1, O $\Sigma$ 2
1852.46	196	199.3	0.27	0.34	-3.3	-0.07	1	O Struve
1853.41	201.3	202.7	0.33	0.33	-1.4	$\pm 0.00$	1	O Struve
1858.36	244	222.1	cert along	0.27	+21.9	-	1	O Struve
1859.40	233	227.0	0.24	0.26	+6.0	-0.02	1	O Struve
1861.26	255.0	237.0	0.28	0.25	+18.0	+0.03	2-1	O Struve
1862.39	260	243.8	oblong	0.24	+16.2	-	1	O Struve
1866.49	283	271.3	oblong	0.24	+11.7	-	1	O Struve

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1870 46	281 8	297 5	<sup>"</sup> <sub>ceit</sub> 0 24	0 24	-15 7	-	1	O Struve
1877 29	328 1	337 3	<sup>"</sup> <sub>oblong</sub> 0 25	0 25	- 9 2	$\pm 0 00$	3	Dembowski 2, O $\Sigma$ 1
1878 28	348 4	343 0	0 27	0 25	+ 5 4	+0 02	2-1	Burnham
1880 37	358 4	375 5	0 18	0 23	+ 0 9	-0 05	1	Burnham
1883	350	18 7	<0 25	0 20	-28 7	+0 05	3	Englemann
1884 10	20	30 2	0 28	0 19	-10 2	+0 09	1	Englemann
1887 42	51 2	68 5	0 18	0 18	-17 3	$\pm 0 00$	6	Schjaparelli
1889 39	98	89 8	cune	0 20	+ 8 2	-	1	O Struve
1891 23	104 2	104 4	0 14	0 23	- 0 2	-0 09	3	Burnham
1892 36	111 6	111 5	0 21	0 25	+ 0 1	-0 04	6-5	Big 2-1, $\beta$ 3, Sch 1
1894 56	121 7	122 6	0 22	0 29	- 0 9	-0 07	3-5	Comstock 2, Barnard 3
1895 20	125 1	125 2	0 33	0 30	- 0 1	+0 03	1-2	Comstock 0-1, See 1

The observation of this star which I made at Madison, is discordant in angle (*AJ* 359), and hence I am led to think that an error of 30° occurred in reading the circle, the unmeduced reading was 94°.3, whereas it doubtless should read 64° 3. As the angle was estimated at 130°, this correction is amply justified.

If good observations can be secured for the next decade, this orbit can be rendered very exact. The following ephemeris will be useful to observers:

$t$	$\theta_o$	$\rho_o$	$t$	$\theta_o$	$\rho_o$
1896 40	127 0	0 31	1899 40	136 8	0 36
1897 40	130 4	0 33	1900 40	139 5	0 37
1898 40	133 7	0 34			

O $\Sigma$ 235.

$\alpha = 11^h 20^m 7$  ,  $\delta = +61^\circ 38'$   
6, yellowish , 7 8, yellowish

*Discovered by Otto Struve in 1843*

## OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1844 33	289 3	0 67	1	O Struve	1852 46	329 5	0 57	1	O Struve
1845 47	296 7	0 54	1	O Struve	1853 41	333 5	0 51	1	O Struve
1846 42	306 8	0 57	1	O Struve	1855 47	345 6	0 51	1	O Struve
1847 45	315 8	0 53	1	O Struve	1856 55	350 3	0 52	1	O Struve
1849 47	320 8	0 49	1	O Struve	1857 51	350 4	0 55	1	O Struve
1850 31	316 5	0 56	1	O Struve	1858 44	358 7	0 75	1	O Struve
1851 42	328 0	0 54	2	O Struve	1859 41	358 7	0 62	1	O Struve

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1861 42	13 3	0 65	2	O Struve	1879 44	55 5	1 07	3	Hall
1862 38	20 3	0 76	1	O Struve	1882 59	64 8	1 26	6	Englemann
1864 43	25 3	0 80	1	O Struve	1887 43	73 0	0 93	5-3	Schiaparelli
1866 49	33 3	0 83	1	O Struve	1888 43	69 4	1 12	1	O Struve
1867 45	40 1 separated		1	Dembowski	1888 69	72 6	1 32	4	Tarrant
1868 13	31 0	0 84	1	Dembowski	1889 35	76 9	1 07	5	Hall
1870 18	42 6	0 9	1	Dembowski	1889 39	67 3	0 90	1	O Struve
1870 46	37 4	0 98	1	O Struve	1891 29	81 7	1 04	1	Bigoudan
1872 40	42 0	0 8	1	Dembowski	1892 12	84 3	0 97	3	Burnham
1872 60	43 1	1 00	1	O Struve	1892 44	88 1	1 03	1	Bigoudan
1876 63	51 0	0 95	1	O Struve	1892 45	85 4	0 80	2	Lv
1877 26	55 5	1 07	2	Dembowski	1892 54	84 2	0 94	3-2	Comstock
1877 32	54 7	1 04	1	O Struve	1893 37	90 2	0 92	1	Comstock
1878 35	58 1	1 18	4	Dembowski	1893 41	86 6	0 85	6-9	Bigoudan
1879 44	58 2	0 76	1	O Struve	1894 24	90 1	0 75	3	Comstock
					1895 27	93 9	0 79	3	Comstock
					1895 74	97 3	0 81	2	See

For a number of years after the discovery of this pair, OTTO STRUVE alone noted the position of the companion, but as his measures soon established the rapid motion of the system, DEMBOWSKI, HALL, SCHIAPARELLI, and other subsequent observers have contributed to the material now available for the investigation of the orbit.

The observations are not very numerous, but for an object of this difficulty, they are comparatively good

The arc described by the companion since 1844 is only  $166^\circ$ , and yet the motion around the apastron of the apparent orbit defines the elements with considerable precision DOBERCK is the only astronomer who has previously investigated the motion of this pair; his elements are as follows.—

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
94 4	1839 1	0 500	0 98	99 6	54 5	134 9	Doberck, 1879	A N 2294
94 406	1839 10	0 5870	1 066	96 28	60 22	129 92	Doberck, 1879	

A careful study of all the observations leads to the following elements:

$$\begin{aligned}
 P &= 80.0 \text{ years} & \Omega &= 81^\circ 7' \\
 T &= 1834.30 & i &= 49^\circ 32' \\
 e &= 0.324 & \lambda &= 137^\circ 78' \\
 a &= 0''.8690 & n &= +4^\circ 5'
 \end{aligned}$$

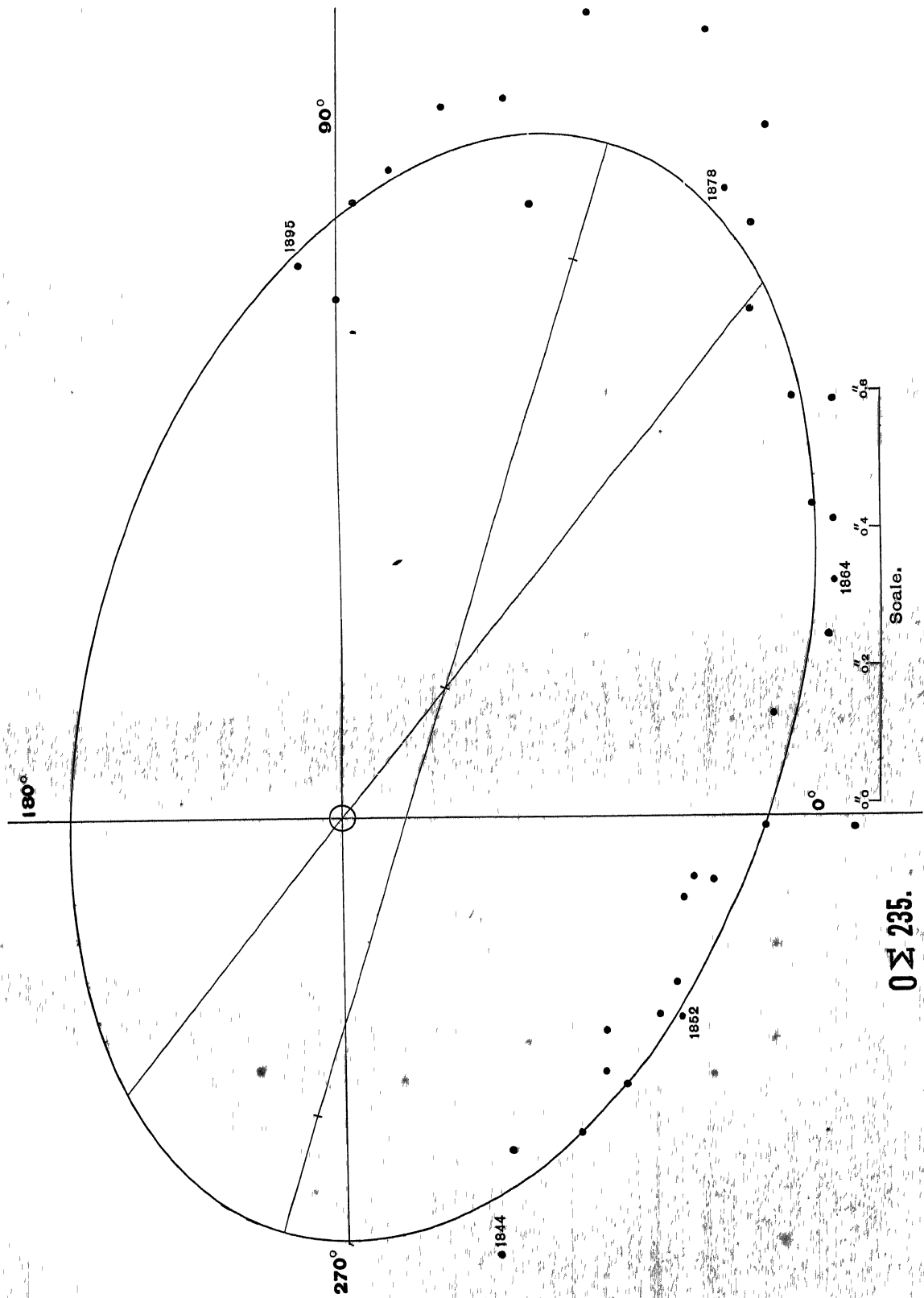
## Apparent orbit

Length of major axis	= 1" 682
Length of minor axis	= 1" 02
Angle of major axis	= 72° 8
Angle of periastron	= 231° 1
Distance of star from centre	= 0" 242

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1844 33	289° 3	288 6	0 67	0 60	+ 0 7	+0 07	1	O Struve
1845 47	296 7	293 5	0 54	0 59	+ 3 2	-0 05	1	O Struve
1846 42	306 8	298 1	0 57	0 58	+ 8 7	-0 01	1	O Struve
1847 45	315 8	303 7	0 53	0 57	+12 1	-0 04	1	O Struve
1849 47	320 8	314 9	0 49	0 56	+ 5 9	-0 07	1	O Struve
1850 31	316 5	318 7	0 56	0 56	- 2 2	$\pm 0 00$	1	O Struve
1851 42	328 0	324 7	0 54	0 56	+ 3 3	-0 02	2	O Struve
1852 46	329 5	330 2	0 57	0 56	- 0 7	+0 01	1	O Struve
1853 41	333 5	335 5	0 54	0 57	- 2 0	-0 03	1	O Struve
1855 47	346 6	346 3	0 51	0 59	+ 0 3	-0 08	1	O Struve
1856 55	350 3	351 8	0 52	0 60	- 1 5	-0 08	1	O Struve
1857 51	350 4	356 6	0 55	0 61	- 6 2	-0 06	1	O Struve
1858 44	358 7	1 0	0 75	0 63	- 2 3	+0 12	1	O Struve
1859 41	358 7	5 5	0 62	0 65	- 6 8	-0 03	1	O Struve
1861 42	13 3	13 7	0 65	0 69	- 0 4	-0 04	2	O Struve
1862 38	20 3	17 5	0 76	0 71	+ 2 8	+0 05	1	O Struve
1864 43	25 3	24 8	0 80	0 76	+ 0 5	+0 04	1	O Struve
1866 49	33 3	30 8	0 83	0 81	+ 2 5	+0 02	1	O Struve
1867 45	40 1	34 2	separated	0 84	+ 5 9	-	1	Dembowski
1868 13	31 0	36 0	0 84	0 86	- 5 0	-0 02	1	Dembowski
1870 32	40 0	40 4	0 94	0 90	- 0 4	+0 04	2	Dembowski 1, O Struve 1
1872 50	42 6	47 1	0 90	0 96	- 4 5	-0 06	2	Dembowski 1, O Struve 1
1876 63	51 0	55 9	0 95	1 02	- 4 9	-0 07	1	O Struve
1877 29	55 1	57 3	1 05	1 03	- 2 2	+0 02	3	Dembowski 2, O Struve 1
1878 35	58 1	59 3	1 18	1 04	- 1 2	+0 14	4	Dembowski
1879 44	58 2	61 5	1 07	1 05	- 3 3	+0 02	1-3	O Struve 1, Hall 0-3
1882 59	64 8	67 3	1 26	1 05	- 2 5	+0 21	6	Englemann
1887 43	72 5	76 1	0 93	1 02	- 3 6	-0 09	4	Schiaparelli
1888 56	72 6	78 4	1 22	1 00	- 5 8	+0 22	4-5	O $\Sigma$ 0-1, Tarrant 4
1889 37	76 9	79 8	1 07	0 98	- 2 9	+0 09	5	Hall
1891 29	81 7	83 6	1 04	0 94	- 1 9	+0 10	1	Bigouldan
1892 39	85 5	85 9	0 94	0 92	- 0 4	+0 02	9-8	$\beta$ 3, Big 1, Lv 2, Com 3 2
1893 39	88 4	88 2	0 89	0 89	+ 0 2	$\pm 0 00$	7-10	Comstock 1, Bigouldan 6 9
1894 24	90 1	90 1	0 75	0 87	$\pm 0 0$	-0 12	3	Comstock
1895 50	93 9	93 3	0 80	0 83	+ 0 6	-0 03	3	Comstock

A comparison of the computed with the observed places shows a very satisfactory agreement, and we cannot doubt that the elements given above will be found to approximate the truth. The period remains uncertain by perhaps five years, and the eccentricity may be varied by  $\pm 0.05$ , but larger alterations in these elements are not to be expected. The motion of this pair will be accelerated in approaching periastron, and hence for a good many years will







deserve the regular attention of observers. If good measures can be secured during the next twenty years, the elements can be determined with great accuracy. The following is a short ephemeris:—

$t$	$\theta_0$	$\rho_0$	$t$	$\theta_0$	$\rho_0$
1896 50	95.9	0.80	1899 50	105.3	0.69
1897 50	98.9	0.76	1900 50	109.0	0.66
1898 50	102.0	0.73			

$\gamma$  CENTAURI = H, 5370.

$\alpha = 12^h 30^m$ ,  $\delta = -48^\circ 25'$

4, yellowish, 4, yellowish

Discovered by Sir John Herschel, March 1, 1835

OBSERVATIONS

I By Sir John Herschel:

MEASURES WITH THE EQUATORIAL.\*

$t$	$\theta_0$	$\rho_0$	$n$	Observer	Remarks
1835.257	351.8	<1	1	Herschel	Extremely close and very difficult, at least as close as $\gamma$ Virgins, 273 barely elongates it.
1835.260	360.3	—	1	Herschel	Certainly double, but far too difficult for this telescope. Distinctly elongated, but the measures of no dependence.
1835.320	351.3	0.67	1	Herschel	Far too difficult for satisfactory measures, yet I must believe these to be somewhere about the truth.
1835.353	346.8	—	1	Herschel	A better set of measures than hitherto got with the equatorial, but it is too difficult for this object-glass.
1835.367	349.6	—	1	Herschel	Certainly seen double, & elongated with parallel fringes.
1836.115	355.3	—	1	Herschel	Excessively close and difficult, but the power No. 4 will act to-night, though not quite so well as I could wish. Field strongly illuminated
1836.156	362.0	—	1	Herschel	
1836.192	355.4	—	1	Herschel	Tolerably elongated with No. 4 Brandishes, dances, and spreads, yet occasionally an elongated centre caught
1836.493	317.1	—	1	Herschel	
1837.140	361.9	1	1	Herschel	

OBSERVATIONS WITH THE REFLECTOR

1835.166	—	—	—		$\gamma$ Centauri, a star 4 <sup>m</sup> , which I am very much inclined to believe close double, but could not verify it owing to bad definition. Tried 320, but it will not bear that power
1835.250	310.8	0.67	1	Herschel	180 with triangular aperture shows it elongated, 320 faintly double and almost divided. Pos. with 320 = $338^\circ 3'$ , with 480 (which shows a black division) = $343^\circ 3'$ . Both stars of 4th magnitude
1836.382	310 $\pm$	—	1	Herschel	Seen decidedly elongated with 320 and diminished aperture, but so violently agitated and ill defined that no measure could be got. That set down may err 20°
1837.074	310 $\pm$	—	1	Herschel	( $\gamma$ Centauri). [Pos. estim. from diag]. Seen decidedly elongated in a position as per diagram, with 320 and triangular aperture, but all attempt at a measure confounded by constant boiling and working of the star.

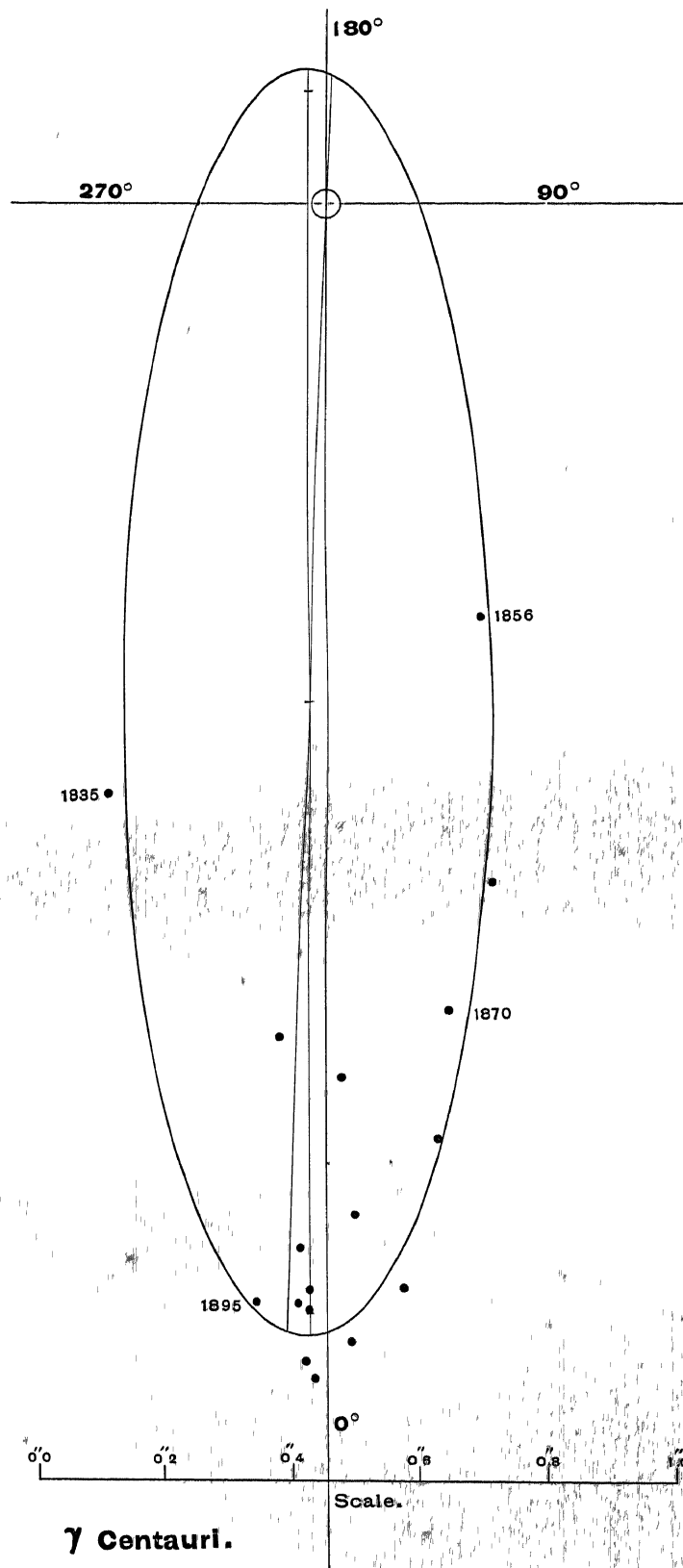
\* *Astronomische Nachrichten*, 3330.

## II By other observers

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1856 20	20° 6'	0.7 ±	3	Jacob	1887 58	359.1	1.76	2 1	Tebbutt
1857 97	13.7	1.11	5	Jacob	1887 53	358.5	1.75	6	Pollock
1860 68	12.8	—	10 obs	Powell	1888 17	359.5	1.87	1 6	Tebbutt
1870 23	6.9	1.5 ±	6	Powell	1889 32	359.1	1.73	1	Pollock
1871 38	3.8	1.18	1	Russell	1890 36	1.2	1.81	1	Sellers
1873 36	4.2	2.29	1	Russell	1890 36	359.0	1.81	2 1	Tebbutt
1874 26	1.6	1.61	1	Russell	1891 10	357.0	1.33	1	Sellers
1876 63	8.5	1.30	—	Elleey	1892 32	357.3	1.21	5	Sellers
1880 44	1.3	1.39	1	Russell	1892 18	358.7	1.66	7 8	Tebbutt
1882 22	2.1	—	1	Tebbutt	1893 36	356.7	1.40	3	Sellers
					1894 40	356.6	1.21	3	Sellers
					1895 33	356.4	1.75	11 7	Tebbutt

In the course of the three years following the discovery, HERSCHEL secured several micrometrical measures with his seven-inch equatorial, but it appears that the records he has left us in his sweeps with the 20-foot reflector are much nearer the truth as regards the position-angle of the stars at that epoch. It is singular that his measures with the equatorial give angles almost identical with that of the pair at the present time ( $356^\circ 4$ ), while his estimates made under the superior power of the reflector give the angle as  $340^\circ 4$ . A careful study of all of his observations of  $\gamma$  Centauri (*Results of Observations at the Cape of Good Hope*, pp. 211, 256, 269), and of the other measures by subsequent astronomers leaves no doubt that his estimates with the reflector are essentially correct, while for some reason the measures taken with the equatorial are vitiated by systematic errors which render them worthless. In the above list of measures I have inserted HERSCHEL's notes, with a view of throwing light upon this interpretation of his observations.

Contrary to the opinion of HERSCHEL, it is now evident that the motion of  $\gamma$  Centauri is retrograde; and hence we perceive that the radius vector has swept over nearly an entire revolution since 1835. The recent measures of TEBBUTT, to whom we are so much indebted for observations of this star, prove beyond doubt that the distance of the components in angle  $350^\circ$  must be at least  $1''.48$ ; and hence it could easily have been divided by HERSCHEL with his seven-inch equatorial. He says, however, that the object was "extremely close and very difficult, at least as close as  $\gamma$  Virginis;" and since it is known that  $\gamma$  Virginis, to which HERSCHEL gave regular attention, was less than  $0''.7$ ,





we may conclude that the distance of  $\gamma$  Centauri did not surpass  $1'' 0$ . If this be the approximate distance at the epoch 1835 25 we see that the angle must have been substantially what HERSCHEL estimated with the reflector, and we are thus enabled to reconcile his measures with those of later observers. His estimate of  $340^\circ \pm$  for the angle is based on three nights' work and can hardly be in error by more than two degrees. If we adopt the position thus indicated

$$1835\ 25 \quad 340^\circ \pm \quad 1'' \pm$$

and make use of the measures secured since 1856, we shall obtain an orbit which is near the truth, and the resulting elements will never be greatly changed. MR. GORE is the only computer who has previously investigated the orbit of this binary, using HERSCHEL'S equatorial measures, and relying mainly on the angles, he found:

$$\begin{array}{ll} P = 61\ 88 \text{ years} & \Omega = 177^\circ 95 \\ T = 1840\ 84 & i = 84^\circ 1 \\ e = 0\ 6316 & \lambda = 40^\circ 81 \\ a = 1'' 50 & \end{array}$$

Making use of the mean places given in the following table, and basing our work on both angles and distances, we are led to the following elements of  $\gamma$  Centauri:

$$\begin{array}{ll} P = 88\ 0 \text{ years} & \Omega = 4^\circ 6 \\ T = 1848\ 0 & i = 62^\circ 15 \\ e = 0\ 800 & \lambda = 194^\circ 3 \\ a = 1'' 0232 & n = -4^\circ 0911 \end{array}$$

#### Apparent orbit

$$\begin{array}{ll} \text{Length of major axis} & = 2'' 10 \\ \text{Length of minor axis} & = 0'' 58 \\ \text{Angle of major axis} & = 0^\circ 1 \\ \text{Angle of periastron} & = 177^\circ 8 \\ \text{Distance of star from centre} & = 0'' 794 \end{array}$$

The period here found may be uncertain by perhaps three years, and the eccentricity by  $\pm 0\ 03$ , but larger variations in these important elements are not to be expected. The orbit of  $\gamma$  Centauri is remarkable for its considerable inclination and high eccentricity, which renders the pair very difficult in the periastron part of the apparent ellipse. Binaries with equal components are very frequent among double stars, and are types of systems which possess a peculiar interest when studied in respect to their evolution.

It is clear that  $\gamma$  Centauri will move rather slowly for a good many years, but it deserves the regular attention of southern observers. The following is a short ephemeris.

$t$	$\theta_o$	$\rho_o$	$t$	$\theta_c$	$\rho_c$
1896 40	356 0	1 75	1899 40	354 8	0 71
1897 40	355 6	1 74	1900 40	354 4	1 70
1898 40	355 2	1 72			

COMPARISON OF THE COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1835 25	340 $\pm$	338 2	1 00	0 88	+1 8	+0 12	3-1	Herschel
1856 20	20 6	19 7	0 7 $\pm$	0 77	+0 9	-0 07	3	Jacob
1857 97	13 7	16 7	1 11	0 91	-3 0	+0 20	5	Jacob
1860 68	12 8	13 4	-	1 10	-0 6	-	10	Powell
1870 23	6 9	6 5	1 5 $\pm$	1 54	+0 4	-0 04	6	Powell
1872 37	4 0	5 6	1 73	1 59	-1 6	+0 14	2	Russell
1874 26	1 6	4 7	1 61	1 64	-3 1	-0 03	1	Russell
1876 63	8 5	3 7	1 30	1 69	+4 8	-0 39	-	Elley
1880 44	1 3	2 2	1 39	1 75	-0 9	-0 36	1	Russell
1882 22	2 1	1 4	-	1 77	+0 7	-	1	Tebbutt
1887 55	358 8	359 5	1 76	1 80	-0 7	-0 04	8-7	Tebbutt 2-1, Pollock 6
1888 47	359 5	359 1	1 87	1 80	+0 4	+0 07	4-6	Tebbutt
1889 32	359 1	358 8	1 73	1 80	+0 3	-0 07	4	Pollock
1890 36	360 1	358 4	1 82	1 80	+1 7	+0 02	2	Sellers 1, Tebbutt 1
1891 40	357 0	358 0	1 33	1 79	-1 0	-0 46	1	Sellers
1892 48	358 7	357 6	1 66	1 79	+1 1	-0 13	7-8	Tebbutt
1895 33	356 4	356 4	1 75	1 77	0 0	-0 02	11-7	Tebbutt

 $\gamma$  VIRGINIS =  $\Sigma$  1670. $\alpha = 12^h 36^m 6$ ,  $\delta = -0^\circ 51'$ 

3, yellow, 2, yellow

Discovered by Bradley and Pound, March 15, 1718

OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1718 20	330 8	—	2	B & P	1819 40	—	3 56	—	Struve
1720 31	319 0	7 49*	1	Cassini	1820 28	284 9	2 76	5	Struve
1756 20	324 4	6 50	—	T Mayer	1822 02	282 8	—	2	Struve
1777 $\pm$	310 $\pm$	9 8	—	C Mayer	1822 25	283 4	3 79	2	II & S
1780 0	—	5 70 $\pm$	—	Herschel	1823 19	—	3 30	—	Amici
1781 89	310 7	—	—	Herschel	1823 32	281 6	2 95	1-3	Struve
1803 37	300 2	—	8 obs	Herschel	1825 32	276 9	3 26	4	South
					1825 32	277 9	2 37	6	Struve

\* Computed from Lunar occultation—of no value

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1828 35	270 5	—	1	Herschel	1839 31	34 6	1 26	2-1	Dawes
1828 38	271 5	2 07	1	Struve	1839 35	35 5	1 30	5	Galle
1829 22	267 7	1 79	2	Herschel	1840 26	27 9	1 30	37-24	Kaiser
1829 39	268 3	1 78	5	Struve	1840 38	25 5	1 24	11-7	Dawes
1830 31	262 1	2 22	6-4	Herschel	1840 45	26 4	1 31	5	O Struve
1830 59	262 2	1 59	7	Bessel	1841 19	20 9	1 42	2	Challis
1831 30	258 4	1 99	6-2	Dawes	1841 34	20 0	1 58	7-5	Dawes
1831 32	257 2	1 74	10-6	Herschel	1841 35	20 1	1 73	12-11	Mädler
1831 36	260 9	1 49	5	Struve	1841 41	22 4	1 63	4	O Struve
1832 27	250 2	1 21	18-1	Herschel	1842 21	16 6	1 58	7-5	Mädler
1832 30	249 9	1 33	9-4	Dawes	1842 34	17 4	1 67	—	Mann
1832 33	—	1 94	—	Cooper	1842 35	17 6	1 83	—	Any
1832 52	253 5	1 26	4	Struve	1842 35	12 2	1 85	2	Challis
1833 20	241 8	1 41	12-3	Herschel	1842 38	14 9	1 73	9-5	Dawes
1833 24	64 9	1 14	1	Bessel	1842 41	17 1	1 86	4	O Struve
1833 35	236 4	—	1	Mädler	1842 82	14 5	1 76	—	Kaiser
1833 36	240 1	1 14	8-2	Dawes	1842 88	14 7	1 81	6-1	Mädler
1833 37	245 5	1 05	7	Struve	1843 30	0 7	2 05	1	Challis
1834 29	227 3	—	8	Dawes	1843 35	12 0	1 77	7	Mädler
1834 34	214 8	—	1	Mädler	1843 39	13 6	2 08	—	Mann
1834 37	223 1	1 51	8-1	Herschel	1843 40	12 2	1 83	10-5	Dawes
1834 38	231 6	0 91	5	Struve	1843 48	11 4	2 45	—	Encke
1834 54	214 9	—	6	Herschel	1844 33	9 0	2 63	1	Challis
1834 84	213 6	—	1	Struve	1844 34	2 9	2 20	—	Richardson
1835 11	201 5	—	8	Herschel	1844 36	8 9	2 06	8-7	Mädler
1835 38	195 5	0 51	9	Struve	1844 38	8 6	2 27	—	Encke
1835 39	195 2	0 57	1	Senff	1845 28	8 9	2 41	—	Encke
1835 42	197 1	—	1	O Struve	1845 37	7 0	—	—	Mädler
1836 28	169 5	—	2	Dawes	1845 46	4 5	2 23	2	O Struve
1836 41	151 6	0 26	3	Struve	1846 28	5 0	—	—	Hind
1836 41	158 7	—	2	O Struve	1846 32	2 2	2 91	2	Jacob
1836 41	153 8	—	1	Sabier	1846 39	6 3	2 25	—	Mann
1836 59	113 9	—	—	Encke	1846 39	2 9	2 35	2	O Struve
1836 59	117 5	—	—	Mädler	1846 49	4 1	1 83	1	Mitchell
1837 41	78 3	0 58	1	Mädler	1846 90	3 8	2 45	2	Dawes
1837 41	77 9	0 58	6	O Struve	1847 07	1 9	2 62	—	Hind
1837 41	78 5	0 67	1	Encke	1847 35	2 5	2 40	8	Dawes
1837 41	77 9	—	1	Angelander	1847 41	13 0	2 37	—	Mann
1838 08	57 5	0 67	1	Herschel	1847 42	2 5	2 40	3	O Struve
1838 32	53 4	—	1	Dawes	1847 56	2 5	3 09	1	Mitchell
1838 36	—	1 24	—	Lamont	1847 94	359 9	2 88	2-1	Jacob
1838 40	51 9	0 86	—	Struve	1848 34	360 8	2 71	7-6	Mädler
1838 43	51 1	0 80	—	O Struve	1848 37	360 6	2 62	9	Dawes
1838 43	49 2	0 83	3±	Ga & Ma	1848 43	359 1	2 55	3	O Struve



$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1848 45	360 4	2 60	2	W C & G P B	1855 18	351 6	3 30	4	O Struve
1848 45	360 6	2 80	1	Mitchell	1855 19	351 3	3 51	4	Dembowski
1848 48	360 5	2 60	2-3	Mann	1855 30	353 4	—	4	Powell
					1855 39	353 5	3 45	—	Mann
1849 37	359 0	2 85	5-4	Dawes	1855 40	352 6	3 37	1	Secchi
1849 41	352 9	2 64	2	O Struve	1855 45	354 1	3 42	2	Madler
1849 45	359 8	3 0	2	W C & G P B	1855 46	351 2	3 31	4-3	Dawes
1849 50	357 0	2 92	3	Mann	1855 53	353 3	3 51	3	Morton
1850 23	359 7	2 85	8	Johnson	1856 10	350 5	3 45	4	Jacob
1850 30	358 0	2 90	2	Jacob	1856 29	349 0	3 51	—	Mann
1850 30	357 5	2 90	3	Hartnup	1856 38	351 7	3 55	6	Secchi
1850 36	356 7	2 95	6-3	Fletcher	1856 39	350 5	3 56	5	Dembowski
1850 39	355 2	2 74	4	O Struve	1856 39	351 7	3 59	6	Madler
1850 42	359 1	—	1	Madler	1856 43	172 1	3 31	4	Winnecke
1850 48	359 7	2 94	4	Mann	1856 96	353 0	3 64	—	Carpenter
					1856 97	351 6	3 66	3	Morton
1851 17	356 8	2 92	4	Philpot	1857 07	—	4 50	—	Schmidt
1851 19	357 7	3 12	2	Jacob	1857 09	348 4	3 76	6	Dembowski
1851 28	357 9	2 99	4	Madler	1857 35	350 1	3 59	7	Dawes
1851 36	356 3	3 04	3	Mann	1857 39	350 8	3 74	7	Secchi
1851 40	356 0	3 05	6	Fletcher	1857 40	352 9	3 58	6 $\pm$	Baxendell
1851 40	356 5	2 99	5	Dawes	1857 41	351 6	3 54	—	Fletcher
1851 42	353 0	2 88	3	O Struve	1857 42	350 2	3 59	9-8	Madler
1851 47	355 9	3 04	3-1	Miller	1857 42	349 9	3 56	6	Dawes
1851 98	356 4	3 30	4-3	Madler	1857 44	350 2	3 63	2	O Struve
					1857 96	350 7	3 50	5	Jacob
1852 24	355 5	3 12	3	Jacob	1858 34	348 5	3 80	6	Dembowski
1852 26	355 5	3 12	6-3	Miller	1858 37	349 9	4 01	2	Madler
1852 32	355 3	3 02	2	Dawes	1858 39	350 0	3 57	—	Fletcher
1852 42	355 4	3 15	5	Fletcher	1858 40	352 0	3 62	3	Secchi
1852 43	354 6	3 17	2	Madler	1858 44	349 3	3 67	2	O Struve
1852 43	353 0	3 00	3	O Struve	1858 45	348 8	3 68	8	Dawes
1852 45	356 9	3 05	—	Fearnley	1858 47	348 0	3 85	—	Carpenter
1852 47	359 7	3 20	3	Mann	1858 48	350 7	3 40	3	Morton
1853 24	353 2	3 12	2	Jacob	1859 15	350 7	3 95	4	Morton
1853 24	354 4	—	7	Powell	1859 37	349 2	3 88	9-8	Madler
1853 27	354 9	3 10	7-5	Miller	1859 38	347 9	3 76	3	O Struve
1853 32	354 6	3 18	6	Fletcher	1859 39	350 0	4 18	—	Wakelin
1853 36	354 1	3 06	3-2	Dawes	1859 44	349 5	3 91	3	Secchi
1853 38	357 4	3 30	2	Mann	1859 46	348 2	3 77	5	Dawes
1853 39	354 2	3 25	6	Madler					
1853 40	352 0	3 13	4	O Struve	1860 24	347 9	3 95	1	Auwers
1853 91	353 0	3 06	2	Jacob	1860 30	358 0	2 90	—	Jacob
					1860 35	345 9	3 90	1	Madler
1854 39	352 0	3 45	8	Madler	1860 36	350 2	—	1	Schiaparelli
1854 39	352 7	3 21	8	Dawes	1860 36	347 1	—	1	Wagner
1854 40	352 1	3 40	3	Morton	1860 36	347 3	—	1	Oblomievsky
1854 47	353 6	3 23	7	Dembowski	1860 44	349 3	4 05	2	Knott

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1861 15	347 0	3 93	4	O Struve	1869 22	344 9	4 77	—	Brunnow
1861 19	357 7	3 12	—	Jacob	1869 22	340 9	5 27	2	Leyton Obs
1861 28	347 8	3 99	4	Main	1869 49	339 8	4 74	3	Main
1861 31	346 1	3 93	5	Powell	1869 98	341 8	4 43	17	Duner
1861 36	348 5	4 12	7	Auweis	1870 33	342 6	4 65	2	Gledhill
1861 41	347 8	4 11	3	Mädler	1870 38	340 6	4 76	6	Main
1862 03	346 5	3 95	5-3	Dawes	1870 39	338 6	—	—	Leyton Obs
1862 33	345 3	3 90	3-2	Powell	1870 72	342 0	4 63	11	Dembowski
1862 38	345 5	4 39	3	Mädler	1870 77	343 4	4 45	3	O Struve
1862 38	349 3	4 31	1	Auweis	1871 21	339 8	5 31	1	Pearce
1862 38	346 6	4 00	—	Main	1871 35	340 9	4 54	5	Main
1862 40	346 9	3 97	2	O Struve	1871 38	343 1	4 76	—	Leyton Obs
1862 42	347 6	3 62	1	Oblomievsky	1871 38	339 8	4 49	3	Knott
1863 25	346 7	4 06	3	Main	1871 38	339 7	5 35	2	W & S
1863 27	345 1	4 34	—	Bamberg	1871 53	341 8	4 77	3	Gledhill
1863 46	347 3	3 90	2	O Struve	1872 12	341 1	4 59	17	Dunér
1863 63	345 6	4 08	2-6	Dembowski	1872 30	339 7	4 4	1	Gledhill
1864 40	345 7	4 27	2	Main	1872 34	342 2	5 59	3	W & S
1864 41	345 5	4 28	2	Secchi	1872 37	338 6	4 80	—	Leyton Obs
1864 42	345 1	4 06	3	O Struve	1872 40	341 5	4 82	1	Knott
1864 44	345 4	4 10	4	Dawes	1872 41	340 0	4 64	3	O Struve
1864 44	345 4	4 27	2	Knott	1872 41	340 3	4 78	3	Main
1864 48	348 3	4 03	3	Englemann	1872 86	340 8	4 59	10	Dembowski
1865 45	345 4	4 02	5	Englemann	1873 40	340 2	4 83	5	Main
1865 36	345 2	4 28	4	Main	1873 41	339 7	4 65	2	Gledhill
1865 37	—	4 18	4	Kaiser	1873 43	340 8	4 55	3	O Struve
1865 42	344 0	4 37	7-6	Dawes	1873 46	340 5	4 96	3	Lindstedt
1865 45	344 3	4 34	3	Knott	1874 27	340 5	5 08	2	Gledhill
1865 74	344 3	4 18	26	Dembowski	1874 30	341 8	5 00	1	W & S
1866 31	344 3	4 39	3	Secchi	1874 32	339 3	5 39	1	Leyton Obs
1866 33	342 8	4 52	3-4	Leyton Obs	1874 33	338 5	5 23	6	Main
1866 37	—	5 00	1	Winlock	1874 41	340 4	4 87	3	O Struve
1866 38	344 6	4 21	6	Kaiser	1875 14	339 1	4 66	14	Dunér
1866 42	344 0	4 29	2	O Struve	1875 22	338 5	4 86	4	Gledhill
1866 45	345 2	4 35	2	Main	1875 29	339 8	5 09	6	Main
1866 46	345 9	4 01	—	Kaiser	1875 30	340 0	4 97	1	Seabroke
1867 24	342 9	5 28	1	Leyton Obs	1875 32	339 2	4 80	11	Dembowski
1867 29	344 3	4 50	5	Harvard	1875 41	339 6	4 86	13	Schiaparelli
1867 38	341 4	4 40	6	Main	1875 44	339 9	4 87	2	O Struve
1867 80	343 2	4 30	12	Dembowski	1876 24	338 7	5 34	5	Doberck
1868 17	344 3	4 58	2	Seale	1876 27	338 7	4 78	13	Gledhill
1868 23	341 0	5 21	2	Leyton Obs	1876 36	340 0	—	1	Leyton Obs
1868 42	341 0	4 63	7-6	Main	1876 38	339 8	5 30	4	Cincinnati
1868 44	343.2	4 30	2	O Struve	1876 40	339 7	4 64	1	Waldo
					1876 41	340 2	5 14	4*	Hall

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1876 42	339 7	4 95	3	O Struve	1883 07	335 6	5 22	7-5	Englemann
1876 45	339 0	4 84	4	Schiaparelli	1883 36	336 8	5 45	5	Hall
1876 48	338 2	5 18	5	Mann	1883 41	335 6	5 23	8	Schiaparelli
1877 07	338 5	—	2	Gledhill	1884 33	335 2	5 65	5-3	H C Wilson
1877 24	340 0	4 65	5-4	Plummer	1884 37	336 1	5 42	5	Hall
1877 28	335 8	5 04	—	Knott	1884 38	335 7	5 43	3	Perrotin
1877 30	338 1	5 19	8-7	Cincinnati	1884 40	337 0	5 53	2	Seabroke
1877 40	339 5	4 91	6	Jedrzejewicz	1884 89	336 1	5 32	4	Englemann
1877 41	337 9	4 91	14	Schiaparelli	1884 40	335 6	5 19	9	Schiaparelli
1877 43	338 4	4 96	—	Flammation	1884 44	336 5	5 32	1	O Struve
1877 43	338 9	4 97	2	O Struve	1885 25	334 4	5 30	1	Cop & Lohse
1877 83	338 1	4 97	8	Dembowski	1885 32	333 7	5 35	2	H C Wilson
1878 26	340 1	5 01	2	W & S	1885 38	336 8	5 35	3	Tarrant
1878 37	337 1	5 06	3-5	Goldney	1885 44	335 2	5 30	16	Schiaparelli
1878 37	337 5	5 03	1	O Struve	1886 28	335 0	5 08	2	Glasenapp
1879 0	336 3	5 07	1	Pritchett	1886 30	336 4	5 38	2	H C Wilson
1879 12	337 3	5 20	20	Cincinnati	1886 36	334 9	5 57	4	Hall
1879 13	337 5	4 97	10	Schiaparelli	1887 26	335 7	5 63	2	Glasenapp
1879 35	338 6	5 00	1	Gledhill	1887 35	334 8	5 58	4	Hall
1879 37	338 3	5 20	3	Hall	1887 38	335 5	5 65	2	Tebbutt
1879 38	338 3	5 04	2	Sea & Smith	1887 41	334 2	5 42	7	Schiaparelli
1879 44	340 0	5 09	1	O Struve	1888 27	333 5	5 93	2	Glasenapp
1880 19	336 7	5 30	1	Burton	1888 33	334 6	5 50	5	Hall
1880 25	337 4	5 35	6	RadcliffeObs	1888 35	334 2	5 33	2	Schiaparelli
1880 26	336 5	5 67	3-2	Tiss & Big	1888 40	335 1	5 29	2	Maw
1880 30	338 2	5 27	5	Hall	1888 43	333 3	5 53	1	O Struve
1880 30	337 5	5 36	2	Burnham	1888 48	334 8	5 74	2	Tebbutt
1880 31	337 3	4 90	—	Gledhill	1888 91	333 8	5 50	9	Leavenworth
1880 32	336 9	5 13	6	Cincinnati	1889 27	333 5	5 93	2	Glasenapp
1880 37	338 1	4 95	3	Doberck	1889 31	333 4	5 72	3	Burnham
1880 40	337 5	4 89	2	Seabroke	1889 39	333 1	5 51	2	O Struve
1880 40	337 1	5 74	2	Tebbutt	1889 43	333 0	5 54	5	Hall
1880 45	337 9	5 24	3	Jedrzejewicz	1889 44	333 8	5 41	3	Schiaparelli
1880 66	337 9	5 22	6	Franz	1890 36	333 3	5 10	4	Glasenapp
1880 70	338 4	5 32	2	Pritchett	1890 43	332 8	5 59	3	Hall
1881 24	336 3	5 40	—	Gledhill	1890 43	333 2	5 53	8	Schiaparelli
1881 24	337 1	5 02	4	Doberck	1890 44	336 0	6 13	1	Hayes
1881 30	336 1	5 57	3	E J Stone	1891 15	330 4	5 75	1	Flint
1881 35	337 7	5 33	4	Hall	1891 32	332 0	5 78	2	Wellmann
1881 39	336 8	5 20	9	Schiaparelli	1891 32	332 9	5 69	11	Knorre
1881 42	338 7	5 28	2	Hough	1891 39	333 1	5 64	3	Hall
1881 44	336 2	5 23	14-13	Bigourdan	1891 42	332 6	5 51	7-6	Schiaparelli
1882 28	335 0	5 13	3	H C Wilson					
1882 28	337 4	5 36	5-4	Doberck					
1882 34	335 8	5 50	2	Sea & Hodges					
1882 41	336 6	5 23	10	Schiaparelli					

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1891 44	331 0	5 64	1	Bigoudan	1893 42	331 9	5 47	6	Schiaparelli
1891 44	332 5	5 70	3	See	1893 43	333 1	5 66	1	Comstock
1892 40	332 6	5 55	6	Schiaparelli	1893 46	331 7	5 64	4	Bigoudan
1892 43	332 2	5 67	2	Leavenworth	1894 40	332 1	5 50	2	Comstock
1892 49	333 6	5 55	3	Comstock	1894 42	332 2	5 62	2	Schiaparelli
1892 51	332 3	5 56	2	Tebbutt	1894 47	328 9	5 71	6	Bigoudan
1892 52	331 8	5 61	3	Bigoudan	1895 30	331 1	5 84	5-4	See
1892 96	332.1	5 83	2	Jones	1895 43	332 0	5 65	3	Comstock

The observations of this celebrated system date back almost to the beginning of double-star Astronomy. The only double star previously recognized which has proved to be binary is  $\alpha$  Centauri †. It was resolved into its components in December, 1689, by FATHER RICHAUD, at Pondicherry, India. On putting one eye to the telescope, and looking at the heavens with the other, BRADLEY found the two components of  $\gamma$  Virginis to be approximately in line with the naked-eye stars  $\alpha$  and  $\delta$  Virginis; this allineation gives a position-angle of  $330^\circ 8$  at the epoch 1718 20. Such an observation has of course some historical interest, but is worthy of little consideration in the discussion of a modern double-star orbit. Neither can any confidence be placed in the position for 1720, which was calculated from a lunar occultation observed by CASSINI while searching for evidence of an atmosphere surrounding the Moon.

The observation which results from the Catalogue of TOBIAS MAYER would be entitled to more weight were it not for the uncertainty of double-star positions deduced from differences of right ascension and declination.

Therefore in the present discussion of the orbit I have relied principally upon observations since the time of WILLIAM STRUVE, but have not entirely ignored the measures of SIR WILLIAM HERSCHEL, which appear to be as good as could be expected from the means at his disposal. After an examination of all the observations, it appeared advisable to base the orbit mainly upon the work of the great standard observers. This sifting of the observational material is rendered the more necessary by virtue of the great number and miscellaneous character of the observers who have occupied themselves with an easy ‡ and celebrated star like  $\gamma$  Virginis. It is probable that more orbits have been computed for this star than for any other binary in the heavens, but as all of these are defective, according to trustworthy recent observations, a new determination of the elements based upon the best measures now available, would seem to be desirable. In dealing with an orbit which has long occupied the

† *Astronomical Journal*, 352

‡ Some of the observations here omitted are good, but in working with the graphical method I have not thought it necessary to use all of the super-abundant material.

attention of eminent men, including SIR JOHN HERSCHEL and the illustrious ADAMS, we could hardly hope for material improvement over the results already obtained, were not the investigation rendered more complete by recent observations, and by the use of the observed distances, which have generally been rejected, but which here acquire a high importance owing to the slow angular motion. The nature of the motion of  $\gamma$  *Virginis* is such that some of the elements, especially the periastron passage and the eccentricity, are determined with great precision; but the period has been underestimated by nearly all recent investigators, and will still remain slightly uncertain, perhaps to the extent of one year

ELEMENTS DERIVED FROM PREVIOUS INVESTIGATIONS

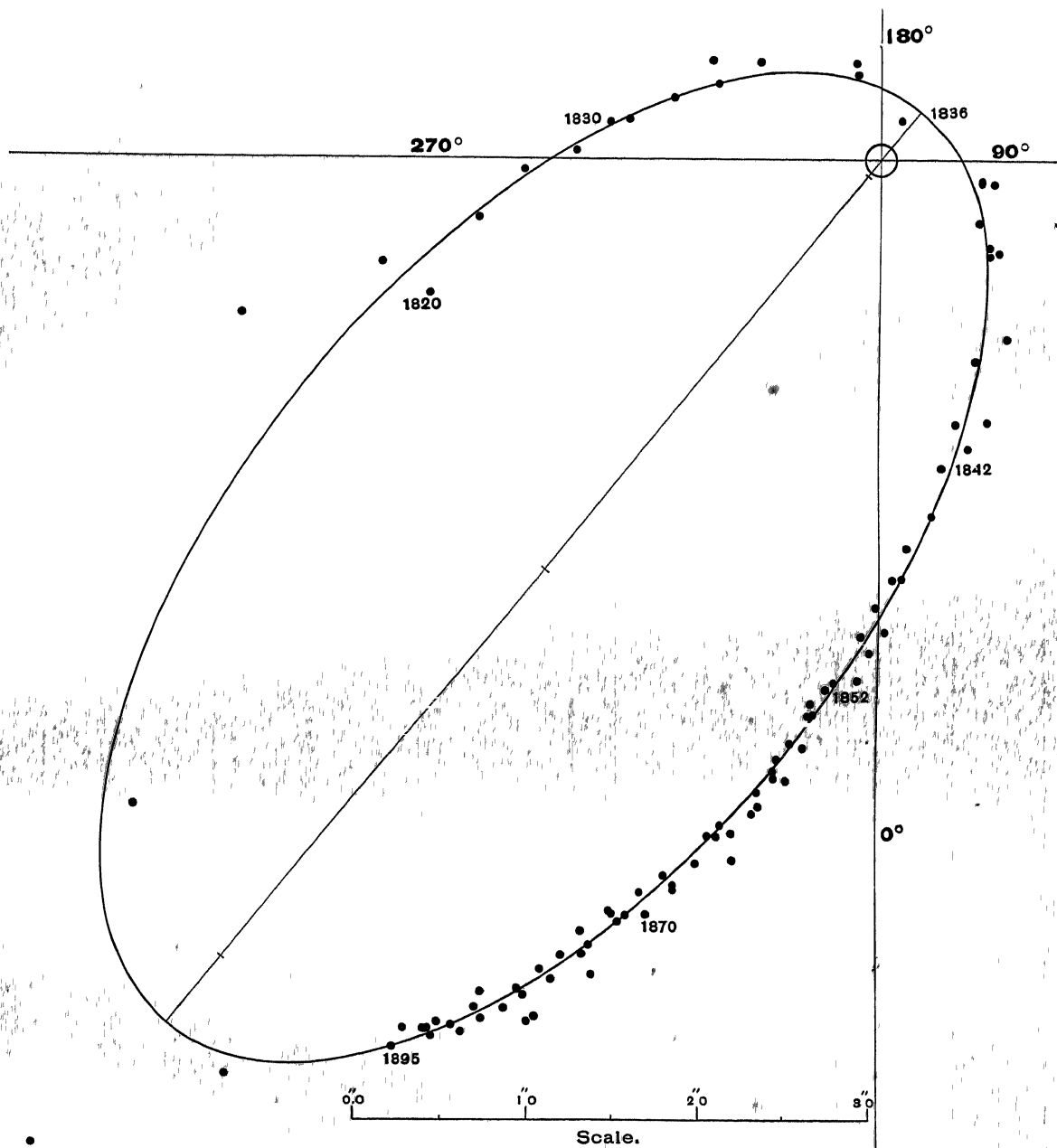
$P$	$T$	$e$	$\alpha$	$\Omega$	$i$	$\lambda$	Authority	Source
<sup>YRS</sup>			<sup>"</sup>	<sup>°</sup>	<sup>°</sup>	<sup>°</sup>		
513 28	1834 01	0 8872	11 830	87 83	68 0	290 0	Herschel, 1831	Mem R A S vol V p 193
628 90	1834 63	0 8335	12 09	97 4	67 03	282 35	Herschel, 1833	Mem R A S, vol VI p 152
145 409	1836 313	0 8681	3 402	60 63	24 65	78 37	Madler, 1841	Dorpat Obs, 1841 p 174
157 562	1836 103	0 8680	3 638	58 38	35 6	94 0	Madler, 1841	A N 368
143 44	1836 29	0 8590	—	70 6	23 1	319 38	Hend'n, 1843	'Spec Hartw', p 345
141 297	1836 228	0 8566	—	78 47	25 23	319 77	Hind, 1845	Mem R A S, vol XVI,
133 5	1836 30	0 8525	3 499	69 67	24 6	249 3	Jacob, 1846	[p 461]
169 445	1836 279	0 8806	—	62 15	25 42	79 07	Madler, 1847	Die Fixs-Syst II p 240
182 12	1836 43	0 8795	—	5 55	23 6	313 75	Herschel, 1847	'Results,' p 297 [p 67]
183 137	1836 385	0 8860	4 336	28 7	30 65	290 5	Herschel, 1850	Mem R A S, vol XVIII,
171 54	1836 40	0 8804	—	20 57	27 38	300 2	Hind, 1851	M N, vol XI, p 136
174 137	1836 34	0 8796	—	34 75	25 45	284 9	Adams, 1851	
184 53	1836 40	0 8794	—	19 12	27 6	295 2	Fletcher, 1853	M N, vol XIII, p 258
148 2	1836 2	0 8725	3 617	41 67	31 95	269 3	Smyth, 1860	'Cycle,' p 356
177 7	1836 50	0 8878	4 226	35 62	37 33	281 7	Smyth, 1860	'Cycle' cont, p 451
185 0	1836 68	0 896	3 97	35 6	35 1	283 7	Thiele, 1866	A N, vol XVIII
175 0	1836 45	0 8715	3 385	—	0 0	long per = 320 0	Fl, 1874	'Catalogue,' p 72
180 54	1836 47	0 8978	4 09	45 82	37 0	93 98	Doberck, 1881	Copernicus, vol I, p 143
179 65	1836 45	0 8904	3 94	46 0	33 95	93 92	Doberck, 1881	Copein, vol I, p 143 [193]
192 07	1836 51	0 895	4 144	54 9	34 12	274 23	See, 1893	Astron & Astro-Phys, Dec

From an investigation of the long list of observations, including the very careful measures recently secured with the 26-inch refractor of the Leander McCormick Observatory of the University of Virginia, we find the following elements of  $\gamma$  *Virginis*.

$$\begin{aligned}
 P &= 194.0 \text{ years} & \Omega &= 50^{\circ} 4 \\
 T &= 1836.53 & i &= 31^{\circ} 0 \\
 e &= 0.8974 & \lambda &= 270^{\circ} 0 \\
 \alpha &= 3^{\circ} 989 & \omega &= -1^{\circ} 8557
 \end{aligned}$$

#### Apparent orbit

$$\begin{aligned}
 \text{Length of major axis} &= 6^{\circ} 824 \\
 \text{Length of minor axis} &= 3^{\circ} 530 \\
 \text{Angle of major axis} &= 140^{\circ} 4 \\
 \text{Angle of periastron} &= 140^{\circ} 4 \\
 \text{Distance of star from centre} &= 3^{\circ} 062
 \end{aligned}$$



$\gamma$  Virginis =  $\Sigma$  1670.



The accompanying table of computed and observed places shows that these are perhaps the most exact elements yet determined for any star. For although all the measures have not been used in forming the mean observations on which the orbit is based, yet those measures which have been employed have been so combined as fairly to represent the best material for each year. Accordingly, the residuals are uniformly small, except just before periastron passage, when the object was extremely difficult, and, as no variation of the elements will materially improve the representation of the observations in this part of the orbit without a corresponding damage elsewhere, we infer that the differences are due mainly to systematic errors in STRUVE's measures.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1718 20	330° 8	326° 2	—	6 27	+ 4 6	—	2	Bradley and Pound
1720 31	319 0	325 0	7 49	6 34	— 6 0	+1 15	1	Cassini
1756 20	324 4	318 7	6 50	6 46	+ 5 7	+0 04	—	Tobias Mayer
1781 89	310 7	308 1	5 70	5 67	+ 2 6	+0 03	1	Herschel
1803 37	300 2	299 6	—	4 60	+ 0 6	—	8 obs	Herschel
1819 40	—	286 9	3 56	3 16	—	+0 40	1+	Struve
1820 28	284 9	284 9	2 76	2 97	0 0	—0 21	5	Struve
1822 25	283 4	283 4	3 79	2 85	0 0	—0 06	2	Herschel and South
1823 32	281 6	281 8	2 95	2 70	— 0 2	+0 25	1, 3	Struve
1825 32	277 9	278 2	2 37	2 43	— 0 3	—0 06	6	Struve
1828 38	271 5	271 4	2 07	2 01	+ 0 1	+0 06	1	Struve
1829 30	268 0	268 8	1 78	1 86	— 0 8	—0 08	7	H 2, $\Sigma$ 5
1830 59	262.2	264.1	1.59	1.63	— 1.9	—0 04	7	Bessel
1831.36	260.9	260 8	1 49	1 50	+ 0.1	—0 01	5	Struve
1832 52	253 5	253 8	1 26	1 26	— 0 3	0 00	4	Struve
1833 36	240 1	247 2	1 14	1 09	— 7 1	+0 05	8, 2	Dawes
1833 37	245 5	247 1	1 05	1 08	— 1 6	—0 03	7	Struve
1834 38	231 6	235 0	0 91	0 84	— 3 4	+0 07	5	Struve
1834 84	213 6	226 5	—	0 72	—12 9	—	1	Struve
1835 38	195 5	212 2	0 51	0 58	—16 7	—0 07	9	Struve
1835 39	195 2	212 0	0 57	0 57	—16 8	0 00	1	Senff
1835 42	197 1	211 3	—	0 56	—14 2	—	1	O Struve
1836.41	151 6	150 2	0 26	0 36	+ 1 4	—0 10	3	Struve
1836 41	158 7	150 2	—	0 36	+ 8 5	—	2	O Struve
1836 41	153 8	150 2	—	0 36	+ 3 6	—	1	Sabler
1837 41	77 9	78 2	0 58	0 52	— 0 3	+0 06	6	O Struve
1837 41	78 5	78 2	0 67	0 52	+ 0 3	+0 15	1	Encke
1838 08	57 5	58 0	0 67	0 70	— 0 5	—0 03	1	Herschel
1838 40	51 9	50 8	0 86	0 78	+ 1 1	+0 08	—	Struve
1838 43	51 1	50 0	0 80	0 79	+ 1 1	+0 01	—	O Struve
1838 43	49 2	50 0	0 83	0 79	— 0 8	+0 04	3±	Galle and Mädler
1839 33	35 5	37 3	1 26	1 01	— 1 8	+0 25	5, 1	Galle 5-0, Dawes 0-1
1840 36	26 3	28 1	1 28	1 23	— 1 8	+0 05	16, 12±	Kaiser 1±, Dawes 11-7, $O\Sigma$ 5
1841 41	22 4	22 0	1 63	1 44	+ 0 4	+0 19	4	O Struve
1842 21	16 6	17 7	1 58	1 60	— 1 1	—0 02	7, 5	Mädler
1842 41	17 1	16 1	1 73	1 67	+ 1 0	+0 06	4, 5	$O\Sigma$ 4-0, Dawes 0-5
1843 37	12 1	13 7	1 80	1 78	— 1 6	+0 02	17, 12	Mädler 7, Dawes 10-5
1844 36	8 9	10 1	2 06	1 97	— 1 2	+0 09	8, 7	Mädler
1845 46	4 5	7 2	2 23	2 15	— 2 7	+0 08	2	O Struve
1846 59	3 6	4 6	2 21	2 31	— 1 0	—0 10	5	$O\Sigma$ 2, Dawes 2, Mitchell 1



$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1847 38	2 5	3 0	2 40	2 42	- 0 5	-0 02	11	Dawes 8, $O\Sigma$ 3
1848 34	0 8	1 3	2 71	2 55	- 0 5	+0 16	7, 6	Madler
1848 40	359 8	1 1	2 57	2 56	- 1 3	+0 01	12	Dawes 9, $O\Sigma$ 3
1849 37	359 0	359 5	2 84	2 67	- 0 5	+0 17	5, 4	Dawes
1850 40	358 0	357 9	2 74	2 80	+ 0 1	-0 06	11, 4	Jacob 2-0, $O\Sigma$ 4, Madler 1 0,
1851 28	357 9	356 8	2 99	2 90	+ 1 1	+0 09	4	Madler [Madler 1 0]
1851 40	356 5	356 4	2 99	2 95	+ 0 2	+0 04	5	Dawes
1852 38	354 6	355 4	3 06	3 01	- 0 8	+0 05	7	Dawes 2, Madler 2, $O\Sigma$ 3
1853 30	353 6	354 3	3 21	3 13	- 0 7	+0 08	5, 4	Jacob 2, Dawes 3 2
1853 56	353 1	354 0	3 15	3 16	- 0 9	-0 01	12	Madler 6, $O\Sigma$ 4, Jacob 2
1854 43	353 2	353 0	3 22	3 26	+ 0 2	-0 04	15	Dawes 8, Dembowski 7
1855 18	351 4	352 3	3 40	3 33	- 0 9	+0 07	8	$O\Sigma$ 3, Dembowski 1
1855 67	352 8	351 8	3 40	3 40	+ 1 0	0 00	10, 9	Senff 1, Madler 3, Dawes 4 3,
1856 39	350 5	351 3	3 56	3 44	- 0 8	+0 12	5	Dembowski [Morton 3]
1857 28	349 1	350 2	3 70	3 56	- 1 1	+0 14	20	Dembowski 6, Dawes 7, Senff 7
1857 56	350 2	350 1	3 57	3 57	+ 0 1	0 00	22, 21	Ma 9 8, Da 6, $O\Sigma$ 3, Ja 5
1858 36	349 2	349 3	3 80	3 65	- 0 1	+0 15	8, 6	Dembowski 6, Madler 2 0
1858 44	350 2	349 3	3 59	3 66	+ 0 9	-0 07	16	Senff 3, $O\Sigma$ 2, Da 8, Mo 3
1859 36	349 1	348 6	3 83	3 72	+ 0 5	+0 11	24, 23	Mo 4, Ma 9 8, $O\Sigma$ 3, Senff 3
1860 40	347 6	347 6	3 97	3 84	0 0	+0 13	3	Madler 1, Knott 2 [Dawes 5]
1861 23	346 6	347 1	3 93	3 90	- 0 5	+0 03	9	$O\Sigma$ 4, Powell 5
1861 38	348 1	347 0	4 11	3 91	+ 1 1	+0 20	3+	Madler 3, Auwers —
1862 28	346 0	346 3	4 01	3 99	- 0 3	+0 02	13, 10	Da 5-3, Po 3-2, Ma 3, $O\Sigma$ 2
1863 54	346 4	345 5	3 99	4 06	+ 0 9	-0 07	28	$O\Sigma$ 2, Dembowski 26
1864 43	345 3	344 9	4 18	4 14	+ 0 4	+0 04	11	Senff 2, $O\Sigma$ 3, Da 4, Kn. 2
1865 54	344 2	344 2	4 36	4 22	0 0	+0 14	36, 35	Da 7-6, Kn 3, Dem 26
1866 36	344 1	343 7	4 34	4 28	+ 0 4	+0 06	5	Senff 3, $O\Sigma$ 2
1867 80	343 2	342 8	4 30	4 40	+ 0 4	-0 10	12	Dembowski
1868 43	342 2	342 4	4 47	4 45	- 0 2	+0 02	9	O Struve 2, Mann 7
1869 98	341 8	341 6	4 43	4 53	+ 0 2	-0 10	17	Dunér
1870 74	342 7	341 2	4 54	4 60	+ 0 5	-0 06	14	Dembowski 11, $O\Sigma$ 3
1871 43	340 5	340 9	4 87	4 65	- 0 4	+0 22	8	Kn 3, Gled 3, W & S 2
1872 12	341 1	340 5	4 59	4 68	+ 0 6	-0 09	17	Dunér
1872 63	340 4	340 0	4 61	4 71	+ 0 4	-0 13	13	$O\Sigma$ 3, Dembowski 10
1873 43	340 3	339 9	4 77	4 76	+ 0 4	+0 01	13	Gled 2, $O\Sigma$ 3, Ma 5, Ian 3
1874 64	340 4	339 3	4 97	4 84	+ 1 1	+0 13	5	Gledhill 2, $O\Sigma$ 3
1875 18	338 8	339 0	4 76	4 88	- 0 2	-0 12	18	Dunér 14, Gledhill 4
1875 36	339 4	338 9	4 83	4 89	+ 0 5	-0 06	25	Dembowski 11, Schnaparelli 13
1876 34	339 1	338 5	5 02	4 95	+ 0 6	+0 07	26	Gled 13, Ill 4, Sch 4, Dk 5
1877 62	338 0	337 9	4 94	5 01	+ 0 1	-0 07	22	Schnaparelli 14, Dembowski 8
1878 37	337 1	337 6	5 06	5 06	- 0 5	0 00	3, 5	Goldney
1879 25	337 9	337 2	5 08	5 12	+ 0 7	-0 04	13	Schnaparelli 10, Hall 3
1880 30	337 5	336 8	5 36	5 17	+ 0 7	+0 19	2	Burnham
1881 44	336 2	336 3	5 28	5 22	- 0 1	+0 06	14, 17	Hall 0 4, Bigourdan 14 13
1882 41	336 6	335 9	5 23	5 28	+ 0 1	-0 05	10	Schnaparelli
1883 28	335 6	335 6	5 30	5 31	0 0	-0 01	20, 18	En 7-5, Hall 0 5; Sch 8
1884 38	335 8	335 1	5 34	5 38	+ 0 7	-0 04	17	Hall 5, Per 3, Sch 9
1885 35	334 1	334 8	5 32	5 40	- 0 7	-0 08	19	Cop 1, H C W 2, Sch 16
1886 36	334 9	334 4	5 45	5 45	+ 0 5	0 00	4, 6	Hall 4, H C W 0-2
1887 38	334 5	334 0	5 50	5 50	+ 0 5	0 00	11	Schnaparelli 7; Hall 4
1888 32	334 1	333 6	5 58	5 55	+ 0 5	+0 03	9	Glas 2, Hall 5, Sch 2
1889 40	333 4	333 3	5 56	5 60	+ 0 1	-0 04	11	Burnham 3, Hall 5, Sch 3
1890 43	332 8	332 9	5 59	5 64	- 0 1	-0 05	3	Hall
1891 44	332 5	332 6	5 70	5 67	- 0 1	+0 03	3	See
1892 56	332 3	332 2	5 64	5 71	+ 0 1	-0 07	16	Sch 6, Lv 2, Com 3, Big 3,
1893 44	332 2	331 9	5 65	5 75	+ 0 3	-0 10	11, 5	Sch 6, Com 1, Big 4
1894 33	331 1	331 6	5 71	5 79	- 0 5	-0 08	10, 6	Com 2-0, Sch 2-0, Big 6
1895 30	331 1	331 3	5 84	5 83	- 0 2	+0 01	5, 4	See

It will be seen that in this orbit the line of nodes coincides with the minor axis of the real ellipse, which is also the minor axis of its projection; and owing to the small inclination the apparent ellipse is only slightly less eccentric than the real ellipse, so that the foci of the two ellipses very nearly coincide. This renders the motion of the radius vector in the apparent orbit very nearly the same as in the real orbit, and makes  $\gamma$  *Virginis* an object of peculiar interest from the point of view of the study of the law of attraction in the stellar systems. From direct observation we are enabled to say that if there is any deviation from the Keplerian law of areas, it must be extremely slight. Therefore the force is certainly central, and the probabilities are overwhelming that the principal star, which is so near the focus of the apparent orbit, occupies the focus of the real orbit, or that the law of attraction is Newtonian gravitation. Other researches in double-star Astronomy increase the probability of the law of gravitation, and leave no adequate ground for doubt as to its absolute universality. Yet a prolonged study of the motion of  $\gamma$  *Virginis* will eventually give a very precise criterion for the rigor of this law, as well as throw light upon the question of the existence of disturbing bodies in binary systems.

The orbit of  $\gamma$  *Virginis* is very remarkable for its high eccentricity, which surpasses that of any other known stellar orbit. This characteristic of  $\gamma$  *Virginis*, which SIR JOHN HERSCHEL recognized when he declared the eccentricity to be "physically speaking, the most important of all the elements" (*Results at Cape of Good Hope*, p. 294), seems to preclude the permanent existence of a third body in the system; for if a companion to either of the components existed, its motion would be affected by an equation of enormous magnitude, analogous to the annual equation in the moon's motion, and at the time of periastron passage would probably soon cause the body to come into collision with one of the stars, or be driven off in an orbit analogous to a hyperbola.

Thus, although the above orbit is exact to a very high degree, the system will still deserve the occasional attention of astronomers.

Since the angular motion for many years to come will be extremely slow, observations of distance will be more valuable than angular measures in effecting a further improvement of the elements.

42 COMAE BERENICES =  $\Sigma 1728$ .

$\alpha = 13^h 5^m 1$  ,  $\delta = +18^\circ 4'$   
 6, orange , 6, orange

*Discovered by William Struve in 1827*

## OBSERVATIONS

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1827 83	189 5	obl	2-1	Struve	1853 09	194 2	0 62	4	Dawes
1829 40	191 6	0 64	3	Struve	1853 35	194 1	0 61	14-12	Mädler
1833.37	170 7?	obl	1	Struve	1853 40	190 8	0 57	3	O Struve
1834 43	228 3	obl	1	Struve	1854 38	194 1	0 60	1	O Struve
1835 39	11 2	—	4	Struve	1854 39	193 6	0 61	8-7	Mädler
1836 41	10 2	0 30	3	Struve	1854 39	192 8	0 55	5	Dawes
1837 40	11 0	0 39	6	Struve	1855 38	198 7	0 55	2-1	Mädler
1838 41	11 5	0 36	3	Struve	1855 44	189 1	0 62	2	O Struve
1839 42	12 2	0 59	—	Galle	1856 40	192 7	0 52	5-4	Mädler
1840 45	15 7	0 55	3	O Struve	1856 42	192 0	0 78	3	Winnecke
1840 74	18 5	0 4±	3	Dawes	1856 96	192 5	0 47	6	Secchi
1841 40	14 7	0 32	12-5	Mädler	1857 39	198 3	0 50	3-1	Mädler
1841 41	14 5	0 49	2	O Struve	1857 49	187 7	0 44	2	O Struve
1842 40	13 9	0 32	3	O Struve	1858 40	196 3	0 4±	6	Mädler
1842 45	15 6	—	4	Mädler	1858 44	188 5	0 38	2	O Struve
1842 53	single	—	—	Dawes	1859 36	215 8	0 2±	3	Mädler
1843 28	single	—	1	Mädler	1859 37	single	—	—	O Struve
1843.45	single	—	—	Dawes	1860 34	3 5?	0 2±	1	Dawes
1844 32	189 5	—	2	Mädler	1861 37	10 7	—	2	Mädler
1845 47	single	—	—	O Struve	1861 40	182 8	0 50	—	Winnecke
1846 40	66 8?	obl ?	3	O Struve	1861 42	15 6	0 43	2	O Struve
1847 42	195 5	0 20	1	O Struve	1862 26	9 1	cuneo	7	Dembowski
1848 42	192 7	0 27	3	O Struve	1862 37	16 5	—	2	Mädler
1849 42	188 6	0 42	3.	O Struve	1862 40	11 6	0 54	2	O. Struve
1850 39	191 4	0 48	3	O Struve	1862 42	2 9	—	—	Oblomievsky
1850 99	193 3	0 40	1	Mädler	1863 25	11 0	0 5±	1	Dawes
1851 27	191 3	0 35	1	Mädler	1863 44	9 3	0 55	1	O Struve
1851 42	187 0	0 49	4	O Struve	1864 42	10 9	0 3±	2	Secchi
1851 96	194 5	0 45	3-2	Mädler	1864 42	12 5	0 51	3	O. Struve
1852.42	191 0	0 54	6-5	Mädler	1864 43	13 4	0 45	1	Dawes
1852 43	190 9	0 56	3	O Struve	1865 53	13 9	0 25±	2	Secchi
1852.45	12 2	0 48	—	Fearnley	1865 57	9 5	cuneo	5	Dembowski
					1865 59	13 7	0 54	6	Englemann
					1866 64	8.5	0.40	3	O Struve

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1867 32	21 4	—	1	Winlock	1881 25	192 2	0 70	2	Bigourdan
1867 32	24 7	—	1	Searle	1881 25	190 9	0 60	4-3	Doberck
1867 47	13 0	0 36	2	O Struve	1881 37	193 0	0 64	4	Burnham
1867 77	14 8	cuneo	2	Dembowski	1881 38	191 6	0 6 $\pm$	5	Schiaparelli
1868 44	15 8	0 21	2	O Struve	1881 39	192 6	0 53	4	Hall
1869 24	11 6	—	1	Leyton Obs	1881 41	193 5	0 5 $\pm$	7-0	Perry
1869 40	19 ?	obl	3	Dunér	1882 35	194 4	1 00	4-2	Seabroke
1869 47	15 ?	obl ?	1	O Struve	1882 38	191 9	0 54	4	Hall
1870 44	single	—	—	O Struve	1882 42	191 4	0 6 $\pm$	6	Schiaparelli
1870 45	16	obl	4	Dunér	1882 46	184 6	0 51	1	O Struve
1871 40	194 6	obl	3	Dembowski	1882 93	192 1	0 56	7	Englemann
1871 43	single	—	—	O Struve	1883 42	193 2	0 50	4	Hall
1872 42	200	obl	1	O Struve	1883 42	191 1	0 5 $\pm$	8	Schiaparelli
1872 52	200	obl	2	Dunér	1883 48	193 4	0 55	5-4	Küstner
1873 36	single	—	1	J M Wilson	1883 51	191 5	0 53	2	Perrotin
1873 46	189 0	0 20	2	O Struve	1884 39	195 8	0 3 $\pm$	4	Schiaparelli
1873 74	200 5	obl	3	Dembowski	1884 40	189 7	0 36	3	Hall
1874 41	189 2	0 30	2	O Struve	1885 41	single	—	1	Perrotin
1875 30	192 5	0 5 $\pm$	1	Seabroke	1885 42	single	—	4	Schiaparelli
1875 43	192 2	0 4 $\pm$	10	Schiaparelli	1885 49	10 2	0 35	1	Hall
1875 43	190 4	0 51	5	Dembowski	1886 42	10 0	0 27	3	Hall
1875 46	189 7	0 39	3	O Struve	1886 51	15 8	0 26	6	Schiaparelli
1875 53	191 5	0 32	7-6	Dunér	1887 42	13 1	0 38	9	Schiaparelli
1876 36	186 4	0 5 $\pm$	1	W Smith	1887 44	13 6	0 42	4	Hall
1876 38	191 2	0 58	4	Dembowski	1888 27	12 0	0 48	3	Schiaparelli
1876 40	193 4	0 40	4	Hall	1888 40	13 8	0 45	3	Hall
1876 42	188 0	0 50	3	O Struve	1888 43	8 7	0 42	1	O Struve
1876 45	193 1	0 5 $\pm$	4	Schiaparelli	1889 08	10 5	0 56	1	Leavenworth
1877 41	190 4	0 52	9-5	Schiaparelli	1889 39	11 8	0 61	1	O Struve
1877 45	191 4	0 51	5	Dembowski	1889 41	10 9	0 49	5	Schiaparelli
1877 46	186 0	0 47	3	O Struve	1890 33	9 3	0 70	4	Burnham
1878 37	191 3	0 65	1	O Struve	1890 43	10 5	0 51	12	Schiaparelli
1878 38	193 7	obl	3	Jedrzejewicz	1891 44	11 4	0 51	3	Hall
1878 38	189 6	0 51	4	Hall	1891 44	10 7	0 49	9	Schiaparelli
1878 43	190 8	0 57	3	Dembowski	1892 37	11 7	0 47	2-1	Leavenworth
1879 37	192 1	0 68	2	Burnham	1892 40	10 7	0 42	6	Schiaparelli
1879 42	193 2	0 51	4	Hall	1892 44	11 7	0 40	8-6	Bigourdan
1879 42	191 4	0 6 $\pm$	5	Schiaparelli	1893 45	10 2	0 32	5	Schiaparelli
1879 44	190 9	0 65	1	O Struve	1894 33	0 1	0 25	3	Comstock
1880 36	191 7	0 52	4	Hall	1894 45	16 6	—	1-0	Bigourdan
1880 41	194 3	obl	4	Jedrzejewicz	1894 46	10 38	0 22	4-5	Schiaparelli
					1895 29	13 9	0 14	3	See

Since the date of discovery this remarkable star has described almost three revolutions. From the first it was given particular attention by WILLIAM and

OTTO STRUVE, and the peculiar and unique character of the system has fully justified the care with which it has been measured. The only previous investigation\* of the orbit is that made by OTTO STRUVE and DUBLAGO in 1874 (*Monthly Notices* 1874-5, p. 367). O. STRUVE's elements are as follows.

$$\begin{array}{ll} P = 25\ 71 \text{ years} & \Omega = 11^{\circ} 0 \\ T = 1869\ 92 & i = 90^{\circ} \\ e = 0\ 480 & \lambda = 99^{\circ} 18 \\ a = 0''\ 657 & \end{array}$$

Some three years ago BURNHAM placed at my disposal a list of measures which was nearly complete, I have since added to it such as were omitted, and besides made new observations during 1895. When scrutinized under the fine definition of the 26-inch Clark Refractor of the University of Virginia the pair proved to be excessively close, and with a power of 1300 could only be elongated. The object has now become single in all existing telescopes and can not again be separated until about 1899.

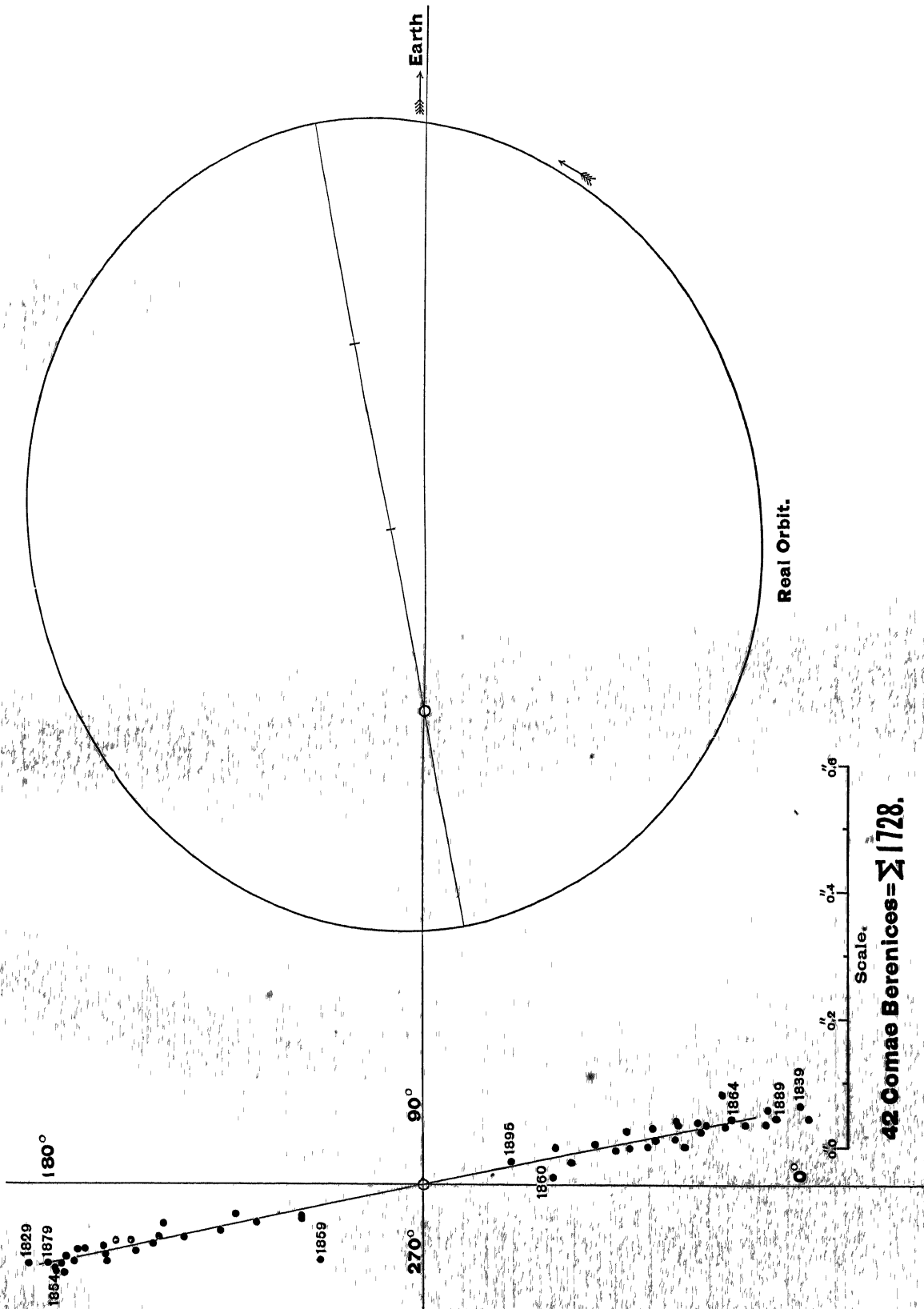
The method followed in the present investigation of the orbit is not very different from that employed by OTTO STRUVE, except that the results are based upon the measures of all reliable observers and are rendered more complete by the observations made since 1874. The list of measures is complete to the occultation of 1896.

It will be seen from an examination of the observations that the motion is to all appearances exactly in the plane of vision, and hence with the exception of the node and inclination, the elements are based wholly on the distances. O. STRUVE's elements are very good, and it would therefore be sufficient to apply differential corrections to his values, but as I had independently discovered a graphical method similar to that employed by him, it seemed of interest to make use of it in deriving approximate values directly from the phenomena. With the elements approximately determined, the observations furnished 52 equations of condition, which were solved for the five unknowns, the weights assigned being proportional to the number of nights. An application of the corrections resulting from the Least Square adjustment gave the following values of the elements

$$\begin{array}{ll} P = 25\ 556 \text{ years} & \Omega = 11^{\circ} 9 \\ T = 1885\ 69 & i = 90^{\circ} \\ e = 0\ 461 & \lambda = 280^{\circ} 5 \\ a = 0''\ 6416 & n = \pm 14^{\circ} 0867 \end{array}$$

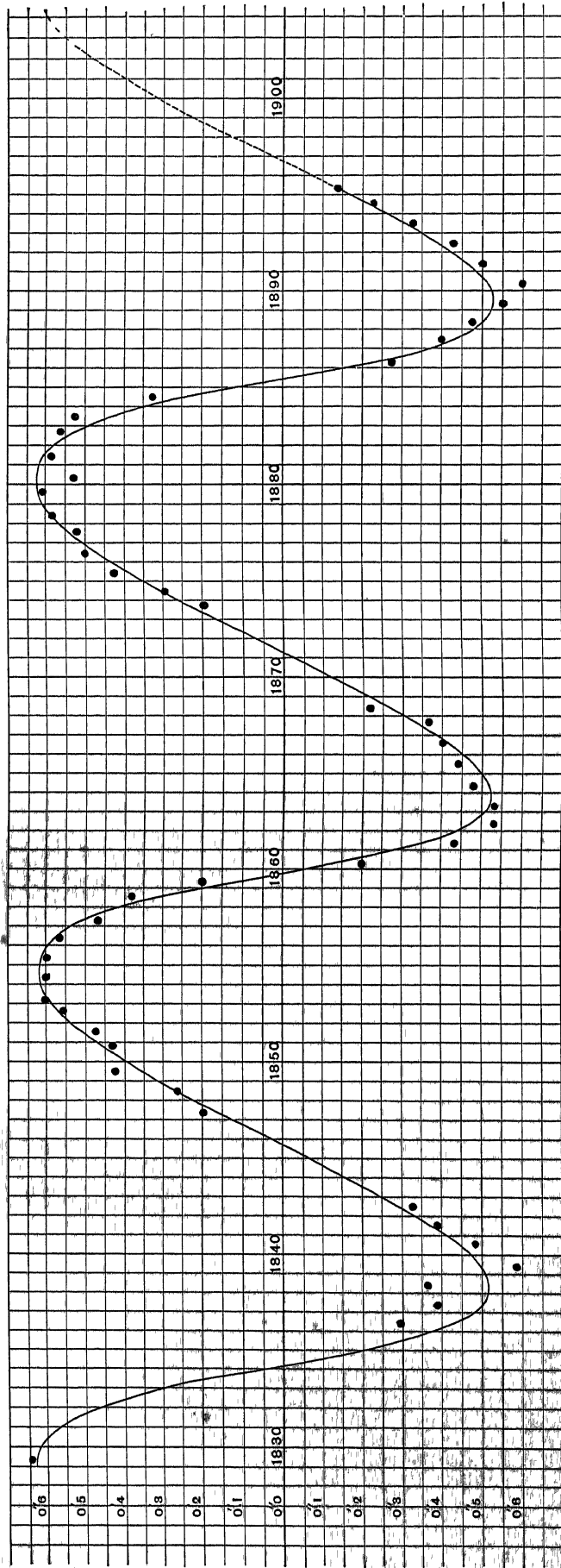
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\**Monthly Notices*, June, 1896



42 Comae Berenices =  $\Sigma$  1728.





**Graphical Illustration of the Motion of 42 Comae Berenices =  $\Sigma$  1728.**





## Apparent orbit

Length of major axis	= $1'' 147$
Length of minor axis	= $0'' 00$
Angle of major axis	= $11^\circ 9$
Angle of periastron	= $11^\circ 9$
Distance of star from centre	= $0'' 054$

The apparent motion is shown in the accompanying diagram, to which is added a figure of the real orbit. A graphical illustration of the motion, obtained by taking the  $x$ -axis to represent the time, while the ordinates represent the distances, was employed in finding the approximate values of the elements, the curve here traced represents the motion according to the elements as corrected. This orbit of 42 *Comae Berenices* is one of the most exact of double-star orbits, and will never require any but very slight modifications. The period can hardly be in error by more than 0.1 year, while a variation of  $\pm 0.01$  in the eccentricity is very improbable.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1827.83	189.5	191.9	obl	0.63	-2.4	+0.01	2-1	Struve
1829.40	191.6	191.9	0.64	—	-0.3	—	3	Struve
1833.37	170.7?	191.9	obl	—	-21.2	—	1	Struve
1834.43	228.3	191.9	obl	—	+36.4	—	1	Struve
1835.39	11.2	11.9	—	—	-0.7	—	4	Struve
1836.41	10.2	11.9	0.30	0.42	-1.7	-0.12	3	Struve
1837.40	11.0	11.9	0.39	0.50	-0.9	-0.11	6	Struve
1838.41	11.5	11.9	0.36	0.51	-0.4	-0.15	3	Struve
1839.42	12.2	11.9	0.59	0.50	+0.3	+0.09	—	Galle
1840.60	17.1	11.9	0.48	0.44	+5.2	+0.04	6	O Struve 3, Dawes 3
1841.40	14.6	11.9	0.40	0.38	+2.7	+0.02	14-7	O Struve 2, Madler 12-5
1842.43	14.7	11.9	0.32	0.30	+2.8	+0.02	7-3	O Struve 3, Madler 4-0
1843.36	—	single	—	—	—	—	2	Madler 1, Dawes—
1844.32	189.5	191.9	—	—	-2.4	—	2	Madler
1845.47	—	single	—	—	—	—	—	O Struve
1846.40	66.8?	191.9	obl?	—	+54.9	—	3	O Struve
1847.42	195.5	191.9	0.20	0.18	+3.6	+0.02	1	O Struve
1848.42	192.7	191.9	0.27	0.27	+0.8	$\pm 0.00$	3	O Struve
1849.42	188.6	191.9	0.42	0.36	-3.3	+0.06	3	O Struve
1850.69	192.3	191.9	0.44	0.45	+0.4	-0.01	4	O Struve 3, Madler 1
1851.55	190.9	191.9	0.47	0.51	-1.0	-0.04	8-6	Madler 1-0, O $\Sigma$ 4, Madler 3-2
1852.42	191.0	191.9	0.55	0.56	-0.9	-0.01	9-8	Madler 6-5, O Struve 3
1853.28	193.0	191.9	0.60	0.60	+1.1	$\pm 0.00$	21-16	Dawes 4, Madler 14-12, O $\Sigma$ 3
1854.39	193.5	191.9	0.60	0.62	+1.6	-0.02	14-13	O $\Sigma$ 1, Madler 8-7, Dawes 5
1855.41	193.9	191.9	0.59	0.61	+2.0	-0.02	4-3	O Struve 2, Madler 2-1
1856.59	192.4	191.9	0.57	0.57	+0.5	$\pm 0.00$	14-13	Madler 5-4; Winn 3, Secchi 6
1857.44	193.0	191.9	0.47	0.51	+1.1	-0.04	5-3	Madler 3-1, O Struve 2
1858.42	192.4	191.9	0.39	0.35	+0.5	+0.04	8	Madler 6, O Struve 2
1859.36	215.8	191.9	0.2 $\pm$	0.14	+23.9	+0.06	3	Madler
1860.34	3.5?	11.9	0.2 $\pm$	0.12	-8.4	+0.08	1	Dawes
1861.40	13.1	11.9	0.43	0.34	+1.2	+0.09	4-2	Madler 2-0, O Struve 2
1862.34	12.4	11.9	0.54	0.46	+0.5	+0.08	11-2	Dem 7-0, Madler 2-0, O $\Sigma$ 2

$t$	$\theta_o$	$\theta_i$	$\rho_o$	$\rho_i$	$\theta_o - \theta_i$	$\rho_o - \rho_i$	$n$	Observers
1863 35	10 2	11 9	0 53	0 52	- 1 7	+0 01	2	Dawes 1, O Struve 1
1864 42	12 3	11 9	0 48	0 51	+ 0 4	-0 03	6-4	Secchi 2-0, O $\Sigma$ 3, Dawes 1
1865 56	12 4	11 9	0 44	0 47	+ 0 5	-0 03	13-8	Secchi 2, Dem 5-0, En 6
1866 64	8 5	11 9	0 40	0 41	- 3 4	-0 01	3	O Struve
1867 62	13 9	11 9	0 36	0 33	+ 2 0	+0 03	4-2	O Struve 2, Dembowski 2-0
1868 44	15 8	11 9	0 21	0 25	+ 3 9	-0 04	2	O Struve
1869 37	15 2	11 9	obl ?	—	—	—	5	Ley 1, Duner 3, O Struve 1
1870 45	16 0	11 9	obl	—	—	—	4	Duner
1871 41	194 6	191 9	obl	—	—	—	3-0	Dembowski
1872 47	200 0	191 9	obl	—	—	—	3	O Struve 1, Duner 2
1873 60	194 7	191 9	0 20	0 23	+ 2 8	-0 03	5-2	Dembowski 3-0, O Struve 2
1874 41	189 2	191 9	0 30	0 30	- 2 7	$\pm$ 0 00	2	O Struve [Du 7-6]
1875 42	191 3	191 9	0 43	0 40	- 0 6	+0 03	26-25	Sea 1, Sch 10, Dem 5, O $\Sigma$ 3,
1876 40	190 4	191 9	0 50	0 47	- 1 5	+0 03	16	Sm 1, Dem 4, Hall 4, O $\Sigma$ 3,
1877 43	190 9	191 9	0 52	0 53	- 1 0	-0 01	17-13	Sch 9-5, Dem 5, O $\Sigma$ 3 [Sch 1
1878 40	191 4	191 9	0 58	0 58	- 0 5	$\pm$ 0 00	11-8	Jed 3-0, Hl 4, Dem 3, O $\Sigma$ 1
1879 40	191 9	191 9	0 61	0 61	$\pm$ 0 0	$\pm$ 0 00	12	$\beta$ 2, Hall 4, Sch 5, O $\Sigma$ 1
1880 38	193 0	191 9	0 52	0 62	+ 1 1	-0 10	8	Hall 4, Jed 4 [Perry 7-0]
1881 34	192 3	191 9	0 59	0 61	- 0 4	-0 02	26-18	Big 2, Dk 4-3, $\beta$ 4, Sch 5, Hl 4
1882 52	190 9	191 9	0 56	0 54	- 1 0	+0 02	22-18	Sea 4-0, Hl 4, Sch 6, O $\Sigma$ 1, En 7
1883 46	192 3	191 9	0 52	0 43	+ 0 4	+0 09	19-18	Hl 4, Sch 8, Ku 5-4, Per 2
1884 40	192 7	191 9	0 33	0 26	+ 0 8	+0 07	7	Schiaparelli 4, Hall 3
1886 46	12 9	11 9	0 27	0 25	+ 1 0	+0 02	9	Hall 3, Schiaparelli 6
1887 43	13 3	11 9	0 40	0 41	+ 1 4	-0 01	13	Schiaparelli 9, Hall 4
1888 33	11 5	11 9	0 47	0 49	- 0 4	-0 02	7-6	Schiaparelli 3, Hall 3, O $\Sigma$ 1-0
1889 25	11 1	11 9	0 55	0 52	- 0 8	+0 03	7	Leavenworth 1, Sch 5, O $\Sigma$ 1
1890 38	9 9	11 9	0 60	0 51	- 2 0	+0 09	16	$\beta$ 4, Schiaparelli 12
1891 44	11 0	11 9	0 50	0 45	- 0 9	+0 05	12	Hall 3, Schiaparelli 9
1892 40	11 4	11 9	0 43	0 39	- 0 5	+0 04	16-13	Lv 2-1, Sch 6, Bigouden 8-6
1893 45	10 2	11 9	0 32	0 31	- 1 7	+0 01	5	Schiaparelli
1894 41	9 0	11 9	0 23	0 22	- 2 9	+0 01	8	Com 3, Big 1-0, Sch 4-5
1895 29	13 9	11 9	0 14	0 14	+ 2 0	$\pm$ 0 00	3	See

O $\Sigma$ 269.

$\alpha = 13^h 28^m 3$  ,  $\delta = +35^\circ 46'$   
 73, yellowish , 77, yellowish

Discovered by Otto Struve in 1844

## OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1844 31	218 0	0 33	1	O Struve	1855 47	223 6	0 27	1	O Struve
1846 38	231 1	0 39	3	O Struve	1861 26	242 8	0 33	1	O Struve
1847 30	222 7	0 25	1	Madler	1865 50	45	oblonga	1	Dembowski
1847 41	215 1	0 18	1	Madler	1868 26	semiplice	—	1	Dembowski
1849 47	218 0?	oblong	1	O Struve	1872 47	257 1	oblong	1	O Struve
1851 30	222 4	0 20	1	Madler	1877 26	oblonga in 180°?	—	1	Dembowski
1851 39	228 9	0 33	1	O Struve					

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1883 41	61 4	0 22	4	Englemann	1891 49	28 9	0 19	2	Schiaparelli
1885 42	195	elong	2	Periotin	1892 40	215 0	0 21	2	Burnham
1889 52	207 7	0 22	3	Schiaparelli	1894 40	210 5	0 30 ±	1	Comstock
1890 41	26 3	0 22	1	Schiaparelli	1895 41	219 0	0 225	2	Schiaparelli
1891 26	213 4	0 22	3	Burnham	1895 74	235 4	0 44	1	See

Since the epoch of discovery in 1844 the companion has described an entire revolution, but the discordance of the observations renders it difficult to define the exact character of the orbit. The measures are frequently very inconsistent, and the most careful selections are necessary in forming the mean places. During the past few years the system has received merited attention from BURNHAM and SCHIAPARELLI, their measures make known the nature of the motion and enable us to fix the elements with considerable precision. BURNHAM was the first to give a proper interpretation of the earlier observations (*Observatory*, July, 1891), and to find a satisfactory apparent ellipse. GORE afterwards attempted an investigation of the orbit based on the angles only; he found the following elements

$$\begin{aligned}
 P &= 47.70 \text{ years} & \Omega &= 51^\circ 93' \\
 T &= 1883.12 & i &= 82^\circ 81' \\
 e &= 0.0575 & \lambda &= 43^\circ 51' \\
 a &= 0''.58
 \end{aligned}$$

The exclusive use of angles in deriving the orbits of close and difficult double stars has frequently led to erroneous results, because when the distance is very small it is even more reliable than the angle. The use of distances becomes not only important but also necessary when the orbit is highly inclined, and the companion therefore has an angular motion which is small compared to the errors of observation, as is the case with 0Σ 269. Accordingly in dealing with the orbit of this star we have given rather more attention to the distances than to the discordant and frequently retrograding angles. Using certain selected measures of the best observers we find the elements of 0Σ 269 to be as follows:

$$\begin{aligned}
 P &= 48.8 \text{ years} & \Omega &= 46^\circ 2' \\
 T &= 1882.80 & i &= 71^\circ 3' \\
 e &= 0.361 & \lambda &= 32^\circ 63' \\
 a &= 0''.3248 & n &= +7^\circ 3771
 \end{aligned}$$

## Apparent orbit.

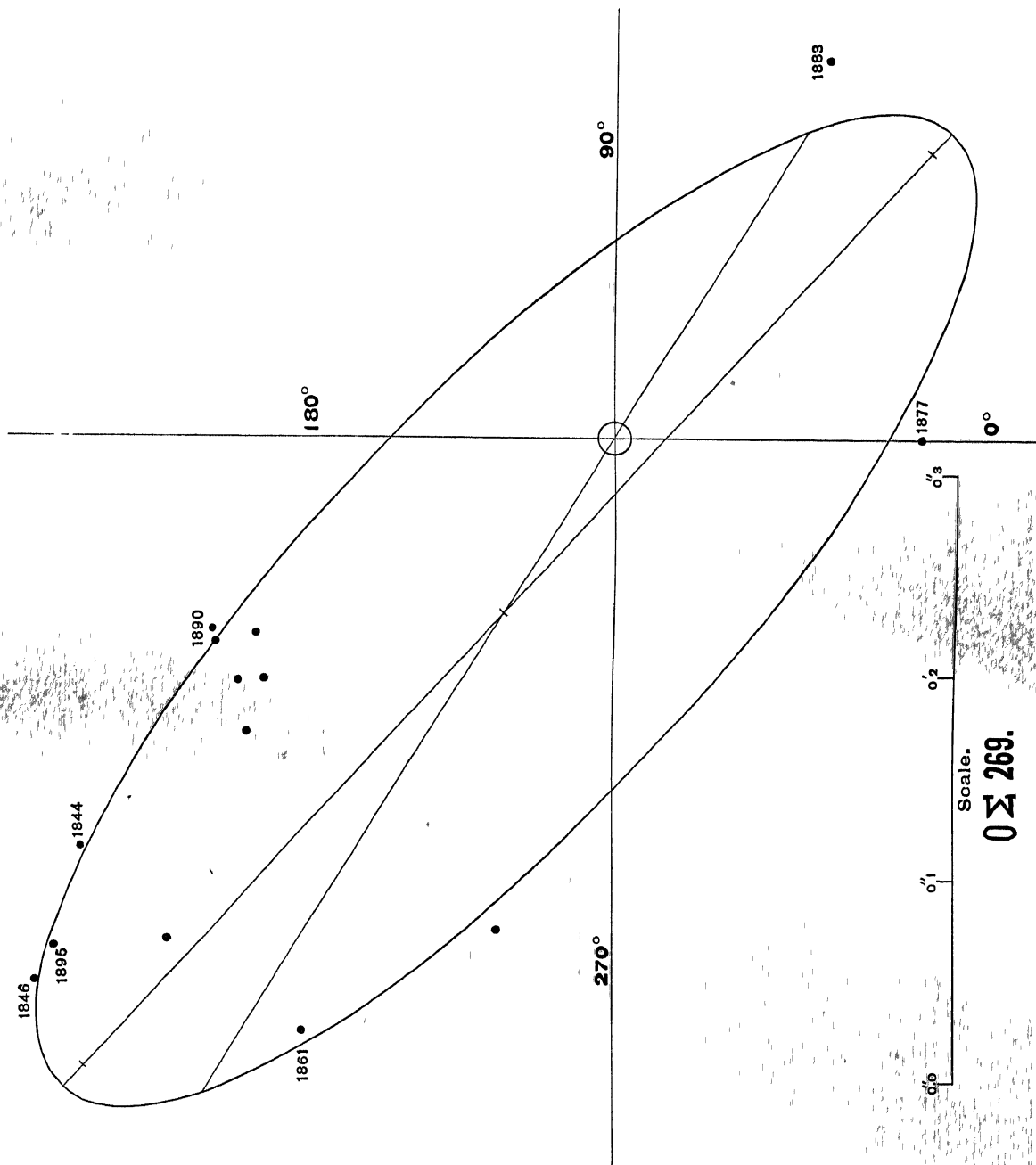
Length of major axis	= 0" 64
Length of minor axis	= 0" 20
Angle of major axis	= 47° 7
Angle of periastron	= 57° 8
Distance of star from centre	= 0" 102

The period here found is undoubtedly very nearly correct, but the other elements are subject to greater uncertainty. However, the observations of ENGLEMAN in 1883 and DEMBOWSKI's estimate in 1877, establish the essential nature of the periastron end of the apparent ellipse, and assure us that a large correction of our apparent orbit will ever be required. The eccentricity is not likely to be altered by more than  $\pm 0.05$ , nor can the node and inclination suffer changes which are proportionately larger. Thus it appears that the orbit is very satisfactory for the scant material now available, and while large corrections are not to be anticipated, it will be desirable to improve upon the elements when more good measures are secured. The ephemeris shows that the star will be comparatively easy for a good many years, and it will therefore commend itself to the regular attention of observers.

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 40	222 4	0 37	1899 40	226 9	0 41
1897 40	224 0	0 39	1900 40	228 2	0 41
1898 40	225 5	0 40			

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1844 31	218 0	215 6	0 33	0 30	+ 2 4	+0 03	1	O Struve
1846 39	223 8	219 9	0 39	0 35	+ 3 9	+0 04	1	O Struve
1851 34	228 9	227 9	0 33	0 41	+ 1 0	-0 08	1	O Struve
1861 26	242 8	243 0	0 33	0 34	- 2 0	-0 01	1	O Struve
1872 47	257 1	298 6	oblong	0 12	-41 5	—	1	O Struve
1877 26	0 0 0	28 0	oblonga	0 19	-28 0	—	1	Dembowski
1883 41	61 4	62 8	0 22	0 16	- 1 40	+0 06	4	Englemann
1889 52	207 7	199 5	0 22	0 17	+ 8 2	+0 05	3	Schiaparelli
1890 41	206 3	205 4	0 22	0 21	+ 0 9	+0 01	1	Schiaparelli
1891 26	213 4	209 5	0 22	0 24	+ 3 9	-0 02	3	Burnham
1891 49	208 9	210 4	0 20	0 24	- 1 5	-0 04	2-1	Schiaparelli
1892 40	215 0	213 6	0 21	0 28	+ 1 4	-0 07	2	Burnham
1895 07	222 9	220 0	0 37	0 35	+ 2 9	+0 02	2	Comstock 1, See 1
1895 41	219 0	220 7	0 23	0 35	- 1 7	-0 12	2	Schiaparelli





25 CANUM VENATICORUM =  $\Sigma$ 1768.

$\alpha = 13^{\text{h}} 33^{\text{m}}$  ,  $\delta = +86^{\circ} 48'$   
5, white , 85, blue

*Discovered by William Struve in 1827*

## OBSERVATIONS

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1829 89	79 6	1 05	5	Struve	1872 38	round	—	1	W & S
1833 12	72 4	1 09	5	Struve	1872 47	58 ?	—	1	O Struve
1836 50	71 8	1 07	3	Struve	1875 36	single	—	1	Hall
1841 17	72 6	1 01	4	O Struve	1875 48	167 1	0 63	1	O Struve
1841 37	70 8	1 00	4-3	Madler	1875 49	round	—	1	Dunér
1842 35	67 7	0 99	3-1	Dawes	1876 42	doubtful	—	1	Hall
1843 35	70 2	1 02	2	Dawes	1876 45	161 4	0 4 $\pm$	4	Schiaparelli
1843 52	70 5	0 71	3	Madler	1877 37	154 5	0 4 $\pm$	10	Schiaparelli
1846 80	67 8	0 72	3	O Struve	1877 54	154 7	0 60	1	O Struve
1847 71	55 3	0 40	1	Madler	1878 41	151 8	0 75	4	Dembowski
1849 77	65 6	0 65	3	O Struve	1879 43	155 7	0 5 $\pm$	5	Schiaparelli
1851 28	56 5	0 39	6-4	Madler	1879 49	157 5	0 51	5	Hall
1852 32	45 0	0 3 $\pm$	4	Madler	1880 37	157 5	0 35	2	Hall
1853 32	36 2	0 35 $\pm$	1	Madler	1880 46	155 0	0 60	2	Burnham
1854 43	36 2	0 35 $\pm$	3	Dawes	1881 24	27 6	—	1	Doberck
1854 78	46 2	0 35 $\pm$	2	Madler	1881 32	151 6	0 49	1	Bigourdan
1856 49	25 7	oblonga	—	Secchi	1881 40	153 4	0 60 $\pm$	5	Schiaparelli
1858 65	26 7	0 2 $\pm$	2	Madler	1881 40	157 4	0 53	3	Hall
1859 41	single	—	1	O Struve	1881 43	155 9	0 41	3	Burnham
1860 36	10-15	0 15 $\pm$	1	Dawes	1882 27	16 0	—	1	Doberck
1861 26	single	—	1	O Struve	1882 33	149 3	0 75	5	Englemann
1861 58	44 5	—	1	Madler	1882 43	152 7	0 45	3	Hall
1862 39	single	—	1	O Struve	1882 45	151 3	0 7 $\pm$	8	Schiaparelli
1862 95	180 ?	—	1	Dembowski	1883 42	147 0	0 59	1	Hall
1863 15	315 ?	—	1	Dembowski	1883 43	151 4	0 80	6	Englemann
1865 44	—	round	1	Dawes	1883 46	149 0	0 7 $\pm$	5	Schiaparelli
1868 13	127 ?	—	1	Dembowski	1883 51	149 2	0 53	2	Perrotin
1869 40	178 ?	—	1	Dunér	1884 33	143 8	—	2	Bigourdan
1870 43	186	0 1 $\pm$	1	Dunér	1884 42	145 5	0 63	3	Hall
1871 45	47 ?	—	1	Dunér	1885 32	148 2	0 8 $\pm$	9	Schiaparelli
					1885 37	149 1	0 89	3	Perrotin
					1885 54	149 6	0 77	3	Tarrant
					1886 38	143 1	—	1	Perrotin
					1886 45	145 2	0 78	4	Hall
					1886 51	146 7	0 78	4	Schiaparelli

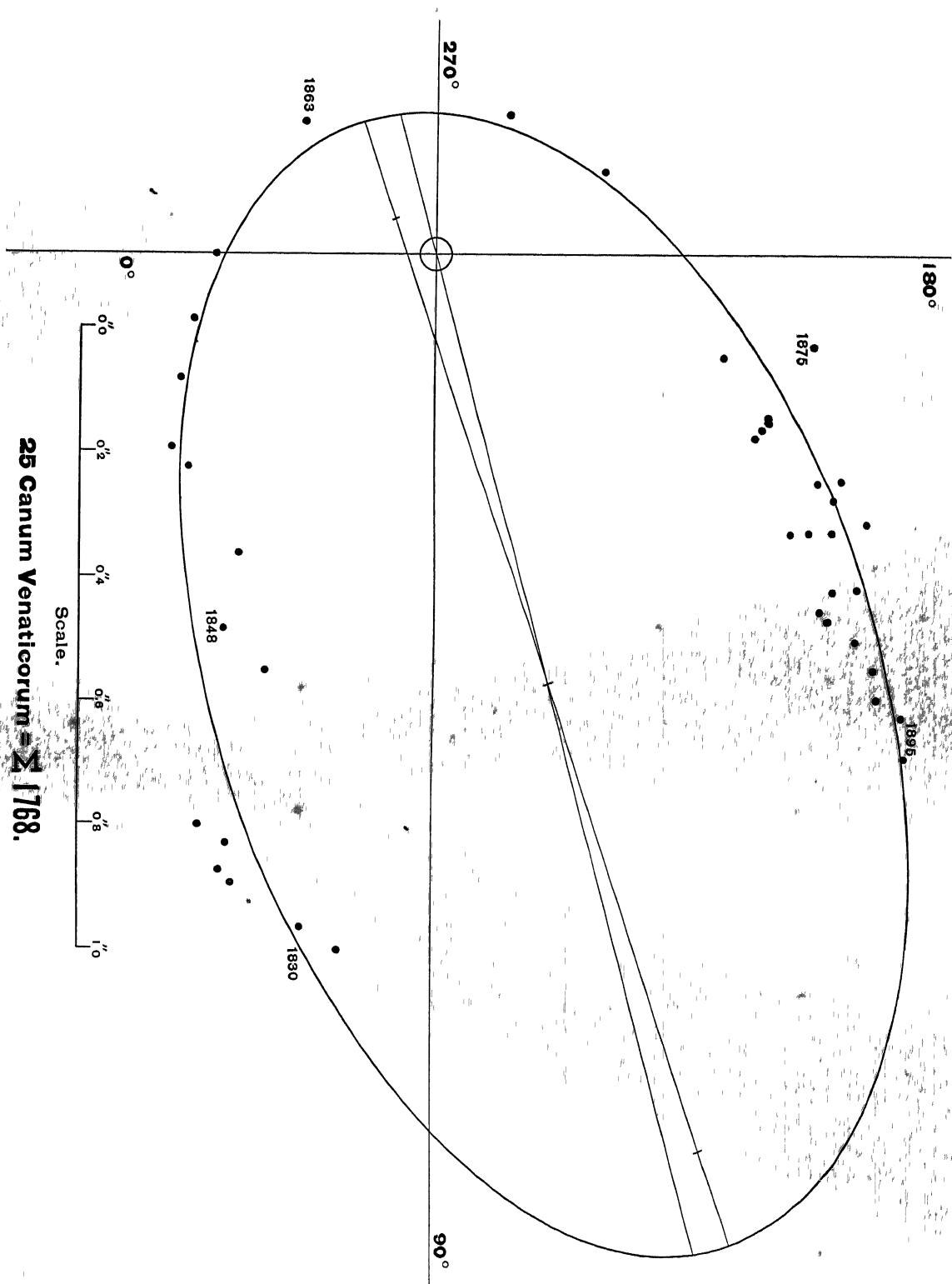


$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1887 41	145 8	0 67	4	Hall	1892 17	137 5	0 98	3	Burnham
1887 46	142 7	0 72	9	Schiaparelli	1892 64	140 0	0 95	3-2	Comstock
1888 44	145 8	0 73	3	Hall	1893 50	138 4	0 81	2	Schiaparelli
1888 54	142 9	0 76	5	Schiaparelli	1893 58	138 9	0 89	1	Comstock
1889 48	140 5	0 84	5-4	Schiaparelli	1894 47	138 1	0 86	1	Schiaparelli
1890 42	137 9	0 81	4	Schiaparelli	1895 11	132 6	1 35	3	Barnard
1891 48	141 4	0 80	4	Schiaparelli	1895 20	134 5	1 11	4-5	Barnard
1891 51	143 6	0 93	3	Maw	1895 28	136 4	1 06	3-4	See
					1895 52	137 4	0 90	2	Comstock

The observations of this remarkable system prior to 1840 gave evidence of a slow retrograde motion, and accordingly it received the attention of OTTO STRUVE, MADLER, DAWES, and subsequent observers. Up to this time the radius vector has swept over  $308^\circ$  of position-angle, while the distance has diminished from  $1'' 13$  to  $0'' 23$  and again increased to about its former value. The data furnished by observation do not suffice to fix the elements of the orbit with great accuracy, but we believe that it is now possible to get a fair approximation to the motion, and that the resulting elements will not be sensibly improved for a great many years.

When the measures of this star are examined it is found that they are far from satisfactory, and therefore we must not expect an agreement such as could be obtained for easier objects, where the components are wider or more nearly equal in magnitude. Some of the recorded measures are so inconsistent that the mean places must be formed with care, and even then the representation of the motion is not entirely satisfactory. The smaller distances have been under-measured, as is clear from the fact that a star of this difficulty could not be seen with small telescopes (such as those used between 1860 and 1875), unless separated by something like  $0'' 3$ . Under these circumstances it seemed proper to increase the measured distances near periastron, in order that when plotted on the diagram of the apparent ellipse they might not convey to the reader an erroneous impression. In the table of computed and observed places, however, we have retained the original values, and it will be seen that the differences are not at all considerable. DOBERCK is the only astronomer who has previously computed an orbit for this pair; using measures up to 1880 he found

$$\begin{aligned}
 P &= 119.9 \text{ years} & \Omega &= 42^\circ 4 \\
 T &= 1863.0 & i &= 33^\circ 3 \\
 e &= 0.72 & \lambda &= 245^\circ 0 \\
 a &= 0'' 81
 \end{aligned}$$





A careful investigation of all the observations leads to the following elements of 25 *Canum Venaticorum*.

$$\begin{array}{ll} P = 184.0 \text{ years} & \Omega = 123^\circ 0 \\ T = 1866.0 & i = 33^\circ 5 \\ e = 0.752 & \lambda = 201^\circ 0 \\ a = 1''.1307 & n = -1^\circ 9565 \end{array}$$

### Apparent orbit

$$\begin{array}{ll} \text{Length of major axis} & = 1''.91 \\ \text{Length of minor axis} & = 1''.08 \\ \text{Angle of major axis} & = 108^\circ 9 \\ \text{Angle of periastron} & = 285^\circ 4 \\ \text{Distance of star from centre} & = 0''.714 \end{array}$$

This orbit is remarkably eccentric, and so far as known is surpassed in this respect by four stars only —  $\gamma$  *Virginis* (0.9),  $\gamma$  *Andromedae* (0.85),  $\gamma$  *Centauri* (0.80) and 99 *Herculis* (0.78). Whatever changes may hereafter be required in these results, it is certain that the eccentricity will remain conspicuous, and will not be varied sensibly from the value here obtained. The period, however, remains uncertain by perhaps 25 years, so that the motion of the system is not so well determined as could be desired. An ephemeris is appended for the use of observers.

### COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1827.28	82.4	79.7	1.13	1.15	+ 2.7	-0.02	1	Struve
1830.54	78.9	77.1	1.10	1.09	+ 1.8	+0.01	4-3	Struve
1833.12	72.4	74.8	1.06	1.04	- 2.2	+0.02	5-4	Struve
1836.50	71.1	71.5	1.05	0.96	- 0.4	+0.09	2	Struve
1841.37	70.8	65.6	1.00	0.85	+ 5.2	+0.15	4-3	Madler
1842.35	67.7	64.2	0.99	0.83	+ 3.4	0.06	3-1	Dawes
1846.80	67.8	56.7	0.72	0.71	+11.1	+0.01	3	O Struve
1848.74	60.5	52.6	0.53	0.66	+ 3.9	-0.13	4	Madler 1, O Struve 3
1851.28	56.5	47.3	0.39	0.60	+ 9.2	-0.21	6-4	Madler
1852.82	40.6	40.1	0.35 $\pm$	0.54	+ 0.5	-0.19	5-1	Madler
1854.43	36.2	35.2	0.35 $\pm$	0.50	+ 1.0	-0.15	3	Dawes
1857.57	26.2	19.8	0.2 $\pm$	0.41	+ 6.4	-0.20	3-2	Secchi 1-0, Madler 2
1860.36	15 $\pm$	356.9	0.15 $\pm$	0.33	+18.1	-0.18	1	Dawes
1862.95	0 ?	330.4	—	0.28	+29.6	—	1	Dembowski
1863.15	315 ?	328.4	oblonga	0.27	-13.4	—	1	Dembowski
1868.76	242.5	236.5	—	0.24	+ 6.0	—	2	Dembowski 1, Dunér 1
1870.94	206.5	205.2	elong	0.29	-24.8	—	2	Dunér
1872.47	238 ?	190.1	—	0.35	+48 ?	—	1	O Struve
1875.48	167.1	171.3	0.63	0.47	- 4.2	+0.16	1	O Struve
1876.45	161.4	167.2	0.5 $\pm$	0.51	- 5.8	-0.01	4-1	Schiaparelli
1877.45	154.6	163.6	0.60	0.55	- 9.0	+0.05	11-1	Schiaparelli 10-0, O Struve 1
1878.41	151.8	160.6	0.75	0.58	- 8.8	+0.17	4	Dembowski
1879.46	156.6	157.7	0.60	0.62	- 5.9	-0.02	10-1	Schiaparelli 5-1, Hall 5-0
1880.41	156.3	155.2	0.60	0.66	- 0.2	-0.06	4-2	Burnham 2, Hall 2-0

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1881 40	153 4	153 3	0 60	0 69	+ 0 1	-0 09	5	Schiaparelli
1882 39	150 3	151 0	0 72	0 73	- 0 7	-0 01	13	Englemann 5, Schiaparelli 8
1883 45	149 1	149 0	0 75	0 76	+ 0 1	-0 02	14-11	Hl 1-0, En 6, Sch 5, Per 2-0
1884 42	145 5	147 4	0 66	0 80	- 1 9	-0 14	3-1	Hall
1885 41	149 0	145 8	0 82	0 82	+ 3 2	$\pm 0 00$	15	Sch 9, Perrotin 3, Tairant 3
1886 48	146 0	144 2	0 78	0 86	+ 1 8	-0 08	8	Hall 4, Schiaparelli 4
1887 46	142 7	143 0	0 73	0 88	+ 1 3	-0 15	13-9	Schiaparelli 9
1888 49	144 3	141 5	0 75	0 92	+ 2 8	-0 17	8	Hall 3, Schiaparelli 5
1889 48	140 5	140 7	0 84	0 94	- 0 2	-0 10	5-4	Schiaparelli
1890 42	137 9	139 3	0 84	0 97	- 1 4	-0 13	4-3	Schiaparelli
1891 50	142 5	138 1	0 87	1 00	+ 4 4	-0 13	7	Schiaparelli 4, Maw 3
1892 17	137 5	137 5	0 97	1 02	$\pm 0 0$	-0 05	6-5	Bunham 3
1893 54	138 6	136 1	0 92	1 05	+ 2 5	-0 13	3	Schiaparelli 2, Comstock 1
1894 47	138 1	135 4	0 86	1 07	+ 2 7	-0 21	1	Schiaparelli
1895 20	133 9	134 7	1 11	1 09	- 0 8	+0 02	7-5	Barnard
1895 28	136 4	134 6	1 06	1 09	+ 1 8	-0 03	3-4	See

## EPHEMERIS

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 50	134 0	1 11	1899 50	131 6	1 17
1897 50	133 2	1 13	1900 50	130 9	1 19
1898 50	132 4	1 15			

 $\alpha$  CENTAURI.

$\alpha = 14^h 32^m 6$  ,  $\delta = -60^\circ 25'$   
 1, orange yellow , 2, orange yellow

*Discovered by Father Richaud at Pondicherry, India, December, 1689*

## OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1690 0	—	—	1	Richaud	1834 33	217 33	17 83	1	Herschel†
1709 5	—	—	1	Feuillee	1834 45	218 78	17 50	2	Herschel
1752 20	218 73	20 51	—	Lacaille	1835 08	218 80	17 33	1	Herschel
1761 5	—	15 6	1	Maskelyne	1835 89	219 59	17 02	11-1	Herschel
1822 00	209 6	28 75	—	Fallows*	1836 61	220 26	16 76	1	Herschel
1824 00	215 41	22 45	35+	Brisbane	1837 22	220 65	16 39	4	Herschel
1826 01	213 18	22 45	—	Dunlop	1840 00	223 2	14 74	—	Maclear
1830 01	215 03	19 95	—	Johnson	1846 21	232 4	10 96	3	Jacob
1831 00	215 97	22 56	—	Taylor*	1846 80	234 3	9 56	4	Jacob
1832 16	216 35	19 85	—	Johnson and Taylor*	1847 09	235 7	9 33	2-3	Jacob
1833 0	217 45	18 67	7±	Henderson	1847 36	234 5	9 31	3	Jacob
					1848 00	237 93	8 05	13-12	Jacob

\*Taken on the authority of SIR JOHN HERSCHEL

†HERSCHEL's means have been formed anew

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1849 63	244 5	6 23	—	Jacob	1854 63	283 44	—	3	Powell
1849 94	245 25	6 96	1	Maclear	1854 66	282 81	4 43	5	Maclear
1849 97	245 42	7 04	3-2	Maclear	1854 93	285 88	3 96	5-4	Maclear
1850 10	246 63	7 01	1	Maclear	1854 96	288 02	—	2	Powell
1850 17	245 75	7 08	6	Maclear	1855 06	289 32	—	10	Powell
1850 20	245 85	6.84	3	Maclear	1855 23	290 19	4 38	3	Maclear
1850 31	247 07	6 75	4	Maclear	1855 29	292 60	—	5	Powell
1850 37	247 52	6 52	7	Jacob	1855 33	293 8	4 11	10	Powell
1850 38	245 74	7 12	1	Maclear	1855 36	291 96	4 38	4	Maclear
1850 41	242 0	7 78	15	Gilliss	1855 54	294 73	—	5	Powell
1850 61	248 84	6 58	3	Maclear	1856 02	301 02	3 99	11-6	Powell
1850 64	249 1	6 20	7	Jacob	1856 02	302 13	3 85	7-6	Maclear
1850 92	250 27	5 88	6	Jacob	1856 10	303 06	3 88	18	Jacob
1850 94	251 84	6 02	3	Maclear	1856 38	306 92	4 05	1	Maclear
1851 02	251 05	5 88	8	Jacob	1856 51	309 84	3 93	10-9	Jacob
1851 08	252 50	6 12	3	Maclear	1856 91	311 26	4 21	4	Mann
1851 20	252 13	5 94	10-8	Jacob	1856 94	311 88	—	11	G Maclear
1851 33	253 92	6 02	5	Maclear	1856 95	310 78	4 05	6	Mann
1851 56	254 42	5 88	3	Maclear	1856 96	315 77	3 96	10-9	Jacob
1851 70	256 38	5 27	8	Jacob	1857 15	318 19	4 02	15	Jacob
1851 94	256 58	5 80	3	Maclear	1857.39	320 60	4 47	2-1	Maclear
1851 94	258 2	5 11	9-8	Jacob	1857 86	326 48	4 14	14	Jacob
1851.99	258 85	5 08	8-7	Jacob	1858.17	330 51	4.39	5	Jacob
1852 25	259.02	5 72	3	Maclear	1858.23	339 42	5 09	3	Maclear
1852 27	261 07	5 03	7	Jacob	1859 34	339 71	5 18	15-12	Powell
1852 38	261 88	4 94	6	Jacob	1859 43	343 44	5 10	5	Mann
1852 43	261 67	5 27	5	Maclear	1859 52	341 8	4 92	4	Powell
1852 53	264 16	5 00	4	Jacob	1859 97	346 08	5 00	3	Mann
1852 56	262 8	5 03	—	Maclear	1860 05	346 55	—	1	G Maclear
1852 58	262 89	5 18	7-9	Maclear	1860 09	345 4	5 65	17-13	Powell
1852 73	262 45	4 95	5-2	Maclear	1860 18	349 34	5 52	4-1	Maclear
1852 79	263 31	—	4	Maclear	1860 35	348 87	—	3	Maclear
1853 05	267 67	4 55	—	Jacob	1860 48	348 7	5 68	1	Powell
1853 13	266 54	4 84	4-6	Maclear	1861 05	351 08	6 07	10-9	Powell
1853 15	268 33	4 59	—	Jacob	1861 09	353 65	6 09	3	Maclear
1853 34	268 72	4 87	5	Maclear	1861 31	353 03	6 21	7	Powell
1853 50	271 03	4 68	6	Maclear	1861 58	354 26	6 32	5-3	Powell
1853 58	272 17	4 57	2-1	Mann	1862 0	0 0	10 0	—	Ellery*
1853 58	270 10	—	—	Powell	1862 20	357 84	6 80	7	Powell
1853 92	275 19	4 44	4-3	Maclear	1862 47	0 0	—	—	Ellery†
1854 00	276 63	4 21	—	Jacob	1862 56	1 38	7 55	3	Maclear
1854 03	276 85	—	7	Powell	1863 03	1 4	7 2	6-4	Powell
1854 24	278 98	4 62	4	Maclear	1863 75	5 2	8 5	—	Ellery
1854 25	279 06	4 16	2	Jacob					
1854 26	279 62	—	4	Powell					

\* Apparently a rough "guess"

† From transit observations

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1864 11	5 7	7 85	7-5	Powell	1878 16	116 98	1 77	1	Russell
1864 72	—	8 1	—	Elleiry	1878 22	119 82	1 95	3	Russell
1865 56	17 3	9 95	1	Elleiry*	1878 28	127 37	1 77	1	Russell
1866 06	11 1	9 3	3	Powell	1878 38	139 10	2 40	—	Maxwell Hall
1868 17	—	9 2	—	Elleiry	1879 25	174 40	3 41	—	Elleiry
1868 18	—	9 6	—	Elleiry	1879 47	173 55	3 41	2	Haigraue
1868 38	13 59	10 29	2	Mann	1880 18	183 9	5 22	4	Tebbutt
1868 51	21 8	11 02	5	Elleiry*	1880 39	185 2	5 56	3	Tebbutt
1869 13	17 97	10 4	2	Powell	1880 45	184 98	5 52	1	Russell
1870 1	20 45	10 24	13-12	Powell	1881 28	189 88	5 07	1	Haigraue
1870 61	21 8	10 09	5-4	Powell	1881 54	190 13	7 52	1	Haigraue
1870 65	—	10 2	—	Elleiry	1881 65	193 15	7 94	2	Tebbutt
1870 65	24 7	10 45	3	Elleiry*	1882 00	194 44	8 23	18	Gill
1870 75	22 53	10 46	4	Russell	1882 22	194 6	8 70	1	Tebbutt
1871 05	23 01	9 89	11	Powell	1882 50	195 82	9 12	52	Elkin
1871 31	23 7	9 8	7	Powell	1884 19	199 0	11 96	—	Russell
1871 48	22 91	10 22	2	Russell	1884 43	199 5	12 32	—	Russell†
1871 51	24 2	9 41	1	Elleiry	1884 53	199 80	12 93	6	Tebbutt
1872 47	25 31	9 73	2	Russell	1885 56	200 8	14 05	4-3	Tebbutt
1872 55	24 1	10 36	1	Elleiry	1886 27	202 5	14 89	5	Pollock
1873 16	—	8 3	—	Elleiry	1886 38	200 4	14 74	1	Russell
1873 33	28 1	9 50	1	Russell	1886 52	201 2	15 19	1	Russell
1874 15	30 5	8 0	—	Elleiry	1886 55	201 02	14 87	4	Pollock†
1874 47	30 0	7 97	2	Russell	1886 56	202 42	15 13	10	Pollock
1874 85	34 17	—	—	Lindsay	1886 58	201 7	15 18	3	Russell
1875 02	34 21	6 82	—	Seeliger	1886 60	201 41	15 16	4	Tebbutt
1875 94	39 3	6 68	1	Elleiry*	1887 39	202 3	16 06	3-5	Tebbutt
1876 41	46 97	4 35	2	Russell	1887 43	202 08	15 83	6-5	Pollock
1876 61	51 05	4 15	2	Elleiry	1887 60	202 35	16 28	3-2	Tebbutt
1876 90	64 3	4 94	1	Elleiry	1887 72	202 16	16 18	2	Tebbutt
1876 94	51 2	4 5	1	Elleiry	1887 74	203 0	15 73	4	Pollock
1877 14	64 4	3 30	—	Maxwell Hall	1888 30	203 4	16 87	3	Tebbutt
1877 25	69 1	3 13	5	Elleiry	1888 63	202 93	17 12	1	Tebbutt
1877 52	72 77	2 60	2-1	Russell	1889 45	204 5	17 91	3	Pollock
1877 56	77 25	2 11	3	Russell	1890 41	205 2	18 58	2	Tebbutt
1877 57	80 50	2 13	2-3	Gill	1890 47	204 75	18 66	4-3	Sellois
1877 59	81 74	1 90	3	Russell	1890 60	205 05	19 06	3-2	Sellois
1877 63	81 49	1 94	3-1	Gill	1890 74	204 6	18 69	1-3	Tebbutt
1877 82	97 12	1 85	2-3	Gill	1891 43	205 62	19 15	5-4	Sellois
1877 89	101 12	1 62	2	Gill	1891 56	207 17	19 25	4-2	Tebbutt

\* From transit observations

† From  $\Delta\alpha$  and  $\Delta\delta$

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1891 57	205° 3	19" 24	2	Sellors	1893 21	206° 75	20" 22	2-1	W H Pickering
1891 64	206 4	19 35	3-6	Tebbutt	1893 34	206 4	19 92	8	Sellors
1892 30	206 45	19 52	2	Gill & Finlay*	1893 42	206 73	20 32	6-4	Tebbutt
1892 40	205 46	19 73	5-4	Sellors	1893 49	206 5	20 24	8	Sellois
1892 45	205 53	19 75	7-4	Tebbutt	1893 50	206 75	20 53	4-2	Tebbutt
1892 58	205 83	19 73	8-5	Tebbutt	1894 47	207 2	20 58	6	Sellors
1892 76	206 9	19 96	1	W H Pickering	1894 78	208 0	20 72	19-11	Tebbutt
1893 21	206 9	20 04	1	A E Douglass	1895 55	207 8	20 97	16-10	Tebbutt

In attempting to investigate the orbit of *a Centauri* it seems desirable to review briefly the work already done on this celebrated system.

The record left us by RICHAUD does not throw much light upon the nature of the orbit, but is of considerable historical interest

“Regardant à l’occasion de la Comète plusieurs fois les pieds du *Centaure* avec une lunette d’environ douze pieds, je remarquai que le pied le plus oriental et le plus brillant étoit une double étoile aussi bien que le pied de la croixade; avec cette différence que dans la croixade, une étoile paraît avec la lunette notablement éloignée de l’autre; au lieu qu’au pied du *Centaure*, les deux étoiles paraissent même avec la lunette presque se toucher; quoique cependant on les distingue aisément.”†

The next record of *a Centauri* was made by FATHER FEUILLÉE, who observed at Lima, Peru, July 4, 1709; in his *Journal des Observations, &c.*, Paris, 1714, tome I, p. 425, we find the following account

“Sur les deux heures du matin, en attendant que je pusse observer l’émersion du premier satellite de *Jupiter*, que des nuages me cachèrent, j’observai avec une lunette de 18 pieds l’étoile de la premier grandeur qui est au pied boréal du devant du *Centaure*, je trouvai cette étoile composée de deux, dont l’une est de la troisième grandeur et l’autre de la quatrième. Celle de la quatrième grandeur est la plus occidentale, et leur distance est égale au diamètre de cette étoile”

From this rather indefinite observation POWELL infers that the distance of the components in 1709 was about 10", and attaches considerable importance to the remark that the companion was “the more westerly” (la plus occidentale) Unfortunately the language is rather ambiguous, and we can not tell whether FEUILLÉE meant that the companion was really to the west of the central star, or whether it merely appeared to the west in the inverted field of view. As

\* By photography

† Publications of the Royal Academy of Sciences, Paris, 1692, or *Monthly Notices*, 1884-5, p 18



$\alpha$  *Centauri* was low in the southwest when the observation was made, it is also possible that the remark may have arisen, as MR ROBERTS has observed, from the position of the heavens at that instant rather than the position-angle of the companion. In any case it follows from the orbit here deduced that the position-angle was  $24^{\circ} 3$ , and the distance  $10''.07$

The third observation of  $\alpha$  *Centauri* was made by LACAILLE at the Cape of Good Hope in 1752. While determining the positions of southern stars he observed the components of  $\alpha$  *Centauri*, and from the resulting  $\Delta\alpha$  and  $\Delta\delta$  we find the values of  $\rho$  and  $\theta$  given in the list of measures. The observations of LACAILLE were first printed in the *Cœlum Australe Stelliferum*, which was published at Paris in 1763, and reprinted in 1847 by the British Association for the Advancement of Science, under the auspices of a Committee composed of HERSCHEL, HENDERSON and BAILY. LACAILLE's observations appear to be as good as could be expected from the instruments and methods employed.

In 1761  $\alpha$  *Centauri* was observed on one night by MASKELYNE while at the island of St Helena, by means of a rough divided-object-glass micrometer he found a distance of  $15''.6$

The observations made early in the present century by FALLOWS, BRISBANE, DUNLOP, JOHNSON, TAYLOR and HENDERSON, rest on measures of  $\Delta\alpha$  and  $\Delta\delta$ . The observation of FALLOWS was made with a small and defective Altitude and Azimuth Instrument, and is entirely erroneous. For a long time this measure was very misleading to computers, as it indicated an eccentricity of about 0.96. The results of BRISBANE, DUNLOP, JOHNSON, TAYLOR and HENDERSON are likewise unworthy of any high degree of confidence. The first observations of conspicuous worth are the micrometrical measures made by SIR JOHN HERSCHEL at the Cape of Good Hope. The measures of HERSCHEL taken in conjunction with others recently made expressly for the purpose have enabled us to determine the orbit of  $\alpha$  *Centauri* with a degree of precision which appears extraordinary when we consider the character of the observations. It will be found on inspecting the list of measures that many of them are vitiated by sensible errors of observation, which are partly systematic and partly accidental. We must remember, however, in judging of the value of results that  $\alpha$  *Centauri* is a very bright star, so that the images are unusually large, and hence if the telescope is not practically perfect, and the atmospheric conditions favorable, we could hardly expect that the measures will be very accordant. It is also to be remembered that the southern observers are not specialists in double-star work, and hence we can not expect results such as could be obtained by the skill of a BURNHAM or a STRUVE. Nevertheless, the measures of  $\alpha$  *Centauri*

taken as a whole, will enable us to obtain one of the best orbits yet deduced for any binary, and we may gratefully acknowledge our deep obligation to the southern observers, who amid many difficulties have measured this star with care and assiduity

In the list of measures given above will be found all the records which are of any value. The observations of T. MACLEAR, G. MACLEAR and W. MANN, which were made about the middle of the century, are taken from DR. ELKIN'S *Inaugural Dissertation*, in which they were first printed; the number of nights was kindly supplied by DR. ELKIN in a private letter. Most of the other measures are taken from the *Memoirs* and *Monthly Notices* of the Royal Astronomical Society. In this connection I take occasion to acknowledge my special obligations to MESSRS. TEBBUTT, PICKERING, DOUGLASS, RUSSELL, SELLORS, GILL and FINLAY for securing sets of measures expressly for this investigation, also to thank HERR HANS LUDENDORFF of the Royal Observatory, Berlin, for confirming from original sources the measures of LACAILLE, BRISBANE, DUNLOP and JOHNSON.

Most of the orbits determined before 1875 have now only historical interest, and among those more recently determined only three are approximately correct; namely, those of ROBERTS (*A.N.*, 3175), SEE (*M.N.*, Dec., 1893), and DOBERCK (*A.N.*, 3330). The following table of the elements found by previous computers is essentially complete:

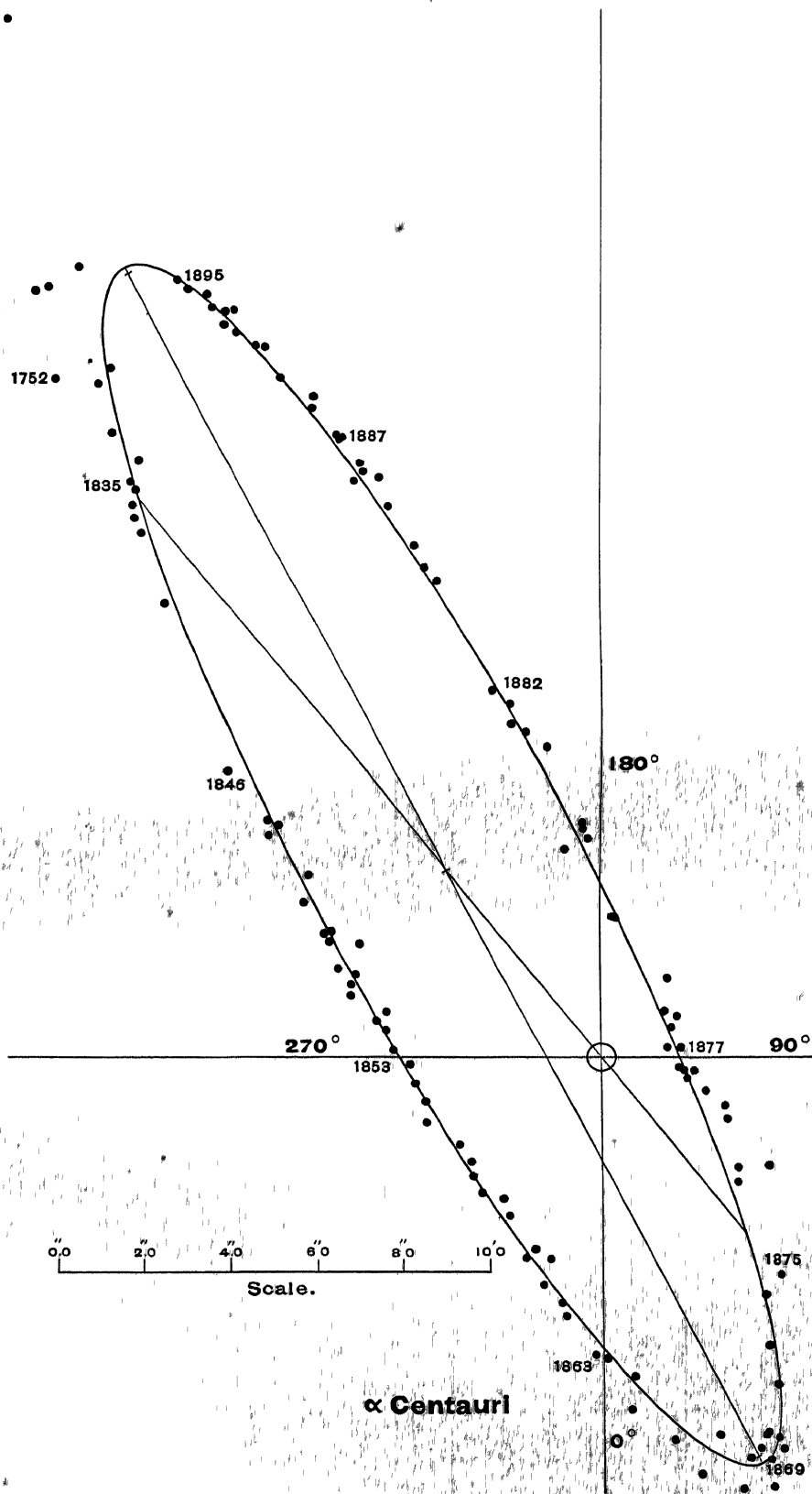
<i>P</i>	<i>T</i>	<i>e</i>	<i>a</i>	$\Omega$	<i>i</i>	$\lambda$	Authority	Source
<sup>YRS</sup> 77 0	1851 50	0 950	15 5	86 12	47 77	291 37	Jacob, 1848	Mem R A S, XVII, p 88
79 0	1863 25	0 818	—	—	—	32 7	Jacob	A N, XLIV, p 48
80 94	1859 42	0 7752	13 57	16 7	62 9	26 03	Hind, 1851	
75 3	1858 012	0 966	30	177 83	77 83	—	Powell, 1854	Mem R A S, XXIV, 93
82 59	1857 012	0 969	31 76	2 6	77 3	27 65	Powell, 1854	Mem R A S, XXIV, 93
77 81	1871 345	0 7033	20 575	22 35	80 95	58 43	Copeland, 1869	
76 25	1874 2	0 63944	20 13	24 3	81 22	59 2	Powell, 1870	M N, XXX, 192
85 042	1874 85	0 6673	21 797	21 8	82.3	59.53	Hind, 1877	M N, XXXVII, 97
88 536	1875 12	0 5332	18 45	25 23	79.4	45 97	Doberck, 1877	A N, 3330
77 42	1875 97	0 5260	17 50	25 78	79 53	54 83	Elkin, 1880	Dissertation, p 8
76 222	1875 951	0 5158	17 33	25 51	79 25	54 98	Downing, 1884	M N, XLIV, 289
87 438	1875 45	0 544	18 89	25 83	79 78	48 98	Powell, 1886	M N, XLVI, 337
80 34	1875 74	0 526	17 20	25 22	79 53	52 5	Gill, 1882	Mem R A S, XLVIII, 15
81 185	1875 715	0 52865	17 71	25 1	79 36	52 02	Roberts, 1893	A N, 3175
81 07	1875 62	0 520	17 705	25 45	79 74	51 56	See, 1893	M N, Dec 1893
79 123	1876 02	0 51184	18 45	25 42	79 23	52 88	Doberck, 1895	A N, 3330
83 565	1875 57	0 52252	18 165	25 9	79 32	49 42	Doberck, 1895	A N, 3330

After careful study of all the observations we have formed mean places and reduced them for precession to 1900.0. These places are given in the

accompanying table, which also contains the comparison resulting from the elements found below.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
	$^{\circ}$	$^{\circ}$	$''$	$''$	$^{\circ}$	$''$		
1690 00	—	258 94	—	6 67	—	—	1	Richaud
1709 50	—	23 86	—	9 94	—	—	1	Feullée
1752 2	217 84	217 21	20 51	18 36	+0 63	+2 15	—	Lacaille
1822 0	209 05	211 17	28 75	22 06	-2 12	+6 69	—	Fallows
1824 0	214 88	212 21	22 45	21 28	+2 67	+1 17	35+	Brisbane
1826 01	212 66	213 07	22 45	21 26	-0 41	+1 19	—	Dunlop
1830 01	214 54	215 01	19 95	19 95	-0 47	$\pm 0 00$	—	Johnson
1831 0	215 49	215 77	22 56	19 28	-0 28	+3 28	—	Taylor
1832 16	215 87	216 47	19 85	18 68	-0 60	+1 17	—	Johnson and Taylor
1833 0	216 98	217 03	18 67	18 42	-0 05	+0 25	7 $\pm$	Henderson
1834 33	216 87	217 92	17 83	17 68	-1 05	+0 15	1	Herschel
1834 45	218 32	217 99	17 50	17 67	+0 33	-0 17	2	Herschel
1835 08	218 35	218 47	17 33	17 63	-0 12	-0 30	2-1	Herschel
1835 89	219 14	219 07	17 02	17 06	+0 07	-0 04	11-1	Herschel
1836 61	219 82	219 67	16 76	16 43	+0 15	+0 33	1	Herschel
1837 22	220 21	220 18	16 39	16 17	+0 03	+0 22	4	Herschel
1840 0	222 78	222 89	14 74	14 42	-0 11	+0 32	—	Maclea
1846 21	232 02	232 87	10 96	9 70	-0 85	+1 26	3	Jacob
1846 80	233 93	234 33	9 56	9 18	-0 40	+0 38	4	Jacob
1847 09	235 33	235 21	9 33	8 90	+0 12	+0 43	2-3	Jacob
1847 36	234 13	235 87	9 31	8 76	-1 74	+0 55	3	Jacob
1848 00	237 57	237 80	8 05	8 35	-0 23	-0 30	13-12	Jacob
1849 63	244 15	243 97	6 23	7 12	+0 18	-0 89	—	Jacob
1849 95	244 98	245 48	7 00	6 83	-0 50	+0 17	4-3	Maclea
1850 20	244 97	246 55	6 92	6 57	-1 58	+0 35	14	Maclea
1850 38	246 28	247 91	6 82	6 46	-1 53	+0 36	8	Jacob 1, Maclea 7
1850 41	241 65	247 96	7 78	6 44	-6 31	+0 34	15	Gilliss
1850 62	248 62	249 22	6 39	6 32	-0 60	+0 07	10	Maclea 3, Jacob 7
1850 93	250 7	250 48	5 95	6 10	+0 22	-0 15	10-9	Jacob 7-6, Maclea 3
1851 10	251 55	251 45	5 98	6 04	+0 10	-0 06	21-19	Jacob 8, Maclea 3, Jacob 10-8
1851 44	253 33	253 90	5 95	5 75	-0 57	+0 20	8	Maclea 5, Maclea 3
1851 87	256 14	256 55	5 53	5 53	-0 41	$\pm 0 00$	11	Jacob 8, Maclea 3
1851 95	258 18	257 19	5 09	5 48	-0 99	-0 39	17-15	Jacob 9-8, Jacob 8-7
1852 33	260 58	259 95	5 24	5 28	+0 63	-0 04	21	Maclea 3, Jacob 7, Jacob 6, Maclea 5
1852 64	262 78	262 40	5 03	5 01	+0 38	+0 02	20-15	Ja 4, Mac -, Mac 7-9, Mac 5-2, Mac 4
1853 27	268 73	268 18	4 69	4 75	+0 55	-0 06	19-18	Ja -, Mac 4-6, Ja -, Mac 5, Mac 6,
1853 75	272 32	272 61	4 44	4 50	-0 29	-0 06	4-3	Powell -, Maclea 4-3 [Mann 2-1
1854 16	277 91	277 60	4 33	4 36	+0 31	-0 03	17-6	Jacob -, Po 7-0, Mac 4, Ja 2, Po 4-0
1854 79	284 72	285 35	4 19	4 16	-0 63	+0 03	15-9	Powell 3-0, Mac 5, Mac 5-4, Po 2-0
1855 25	291 26	290 90	4 29	4 20	+0 36	+0 09	32-17	Po 10-0, Mac 3, Po 5-0, Po 10, Mac 4
1856 13	302 97	302 33	3 94	4 08	+0 64	-0 14	42-31	Po 5, Po 11-6, Mac 7-6, Ja 18, Mac 1
1856 51	309 84	307 72	3 93	4 10	+2 12	-0 17	10-9	Jacob
1856 94	312 11	312 80	4 07	4 16	-0 69	-0 09	31-19	Mann 4, G Mac 11-0, Mann 6, Ja 10-9
1857 27	319 09	317 57	4 24	4 24	+1 52	$\pm 0 00$	17-16	Jacob 15, Maclea 2-1
1857 86	326 18	326 15	4 14	4 42	+0 03	-0 28	14	Jacob
1858 17	330 22	328 09	4 39	4 50	+2 13	-0 11	5	Jacob
1859 33	340 6	339 49	5 12	5 06	+1 11	+0 06	23-20	Maclea 3, Powell 15-12, Mann 5
1859 52	341 52	341 38	4 92	5 15	+0 14	-0 23	4	Powell
1859 97	345 8	344 89	5 00	5 40	+0 91	-0 40	3	Mann
1860 27	347 78	346 70	5 62	5 56	+1 08	+0 06	26-15	G Mac 1, Po 17-13, Mac 4-1, Mac 3-0,
1861 07	352 09	351 96	6 08	6 05	+0 13	+0 03	13-12	Powell 10-9, Maclea 3 [Po 1
1861 44	353 38	354 15	6 26	6 25	-0 77	+0 01	12-10	Powell 7, Powell 5-3
1862 41	359 47	358 15	7 17	7 10	+1 22	+0 07	10	Powell 7, Ellely -, Maclea 3





$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1863 03	1 14	2 64	7 2	7 34	-1 50	-0 14	6-4	Powell
1863 75	4 95	4 43	8 5	7 73	+0 52	+0 77	-	Ellery
1864 11	5 55	5 74	7 85	7 96	-0 19	-0 11	7-5	Powell
1864 72	—	7 70	8 1	8 38	—	-0 28	3±	Ellery
1865 56	17 06	10 14	9 95	8 75	+7 92	+1 20	1 •	Ellery
1866 06	10 86	11 50	9 3	9 22	-0 74	+0 08	3	Powell
1868 38	13 37	15 70	10 29	10 08	-2 33	+0 21	2	Mann
1868 51	21 58	16 09	11 02	10 13	+5 49	+0 89	5	Ellery
1869 13	17 75	18 45	10 4	10 40	-0 70	±0 00	2	Powell
1870 1	20 24	20 42	10 24	10 30	-0 18	-0 06	13-12	Powell
1870 61	21 59	21 47	10 09	10 28	+0 12	-0 19	5-4	Powell
1870 65	24 5	21 55	10 32	10 28	-0 05	+0 05	3-5±	Ellery
1870 75	22 33	21 78	10 46	10 26	+0 55	+0 20	4	Russell
1871 18	23 15	22 74	9 84	10 18	+0 41	-0 34	18	Powell 11, Powell 7
1871 49	23 35	23 30	9 82	10 08	+0 05	-0 26	3	Russell 2, Ellery 1
1872 51	24 51	25 07	10 04	9 60	-0 56	+0 44	3	Russell 2, Ellery 1
1873 25	27 91	28 34	8 90	8 90	-0 43	±0 00	1-3±	Ellery -, Russell 1
1874 31	30 07	31 00	7 98	7 60	-0 93	+0 38	3-4±	Ellery -, Russell 2
1875 02	34 04	34 45	6 82	6 50	-0 41	+0 32	-	Seeliger
1875 94	39 13	40 50	6 68	4 86	-1 37	+1 82	1	Ellery
1876 41	46 80	45 93	4 35	3 98	+0 87	+0 37	2	Russell
1876 61	50 89	50 09	4 15	3 56	+0 80	+0 59	2	Ellery
1876 92	57 09	55 86	4 72	3 00	+1 23	+1 72	2	Ellery
1877 14	64 4	61 50	3 30	2 60	+2 90	+0 70	-	Maxwell Hall
1877 25	68 94	69 95	3 13	2 45	-1 01	+0 68	5	Ellery
1877 52	72 61	77 41	2 60	2 10	-4 80	+0 50	2-1	Russell
1877 56	77 09	79 70	2 11	2 01	-1 61	+0 10	3	Russell
1877 57	80 34	80 21	2 13	2 00	+0 13	+0 13	2-3	Gill
1877 59	81 58	81 37	1 90	1 98	+0 21	-0 08	3	Russell
1877 63	81 33	83 58	1 94	1 92	-2 25	+0 02	3-1	Gill
1877 82	96 97	96 24	1 85	1 75	+0 73	+0 10	2-3	Gill
1877 89	100 97	101 15	1 62	1 70	-0 18	-0 08	2	Gill
1878 16	116 83	122 39	1 77	1 67	-5 56	+0 10	1	Russell
1878 22	119 67	127 51	1 95	1 68	-7 84	+0 27	3	Russell
1878 28	127 12	131 01	1 77	1 71	-3 89	+0 06	1	Russell
1878 38	138 95	138 22	2 40	1 78	+0 73	+0 62	-	Maxwell Hall
1879 25	174 25	172 21	3 41	3 12	+1 96	+0 29	-	Ellery
1879 47	173 41	176 35	3 41	3 56	-2 94	-0 15	2	Hargrave
1880 18	183 76	184 97	5 22	4 93	-1 21	+0 29	4	Tebbutt
1880 39	185 06	186 70	5 56	5 36	-1 64	+0 20	3	Tebbutt
1880 46	184 84	187 05	5 52	5 53	-2 21	-0 01	1	Russell
1881 28	189 75	191 72	5 07	7 11	-1 97	-2 04	1	Hargrave
1881 54	190 00	192 77	7 52	7 64	-2 77	-0 12	1	Hargrave
1881 65	193 02	193 18	7 94	7 82	-0 16	+0 12	2	Tebbutt
1882 00	194 44	194 33	8 23	8 45	+0 11	-0 22	18	Gill
1882 22	194 48	194 87	8 70	8 80	-0 39	-0 10	1	Tebbutt
1882 50	195 82	196 03	9 12	9 29	-0 21	-0 17	52	Elkin
1884 19	198 89	199 37	11 96	12 04	-0 48	-0 08	-	Russell
1884 43	199 39	199 55	12 32	12 38	-0 16	-0 06	-	Russell
1884 53	199 69	199 65	12 93	12 53	+0 04	+0 40	6	Tebbutt
1885 56	200 7	200 82	14 05	13 84	-0 12	+0 21	4-3	Tebbutt
1886 27	202 4	201 60	14 89	14 73	+0 80	+0 16	5	Pollock
1886 38	200 3	201 70	14 74	14 78	-1 40	-0 04	1	Russell
1886 54	201 46	201 85	15 06	15 01	-0 39	+0 05	15	Russell 1, Pollock 4, Pollock 10
1886 59	201 46	202 91	15 17	15 07	-1 45	+0 10	7	Russell 3, Tebbutt 4
1887 41	202 1	202 64	15 95	15 96	-0 54	-0 01	9-10	Tebbutt 3-5, Pollock 6-5
1887 69	202 30	202 87	16 09	16 24	-0 57	-0 14	9-8	Tebbutt 3-2, Tebbutt 2, Pollock 4
1888 30	203 32	203 34	16 87	16 87	-0 02	±0 00	3	Tebbutt

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1888 63	202 85	203 62	17 12	17 14	-0 77	-0 02	1	Tebbutt
1889 45	204 43	204 22	17 91	17 81	+0 21	+0 10	3	Pollock
1890 49	204 93	204 89	18 77	18 64	+0 04	+0 13	9-7	Tebbutt 2, Sellois 4-3, Sellois 3-2
1890 74	204 53	205 05	18 69	18 85	-0 42	-0 16	1-3	Tebbutt
1891 55	206 01	205 52	19 25	19 28	+0 49	-0 03	14	Sel 5-4, T 4-2, Sel 2, T 3-6
1892 30	206 45	205 97	19 52	19 73	+0 48	-0 18	2	Gill and Finlay
1892 43	205 47	206 04	19 74	19 83	-0 57	-0 09	12-8	Sellois 5-4, Tebbutt 7-4
1892 67	205 87	206 18	19 84	19 93	-0 31	-0 09	9-6	Tebbutt 8-5, Pickering 1
1893 25	206 70	206 50	20 06	20 21	+0 20	-0 16	11-10	Douglass 1, Pickering 2-1, Sellois 8
1893 47	206 66	206 59	20 36	20 30	+0 07	+0 06	18-14	Tebbutt 6-4, Sellois 8, Tebbutt 4-2
1894 62	207 6	207 21	20 65	20 81	+0 39	-0 16	25-17	Sellois 6, Tebbutt 19-11
1895 55	207 8	207 67	20 97	21 09	+0 13	-0 12	16-10	Tebbutt

In dealing with this orbit it seems probable that the graphical method will be superior to any process involving a least-square adjustment, because of the undoubted existence of sensible systematic errors in the observations. An adjustment based on both angles and distances will eventually be desirable, but before this definitive determination can be made with advantage, it will be necessary to have an additional revolution. In the present state of the observations it is wholly useless to apply corrections of a very minute character. Basing the work upon all the best observations we find the following elements of  $\alpha$  Centauri:

$$\begin{aligned}
 P &= 81.1 \text{ years} & \Omega &= 25^\circ 15' \\
 T &= 1875.70 & i &= 79^\circ 30' \\
 e &= 0.528 & \lambda &= 52^\circ 00' \\
 a &= 17'' 70 & n &= +4^\circ 43' 89.54
 \end{aligned}$$

#### Apparent orbit

$$\begin{aligned}
 \text{Length of major axis} &= 32'' 18 \\
 \text{Length of minor axis} &= 6'' 16 \\
 \text{Angle of major axis} &= 27^\circ 25' \\
 \text{Angle of periastron} &= 38^\circ 65' \\
 \text{Distance of star from centre} &= 5'' 90
 \end{aligned}$$

If we adopt the parallax of GILL and ELKIN ( $0''.75$ ), we find that the major semi-axis of the orbit is 23.6 astronomical units. It follows that the combined mass of the components is 2.00 times the mass of the sun and earth.

Thus we see that the companion of  $\alpha$  Centauri moves in an orbit with a major axis which is about a mean between those of *Uranus* and *Neptune*. But owing to the eccentricity of the orbit the distance at periastron (11.2) only slightly surpasses that of *Saturn* from the sun, while at apastron it extends considerably beyond *Neptune* (36.0).

According to preliminary researches of STONE in 1875, it was found that the masses of the two components are sensibly equal. MR. A. W. ROBERTS has

recently made a very careful determination of this mass-ratio, and finds (*A N*, 3313) that the masses of  $\alpha^2$  and  $\alpha^1$  (the companion) are as  $51 \ 49 \pm \frac{1}{5}\%$  of the amount. A very similar result was obtained by DR. ELKIN in his *Inaugural Dissertation*, and hence we may conclude that in this case the relative masses are known with almost the desired precision.

MR. ROBERTS has also made a careful discussion of the parallax of  $\alpha$  *Centauri* from the meridian observations of 1879–81 and obtained (*A N*, 3324) results which confirm the work of GILL and ELKIN with the heliometer. Using both right ascensions and declinations MR. ROBERTS finds

$$\pi = +0''.71 \pm 0''.05$$

Our knowledge of this system is therefore far more accurate than that of any other system in the heavens, and it does not seem possible that the results here obtained will ever be sensibly altered. But as some refinement is still possible this glorious object will always merit the attention of observers.

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02285.

$$\alpha = 14^h \ 41^m \ 7^s, \quad \delta = +42^\circ \ 48'$$

7 5, yellowish, 7 6, whitish

*Discovered by Otto Struve in 1845*

OBSERVATIONS

<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1845 80	72 2	0 61	3	O. Struve	1887 60	202 2	0 24	4	Schiaparelli
1847 96	72 2	0 42	3	Mädler	1888 61	187 5	0 22	3	Schiaparelli
1852 71	58 4	0 49	5	Mädler	1889 52	193 2	0 22	1	Schiaparelli
1855 84	53 9	0 51	3	O. Struve	1891 30	168 7	0 24	3	Burnham
1857 50	65 5	0 40	1	Secchi	1891 49	159 2	0 20	1	Schiaparelli
1865 53	36 0	—	1–0	Dembowski	1892 30	162 2	0 24	3–2	Burnham
1876 40	350 0	0 3 ±	1	Burnham	1893 46	156 0	0 24	1	Burnham
1881 50	—	doubtful	1	Burnham	1893 51	158 8	—	1–0	Bigourdan
1883 84	258 3	0 22	5	Englemann	1894 47	136 8	—	1–0	Bigourdan
1885 40	225 0	elong	1	Perrotin	1895 32	147 3	0 30	3	See
					1895 56	143 2	0 35	1	Schiaparelli



This close double star was measured by OTTO STRUVE several times during the few years following its discovery\*. The other early measures were by MADLER and SECCHI, while in later years the pair has been measured only by ENGLEMAN, SCHIAPARELLI, BURNHAM and the writer. Thus, only a small number of observations are available for the determination of an orbit, but it happens that these are distributed so as to give a fairly good set of elements.

The star has always been a difficult object, and hence the measures are necessarily less accurate than in case of easier pairs. BURNHAM was the first to attempt an investigation of the orbit (*Sidereal Messenger*, June, 1891). His apparent ellipse and the resulting elements are not very different from those found in this paper. MR. GORE has since attempted an orbit by a very different process, and obtained results of a wholly different character (*Monthly Notices*, April, 1893). These two sets of elements are.

GORE	BURNHAM
$P = 118.57$ years	62.1
$T = 1881.93$	1885.3
$e = 0.58$	0.429
$a = 0''.46$	$0''.387$
$\Omega = 107^\circ 0$	$54^\circ 3$
$i = 45^\circ 7$	$44^\circ 3$
$\lambda = 161^\circ 4$	$180^\circ 0$

Using all the measures, and basing the work on both angles and distances, I find the following elements of O $\Sigma$ 285:

$P = 76.67$ years	$\Omega = 62^\circ 2$
$T = 1882.53$	$i = 41^\circ 95$
$e = 0.470$	$\lambda = 162^\circ 23$
$a = 0''.3975$	$n = -4^\circ 6953$

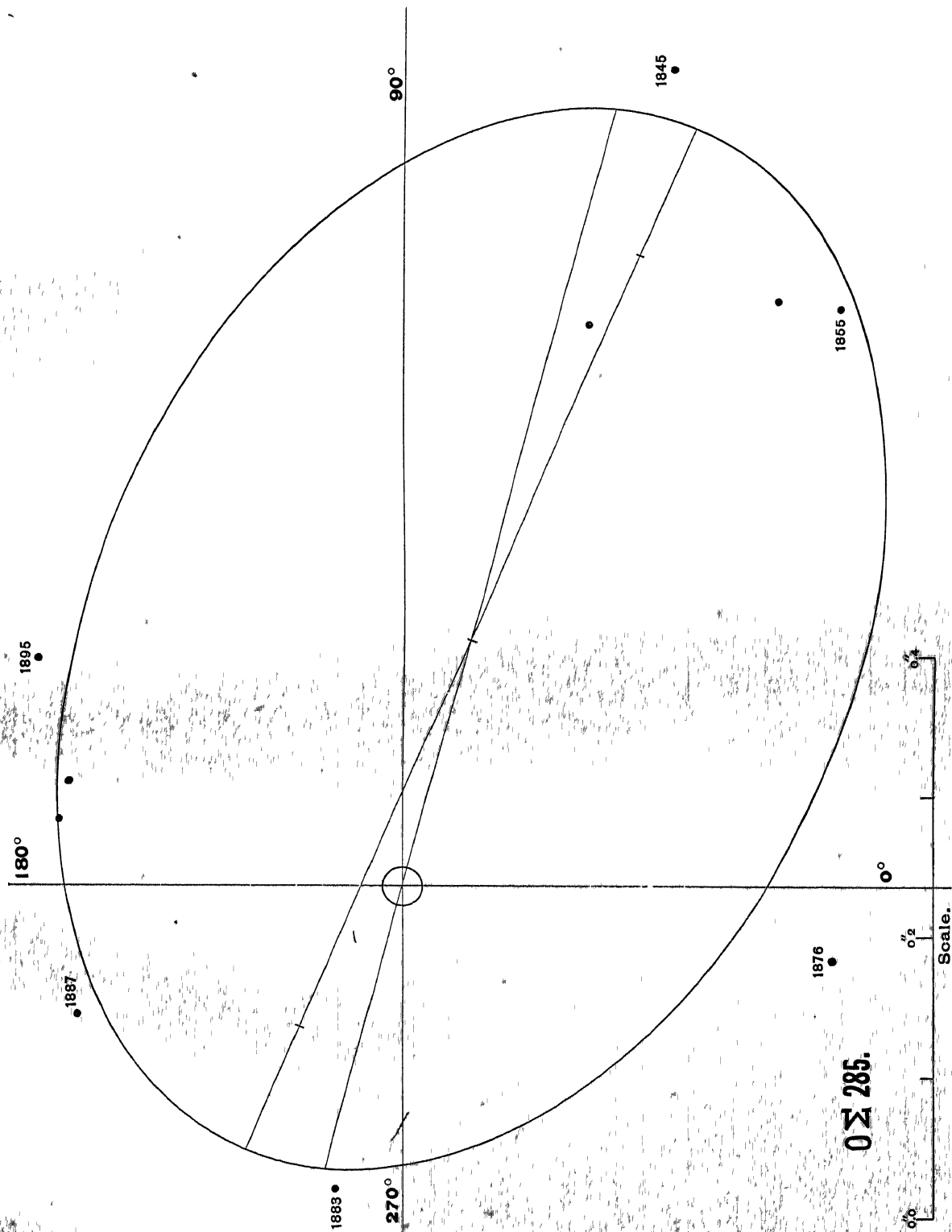
Apparent orbit:

Length of major axis	$= 0''.788$
Length of minor axis	$= 0''.522$
Angle of major axis	$= 67^\circ 1$
Angle of periastron	$= 255^\circ 3$
Distance of star from center	$= 0''.182$

The following table of computed and observed places shows that the measures are represented as well as could be expected in the case of an object of this difficulty.

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\* *Astronomical Journal*, 356



0.2 285.



COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
	$^{\circ}$	$^{\circ}$	$''$	$''$	$^{\circ}$	$''$		
1845 80	72 2	73 2	0 61	0 57	- 1 0	+0 04	3	O Struve
1847 96	72 2	70 0	0 42	0 57	+ 2 2	-0 15	3	Mädler
1852 71	58 4	62 9	0 49	0 56	- 4 5	-0 07	5	Mädler
1855 84	53 9	58 0	0 51	0 54	- 4 1	-0 03	3	O Struve
1857 50	65 5	55 1	0 40	0 52	+10 4	-0 12	1	Secchi
1865 53	36 0	38 4	—	0 42	- 2 4	—	1	Dembowski
1876 40	350 0	357 4	0 3 $\pm$	0 24	- 7 4	+0 06	1	Burnham
1881 50	—	267 6	doubtful	0 20	—	—	1	Burnham
1883 84	258 3	241 0	0 22	0 21	+17 3	+0 01	5	Englemann
1887 60	202 2	203 6	0 24	0 22	- 1 4	+0 02	4	Schiaparelli
1891 30	168 7	170 1	0 24	0 24	- 1 4	$\pm$ 0 00	3	Burnham
1892 30	162 2	162 0	0 24	0 25	+ 0 2	-0 01	3-2	Burnham
1893 46	156 0	153 2	0 24	0 26	+ 2 8	-0 02	1	Burnham
1895 32	147 3	142 0	0 30	0 28	+ 5 3	+0 02	3	See

The only large residual is that of ENGLEMANN, whose small telescope would necessarily render his observations subject to considerable uncertainty. Indeed, he gives the angle as  $78^{\circ} 3$ , but I have assumed that he really saw the companion, and have therefore changed the angle by  $180^{\circ}$ . The estimate of  $36^{\circ}$  for the position-angle in 1865.53 is very nearly correct, and leaves no doubt that the elongation observed by DEMBOWSKI was real.

When I measured the object recently with the 26-inch refractor of the Leander McCormick Observatory in Virginia, the stars were not separated, except on one night, and hence the difficulty of the pair will doubtless account for the error in angle. The star is slowly separating, and ought to be observed annually. The following is an ephemeris for the next five years

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
	$^{\circ}$	$''$		$^{\circ}$	$''$
1896 40	135 6	0 30	1899 40	122 3	0 35
1897 40	130 7	0 32	1900 40	118 4	0 36
1898 40	126 5	0 33			

The comparatively long period of this close star may probably be construed to mean that the system is very remote from the *Earth*, otherwise the mass would be excessively small. The eccentricity of the orbit is fairly well defined, and is near the mean value of this element among double stars.

$\xi$  BOOTIS =  $\Sigma$ 1888.

$\alpha = 14^h 46^m 8$  ,  $\delta = +19^\circ 31'$   
 4 5, yellow , 6 5, purple

*Discovered by Sir William Herschel, April 19, 1780*

## OBSERVATIONS

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1780 69	24 1	3 23	1	Herschel	1841 06	325 1	7 03	5	O Struve
1791 39	nf	—	1	Herschel	1841 42	323 4	7 27	3	Dawes
1792 30	355 7	—	1	Herschel	1841 43	324 7	7 10	4	Madler
1795 32	354 9	—	1	Herschel	1841 65	322 1	6 72	—	Kaiser
1802 25	352 9	—	1	Herschel	1842 30	322 7	7 03	2	Dawes
1804 25	353 9	6 $\pm$	1	Herschel	1842 40	323 4	6 88	3-1	Madler
1821 20	342 4	9 25	1	H and So	1843 33	322 7	6 70	1	Dawes
1822 69	335 8	7 54	—	Struve	1843 35	322 4	6 81	7-5	Madler
1823 30	—	6 67	—	Amici	1843 58	323 8	6 91	7	Schlüter
1823 34	340 2	8 42	1	H and So	1843 68	322 2	6 64	—	Kaiser
1825 37	337 0	7 78	4	South	1844 36	321 6	6 90	3	Madler
1828 54	336 0	7 18	2	Herschel	1845 36	320 9	6 81	8-6	Madler
1829 46	334 2	7 22	4	Struve	1845 37	322 3	6 12	—	Hind
1830 29	333 7	7 62	5-4	Herschel	1845 40	318 6	6 76	28	Morton
1831 40	331 2	7 30	5	Bessel	1846 29	320 4	6 69	5	Madler
1832 40	331 1	7 14	2	Struve	1846 46	319 2	6 75	20	Morton
1833 23	330 7	7 54	2	Herschel	1847 37	319 4	6 68	6	Madler
1834 44	330 4	7 54	3	Dawes	1847 44	318 8	6 80	2	Dawes
1835 43	329 0	7 07	5	Struve	1847 63	317 7	6 48	—	Mitchell
1835 45	330 4	7 63	3-2	Madler	1847 82	319 4	6 53	3	O Struve
1836 37	329 1	7 52	1	Madler	1848 28	318 0	6 63	5-4	Madler
1836 49	328 2	7 09	4	Struve	1848 50	317 9	6 71	2	Dawes
1837 31	327 0	6 79	—	Encke	1850 77	316 5	6 56	1	Madler
1838 22	326 7	6 97	—	Madler	1851 11	317 4	6 56	5	Fletcher
1838 47	327 1	6 85	2	Struve	1851 49	316 1	6 21	5	Madler
1838 54	326 5	7 26	—	Galle	1852 30	316 6	6 51	32	Miller
1839 41	325 8	7 07	—	Galle	1852 56	315 3	6 22	15-13	Madler
1840 26	325 1	6 70	34-25 <sub>obs</sub>	Kaiser	1853 44	314 4	6 31	8-7	Madler
1840 43	324 1	7 16	3	Dawes	1853 54	313 4	6 23	3	O Struve
					1854 46	312 0	6 26	3	Dawes
					1854 48	312 4	6 07	5-4	Madler
					1854 75	311 7	5 99	8	Dembowski
					1855 38	311 7	6 07	2	Madler
					1855 42	310 5	6 00	3	Secchi

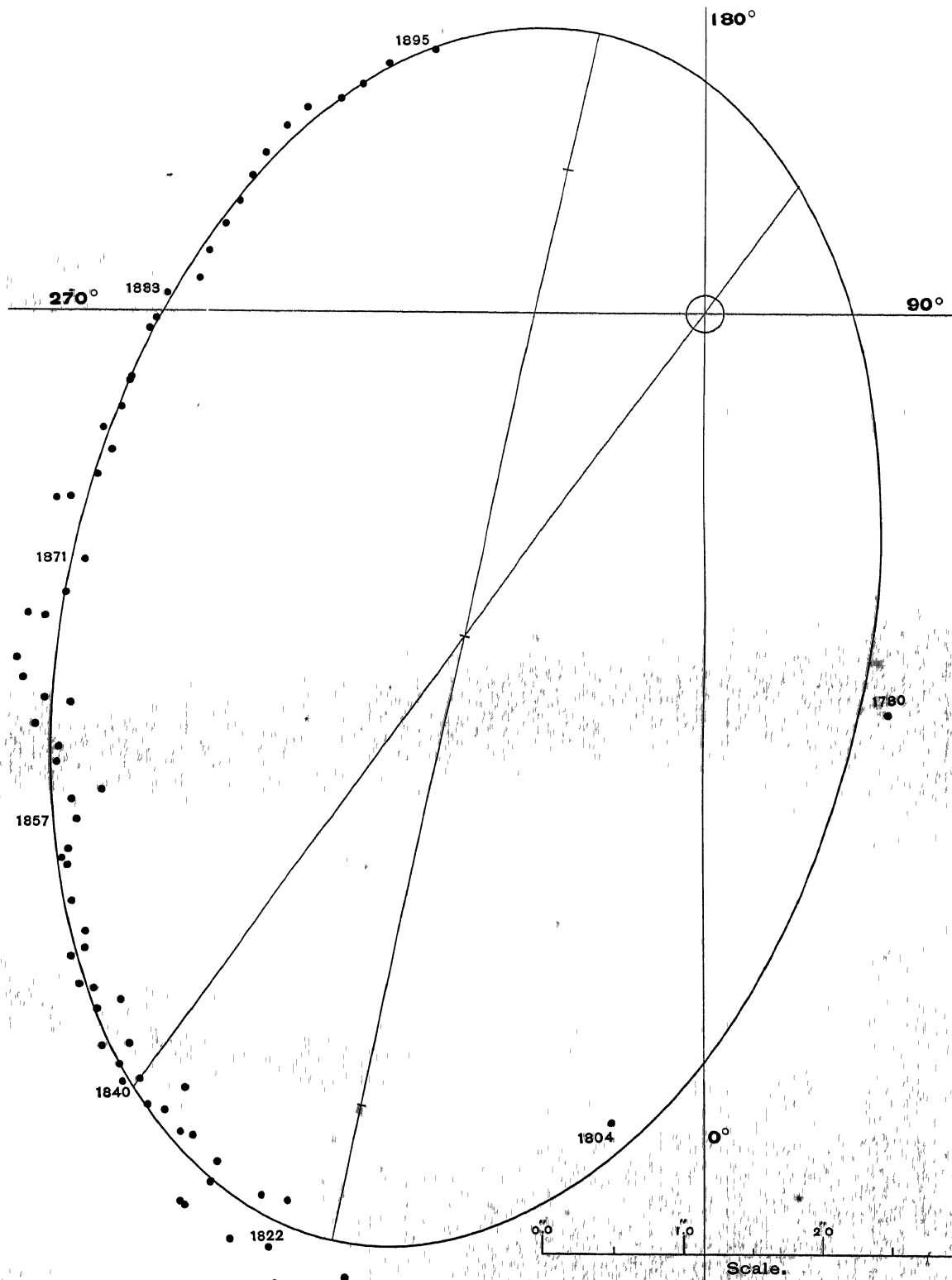
<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1856.39	312.1	5.89	1-3	Mädler	1870.38	293.0	5.41	2	Main
1856.45	310.8	5.95	8	Dembowski	1870.46	295.8	4.66		Leyton Obs.
1856.45	311.9	6.76	2	Luther	1870.56	294.4	4.95	1	Dunér
1856.55	311.7	6.00	3	Winnecke	1871.35	292.8	4.93	2	Main
1856.88	310.0	6.02	12	Secchi	1871.49	293.5	4.73	4	Dunér
1857.40	311.2	5.76	5	Mädler	1871.82	290.9	4.75	9	Dembowski
1857.42	310.0	5.90	1	Dawes	1873.19	286.7	4.62	4	O. Struve
1857.56	308.9	5.90	2	Dembowski	1873.39	286.0	4.93	1	Main
1858.36	308.2	5.76	5	Dembowski	1873.43	287.0	4.84	1	Lindstedt
1858.38	307.8	5.93	12	Morton	1873.48	286.6	4.71	1	Leyton Obs.
1858.54	309.9	5.65	7	Mädler	1873.91	287.8	4.62	8	Dembowski
1859.39	309.4	5.57	3	Mädler	1874.22	289.2	5.0	—	Gledhill
1861.29	305.0	5.52	35	Powell	1874.36	283.9	4.92	4	Main
1861.50	307.1	5.79	10-9	Mädler	1874.43	287.3	4.71	2-1	Leyton Obs.
1861.57	305.0	5.78	5	O. Struve	1874.44	288.4	4.72	5	W. & S.
1862.15	303.1	5.93	6	Anwers	1875.31	286.5	4.76	4	Main
1862.33	305.9	5.68	1	Main	1875.48	283.9	4.43	1	O. Struve
1862.47	304.1	5.59	4	O. Struve	1875.36	285.1	4.60		Gledhill
1862.51	302.9			Anwers	1875.38	286.3		—	Nohle
1862.51	302.2			Winnecke	1875.40	284.3	4.41	5	Schiaparelli
1862.65	306.1	5.27	2	Mädler	1875.51	286.6	4.45	4	Dunér
1863.15	303.0	5.59	14	Dembowski	1875.90	284.7	4.43	8	Dembowski
1863.28	302.4	5.79	—	Leyton Obs.	1876.34	284.8	4.31	5	Dobereck
1863.56	302.0	5.67	5	O. Struve	1876.43	283.4	4.64	3	Hall
1864.46	303.4	5.32	1	Englemann	1876.58	282.0	4.19	1	O. Struve
1864.87	301.6	5.44	16	Dembowski	1877.24	282.9	4.70	3	Dobereck
1865.33	301.6	5.61	3	Englemann	1877.45	283.0	4.35	5	Jedrzejewicz
1865.77	300.8	5.41	1	Secchi	1877.45	280.7	4.23	5	Schiaparelli
1866.39	298.5	5.59	2-4	Leyton Obs.	1877.51	279.4	4.21	1	O. Struve
1866.44	299.6	5.20	—	Kaiser	1877.93	280.9	4.26	8	Dembowski
1866.43	299.8	5.24	2-1	Englemann	1878.40	281.3	4.62	4	Goldney
1866.50	298.0	5.81	3-2	Searle	1878.42	277.4	4.32	2	Hall
1866.50	299.2	6.27	3-2	Winlock	1878.45	281.2	4.13	3	Dobereck
1866.86	299.0	5.30	11	Dembowski	1878.52	278.8	4.01	5	Schiaparelli
1867.30	298.1	5.64	1	Winlock	1878.54	279.4	4.13	1	O. Struve
1867.42	296.7	5.43	2	Searle	1879.51	277.6	4.10	6	Schiaparelli
1868.40	294.7	5.33	1	Main	1879.52	275.7	4.18	5	Hall
1869.09	295.4	5.09	1	O. Struve	1880.16	278.8	4.28	5	Franz
1869.47	295.6	5.07	5	Dunér	1880.48	276.0	4.19	3	Jedrzejewicz
1869.56	292.4	5.35	3	Main	1880.51	276.3	3.97	3	Schiaparelli
1869.61	298.8	5.42	1	Leyton Obs.	1881.40	269.2	4.04	3	Hall
					1881.50	273.2	3.87	3	Schiaparelli
					1881.60	273.3	4.03	3	Seabroke

$t$	$\theta_0$ "	$\rho_0$ "	$n$	Observers	$t$	$\theta_0$ "	$\rho_0$ "	$n$	Observers
1882 33	267 6	4 73	1	Glasenapp	1887 43	256 0	3 51	3	Hall
1882 42	270 4	3 99	3	Hall	1887 50	257 0	3 31	12	Schiaparelli
1882 50	271 4	3 86	7	Schiaparelli	1888 25	250 2	3 51	1	Glasenapp
1883 43	267 1	3 90	3	Hall	1888 42	251 9	3 40	3	Hall
1883 47	268 1	3 72	9	Schiaparelli	1888 54	255 0	3 15	2	O Struve
1883 50	269 4	3 72	3	Jedrzejewicz	1888 62	253 9	3 51	2	Maw
1883 52	267 6	4 14	3	Seabioke	1889 31	250 5	3 83	2	Glasenapp
1883 57	268 1	3 79	4	Perrotin	1889 48	249 1	3 40	3	Hall
1884 42	262 8	4 30	2	Glasenapp	1889 61	249 9	3 31	3	Maw
1884 45	266 6	3 65	6	Englemann	1890 41	246 2	3 15	3	Maw
1884 45	266 1	3 71	2	Perrotin	1890 43	246 3	3 21	3	Hall
1884 49	266 3	3 58	9	Schiaparelli	1890 53	244 4	3 47	2	Hayn
1884 50	266 2	3 56	1	O Struve	1891 44	241 0	3 26	5-4	See
1885 37	264 3	3 44	3	Tarrant	1891 45	242 4	3 18	3	Hall
1885 37	261 4	3 68	3	Hall	1891 48	243 4	3 18	4	Maw
1885 44	262 9	3 51	4	Perrotin	1892 32	240 0	3 08	3	Leavenworth
1885 44	262 1	3 55	5	deBall	1892 41	239 4	3 11	3	Maw
1885 48	263 1	3 61	12	Schiaparelli	1892 49	238 3	2 91	3	Comstock
1885 55	263 1	3 61	7	Englemann	1893 47	235 8	2 96	3	Maw
1885 64	263 6	3 63	4	Jedrzejewicz	1894 53	231 2	2 90	3	Maw
1886 40	259 6	3 56	3	Perrotin	1895 49	226 4	2 88	3	Comstock
1886 43	259 3	3 59	3	Hall	1895 70	223 8	2 57	4	See
1886 51	260 2	3 49	7	Schiaparelli	1895 73	224 4	2 65	2	Moulton
1886 60	259 4	3 32	6	Englemann					

The stars of this system are somewhat unequal in magnitude, and are moreover distinguished by very striking colors. The principal star is yellow, while the companion is reddish purple; and hence the appearance of the system, so far as it depends on contrast in color and inequality of the components, is very similar to those of 70 *Ophiuchi* and  $\eta$  *Cassiopeae*.\* The early observations of HERSCHEL established the physical connection of the stars, and since the time of STRUVE the measures are both sufficiently numerous and sufficiently exact to give the position of the companion with the desired precision. In spite of the fact that since 1780 an arc of only about  $170^\circ$  has been described, we are enabled by the favorable shape of this arc to make a very satisfactory determination of the elements. The companion is now approaching periastron, and in the course of a few years the motion will become very rapid. For the next fifteen years this system will deserve special attention from observers, as the part of the apparent ellipse swept over by the companion during this interval

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\* *Astronomische Nachrichten*, 3334.







will be the most critical, and measures secured near periastron will enable us to render the orbit exact to a very high degree.

The following table gives the elements of this interesting system published by previous computers.

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
<sup>Yrs</sup>			<sup>"</sup>	<sup>°</sup>	<sup>°</sup>	<sup>°</sup>		
117 14	1779 958	0 59374	12 56	0 0	80 1	101 0	Herschel, 1833	Mem R A S vol VI
160 695	1761 71	0 454	5 591	172 7	52 7	315 2	Mädler	Hand D S p 304[p 149
168 91	1779 75	0 7822	9 95	11 4	71 6	96 4	Hind, 1872	M N, vol XXXII, p 250
140 64	1767 76	0 641	5 425	11 6	48 4	124 15	Winogradsky '72	Gore's Catalogue
127 97	1770 44	0 6781	4 813	12 02	37 9	130 9	Doberck, 1876	A N 2118
127 35	1770 69	0 7081	4 86	26 37	36 9	117 77	Doberck, 1877	A N 2129

From an investigation of all the observations we are led to the following elements of  $\xi$  *Bootis*:

$$\begin{aligned}
 P &= 128 \cdot 0 \text{ years} & \Omega &= 10^{\circ} 5 \\
 T &= 1903 \cdot 90 & i &= 52^{\circ} 28 \\
 e &= 0 \cdot 721 & \lambda &= 239^{\circ} 25 \\
 a &= 5'' \cdot 5578 & n &= -2^{\circ} 8125
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 9'' \cdot 07 \\
 \text{Length of minor axis} &= 5'' \cdot 76 \\
 \text{Angle of major axis} &= 167^{\circ} 7 \\
 \text{Angle of periastron} &= 144^{\circ} 7 \\
 \text{Distance of star from centre} &= 2'' \cdot 94
 \end{aligned}$$

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
	<sup>°</sup>	<sup>°</sup>	<sup>"</sup>	<sup>"</sup>	<sup>°</sup>	<sup>"</sup>		
1780 69	24 1	35 3	3 23	2 18	-11 2	+1 05	1	Herschel
1792 30	355 7	2 2	—	5 24	- 6 5	—	1	Herschel
1795 32	354 9	358 5	—	5 71	- 3 6	—	1	Herschel
1802 25	352 9	351 9	—	6 48	+ 1 0	—	1	Herschel
1804 25	353 9	350 1	6 ±	6 66	+ 3 8	-0 66	1	Herschel
1821 20	342 4	337 8	9.25	7 33	+ 4 6	+1 92	1	Herschel and South
1822 69	335 8	336 8	7 54	7 34	- 1 0	+0 20	—	Struve
1823 32	340 2	336 4	7 55	7 35	+ 3 8	+0 20	1 2 ±	Herschel and So 1, Amici 0 2 ±
1825 37	337 0	335 1	7 78	7 35	+ 1 9	+0 43	4	South
1828 54	336 0	332 9	7 18	7 33	+ 3 1	-0 15	2	Herschel
1829 46	334 2	332 2	7 22	7 31	+ 2 0	-0 09	4	Struve
1830 29	333 7	331 6	7 62	7 30	+ 2 1	+0 32	5-4	Herschel
1831 40	331 2	330 9	7 30	7 29	+ 0 3	+0 01	5	Bessel
1832 40	331 1	330 2	7 14	7 27	+ 0 9	-0 13	2	Struve
1833 23	330 7	329 7	7 54	7 25	+ 1 0	+0 29	2	Herschel
1834 44	330 4	328 8	7 54	7 22	+ 1 6	+0 32	3	Dawes
1835 43	329 0	328 0	7 07	7 19	+ 1 0	-0 12	5	Struve
1836 49	328 2	327 2	7 09	7 16	+ 1 0	-0 07	4	Struve
1837 31	327 0	326 6	6 79	7 13	+ 0 4	-0 34	—	Encke
1838 41	326 8	325 8	7 03	7 09	+ 1 0	-0 06	2+	Mädler —; $\Sigma$ 2, Galle —
1839.41	325 8	325 2	7 07	7 06	+ 0 6	+0 01	—	Galle
1840 34	324 1	324 4	6 93	7 02	- 0 3	-0 09	3-6 ±	Kaiser 34-25 obs, Dawes 3

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1841 39	323 1	323 6	7 03	6 97	- 0 5	+ 0 06	7-12+	$O\Sigma$ 0-5, Da 3, Ma 4, Ka -
1842 35	323 0	322 8	6 95	6 93	+ 0 2	+ 0 02	5 3	Dawes 2, Madler 3-1
1843 48	322 8	322 0	6 77	6 88	+ 0 8	- 0 11	15-13±	Ma 7-5, Da 1, Schl 7, Ka -
1844 36	321 6	321 3	6 90	6 83	+ 0 3	+ 0 07	3	Madler
1845 38	320 6	320 4	6 56	6 78	+ 0 2	- 0 22	-	Ma -, H <sub>1</sub> -, Mo 28 obs
1846 37	319 8	319 6	6 72	6 73	+ 0 2	- 0 01	8±	Madler 5, Morton 20 obs
1847 56	318 8	318 6	6 62	6 67	+ 0 2	- 0 05	11+	Ma 6, Da 2, Mit -, $O\Sigma$ 3
1848 39	318 0	317 9	6 67	6 62	+ 0 1	+ 0 05	7-6	Madler 5-4, Dawes 2
1850 77	316 5	315 9	6 56	6 48	+ 0 6	+ 0 08	1	Madler
1851 30	316 7	315 4	6 44	6 44	+ 1 3	0 00	10	Fletcher 5, Madler 5
1852 43	316 0	314 4	6 37	6 37	+ 1 6	0 00	18-16±	Miller 32 obs, Ma 15-13
1853 49	313 9	313 4	6 27	6 31	+ 0 5	- 0 04	11-10	Madler 8-7, $O\Sigma$ 3
1854 56	312 0	312 3	6 11	6 23	- 0 3	- 0 12	16-15	Dawes 3, Madler 5-4, Dem 8
1855 40	311 1	311 6	6 03	6 18	- 0 5	- 0 15	5	Madler 2, Secchi 3
1856 56	311 3	310 4	6 12	6 09	+ 0 9	+ 0 03	29-28	Ma 4-3, Dem 8, Winn 3, Lu 2,
1857 46	310 0	309 5	5 85	6 03	+ 0 5	- 0 18	8	Ma 5, Da 1, Dem 2 [Sec 12
1858 43	308 6	308 5	5 78	5 96	+ 0 1	- 0 18	24	Dem 5, Morton 12, Madler 7
1859 39	309 4	307 5	5 57	5 90	+ 1 9	- 0 33	3	Madler
1861 45	305 7	305 2	5 70	5 74	+ 0 5	- 0 04	18-17±	Po 35 obs, Ma 10-9, $O\Sigma$ 5
1862 40	304 9	304 1	5 62	5 66	+ 0 8	- 0 04	13	Au 6, Main 1, $O\Sigma$ 4, Ma 2
1863 33	302 5	303 0	5 68	5 59	- 0 5	+ 0 09	19+	Dem 14, Leyton obs -, $O\Sigma$ 5
1864 67	302 5	301 4	5 38	5 47	+ 1 1	- 0 09	17	Englemann 1, Dembowski 16
1865 55	301 2	300 3	5 51	5 41	+ 0 9	+ 0 10	7	Englemann 3, Secchi 4
1866 52	299 0	299 1	5 57	5 33	- 0 1	+ 0 24	21-20+	Ley 2-4, Ka -, En 2-1, Sr 3-2,
1867 36	297 5	297 9	5 54	5 25	- 0 4	+ 0 29	3	Wlk 1, Sr 2 [Wlk 3-2, Dem 11
1868 40	294 7	296 5	5 33	5 17	- 1 8	+ 0 16	1	Main
1869 43	295 5	295 0	5 23	5 08	+ 0 5	+ 0 15	13	$O\Sigma$ 4, Du 5, Ma 3, Ley 1
1870 47	294 4	293 5	5 01	4 98	+ 0 9	+ 0 03	3+	Madler, Leyton -, Dunér 1
1871 55	292 4	291 9	4 80	4 89	+ 0 5	- 0 09	15	Ma 2, Du 4, Dem 9 [Dem 8
1873 48	286 4	288 7	4 74	4 71	- 2 3	+ 0 03	15	$O\Sigma$ 4, Ma 1, Ley 1, Lin 1,
1874 36	286 5	287 1	4 84	4 63	- 0 6	+ 0 21	11-10+	Gl -, Ma 4, Ley 2-1, W & S 5
1875 45	285 4	285 1	4 51	4 53	+ 0 3	- 0 02	22+	Ma 4, $O\Sigma$ 1, Gl -, No -, Sch 5,
1876 45	283 4	283 3	4 38	4 45	+ 0 1	- 0 07	9	Dk 5, Hl 3, $O\Sigma$ 1 [Du 4, Dem 8
1877 52	281 4	281 2	4 39	4 34	+ 0 2	+ 0 05	22	Dk 3, Jed 5, Sch 5, $O\Sigma$ 1, Dem 8
1878 46	279 6	279 4	4 24	4 26	+ 0 2	- 0 02	15	Go 4, Hl 2, Dk 3, Sch 5, $O\Sigma$ 1
1879 52	276 7	277 1	4 14	4 16	- 0 4	- 0 02	11	Schiaparelli 6, Hall 5
1880 38	277 0	275 3	4 15	4 09	+ 1 7	+ 0 06	11	Franz 5, Jed 3, Sch 3
1881 50	271 9	272 8	3 98	4 00	- 0 9	- 0 02	9	Hall 3, Sch 3, Sea 3
1882 46	270 9	270 4	3 93	3 90	+ 0 5	+ 0 03	10	Hall 3, Schiaparelli 7
1883 50	268 1	268 1	3 85	3 82	0 0	+ 0 03	22	Hl 3, Sch 9, Jed 3, Sea 3, Per 4
1884 47	266 3	265 2	3 65	3 72	+ 1 1	- 0 09	18	En 6, Per 2, Sch 9, $O\Sigma$ 1
1885 47	262 9	262 6	3 58	3 64	+ 0 3	- 0 06	38	Tar 3, Hl 3, Per 4, Sch 12, deBall 5,
1886 48	259 6	259 5	3 49	3 57	+ 0 1	- 0 08	19	Per 3, Hl 3, Sch 7, En 6 [En 7, Jed 4
1887 47	256 5	256 6	3 43	3 46	- 0 1	- 0 03	15	Hall 3, Schiaparelli 12
1888 46	253 0	253 6	3 39	3 37	- 0 6	+ 0 02	8	Glas 1, Hl 3, $O\Sigma$ 2, Maw 2
1889 45	249 8	250 4	3 35	3 29	- 0 6	+ 0 06	8-6	Glas 2-0, Hall 3, Maw 3
1890 46	245 6	247 0	3 28	3 21	- 1 4	+ 0 07	8	Maw 3, Hall 3, Hayn 2
1891 46	242 3	243 3	3 21	3 13	- 1 0	+ 0 08	12-11	See 5-4, Hall 3, Maw 4
1892 41	239 2	239 5	3 03	3 04	- 0 3	- 0 01	9	Lv 3, Maw 3, Com 3
1893 47	235 8	235 6	2 96	2 96	+ 0 2	0 00	3	Maw
1894 54	231 2	230 8	2 90	2 86	+ 0 4	+ 0 04	3	Maw
1895 59	225 1	225 7	2 72	2 75	- 0 6	- 0 03	7	Comstock 3, See 4

The table of computed and observed places shows that the set of elements given above is extremely satisfactory, and we may confidently conclude that the general nature of the orbit here obtained will never be materially changed.

It is possible that the period may be varied by so much as one year, and that the eccentricity is uncertain to the extent of about  $\pm 0.02$ ; larger alterations in these quantities are not to be expected, and the values of the other elements are correspondingly well determined

The system of  $\xi$  *Bootis* is chiefly remarkable for the great eccentricity of the orbit, and for the wide angular separation of the components. The great length of the major-axis and the comparatively short periodic time would support the belief that the system is not very far from the earth, and this view of relative proximity is rendered the more probable by the brightness of the components. But while these considerations tend to render it probable that the parallax is sensible, such a view is not supported by the small proper motion of the system in space, which is only  $0''.161$  per year. We might, therefore, infer that the system is perhaps very remote from the earth, and hence of enormous dimensions, or comparatively near us, with the proper motion mainly in the line of sight. In any case the parallax of this system is particularly worthy of investigation, and it might be determined either by the ordinary process of direct measurement, or by the spectroscopic method (*A.N.*, 3314, or §5, Ch I), which here seems likely to be entirely practicable.

The following is an ephemeris for the companion for the next ten years :

$t$	$\theta_o$	$\rho_o$	$t$	$\theta_o$	$\rho_o$
1896 50	221.2	2.65	1902 50	173.3	1.55
1897 50	216.2	2.53	1903 50	154.7	1.25
1898 50	210.1	2.40	1904 50	125.5	1.03
1899 50	203.4	2.25	1905 50	90.1	1.05
1900 50	195.7	2.06	1906 50	63.2	1.33
1901 50	186.1	1.83			

### $\eta$ CORONAE BOREALIS = $\Sigma$ 1937.

$\alpha = 15^h 19^m 1$  ,  $\delta = +30^\circ 39'$   
5 5, yellowish , 6, yellowish

*Discovered by Sir William Herschel, September 9, 1781.*

#### OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1781 69	30.7	—	1	Herschel	1826 77	35.3	1.07	4	Struve
1802 69	179.7	—	1	Herschel	1829 55	43.2	0.96	2	Struve
1823 27	25.9	1.58	2-1	H & So	1830 30	44.5	—	8	Herschel

$t$	$\theta_0$ "	$\rho_0$ "	$n$	Observers	$t$	$\theta_0$ "	$\rho_0$ "	$n$	Observers
1831 34	50 8	—	2	Dawes	1849 44	218 3	0 69	2-1	Dawes
1831 47	52 7	1 02	10-1	Herschel	1849 65	220 3	0 60	3	O Struve
1831 63	50 6	0 88	3	Struve	1850 50	221 2	0 46	1	W Struve
1832 50	57 1	0 69	9-2	Herschel	1850 52	230 8	0 49	3	O Struve
1832 55	56 7	—	1	Dawes	1850 56	235 0	0 7 $\pm$	2	Fletcher
1832 76	56 9	0 79	3	Struve	1850 69	228 8	0 42	3	Madler
1833 27	61 9	0 72	8-2	Herschel	1851 31	236 8	0 35	3-2	Madler
1833 39	63 5	—	3	Dawes	1851 42	238 1	0 55	2	Dawes
1834 84	69 1	0 70	1	Struve	1851 56	241 8	0 48	10	O Struve
1835 41	75 7	0 74	5	Struve	1851 83	234 8	0 31	7-5	Madler
1836 49	98 8 (Schatzung)		1	Madler	1852 52	250 1	0 5 $\pm$	2	Dawes
1836 52	88 8	0 56	6	Struve	1852 62	261 2	0 43	6	O Struve
					1852 67	241 1	0 30	13-11	Madler
1839 59	119 8	0 5 $\pm$	2	Dawes	1853 20	257 9	0 4 $\pm$	2	Jacob
1839 82	132 1	0 76	2	O Struve	1853 37	267 8	0 27	5	Madler
1839 82	126 9	0 59	3	W Struve	1853 56	280 9	0 32	5	O Struve
1840 52	137 2	0 51	5	O Struve	1853 64	273 3	0 44 $\pm$	4	Dawes
1840 62	135 9	0 50 $\pm$	2	Dawes	1853 79	270 4	0 3	1	Madler
1841 42	150 4	0 48	5	Madler	1854 04	285 3	0 5 $\pm$	3	Jacob
1841 50	149 7	0 52	5	O Struve	1854 42	301 5	0 47	3	Dawes
1841 65	149 4	0 49	6-1	Dawes	1854 66	313 2	0 33	4	O Struve
					1854 74	317 1	0 26	4-3	Madler
1842 26	157 6	0 55	5	Madler	1855 39	325 6	0 32 $\pm$	2	Secchi
1842 58	156 6	0 5 $\pm$	2	Dawes	1855 50	324 9	0 45	10-6	Winnecke
1842 60	159 1	0 57	2	O Struve	1855 51	322 5	0 45 $\pm$	1-3	Dawes
1843 37	166 9	0 57	6	Madler	1855 62	330 2	0 40	4	O Struve
1843 63	171 6	0 60	7	Madler	1855 77	330 2	—	2	Madler
1844 38	174 0	0 57	3	Madler	1856 35	336 8	0 51	9-6	Winnecke
1845 46	179 3	0 58	6	O Struve	1856 37	341 7	0 45	1-3	Dawes
1845 50	186 1	0 59	19	Madler	1856 39	327 7	0 5 $\pm$	2	Jacob
1845 64	188 3	0 60	1	W Struve	1856 51	341 6	0 55	8-4	Winnecke
					1856 59	344 4	0 47	7	Secchi
1846 61	195 7	0 61	3	O Struve	1856 62	342 6	0 47	3	O Struve
1846 50	194 0	0 56	14-13	Madler	1857 38	347 2	0 47	2	Madler
1847 07	196 6	—	3	Hind	1857 45	350 8	0 60	2	Dawes
1847 24	199 0	0 69	11	Madler	1857 48	351 0	0 58	7	Secchi
1847 64	204 0	0 56	5	O Struve	1857 62	351 8	0 65	4	O Struve
1847 71	204 6	0 62	5	Madler	1857 95	355 8	0 6 $\pm$	3	Jacob
1848 29	205 7	0 62	3	Madler	1858 48	356 5	0 79	1	Winnecke
1848 34	204 4	0 65	2	Dawes	1858 51	359 2	0 53	3	Secchi
1848 47	207 4	0 69	1	Dawes	1858 52	1 1 cuneo		10	Dembowski
1848 62	208 7	0 8 $\pm$	2	W <sup>c</sup> Bond	1858 54	359 6	0 76	5	O Struve
1848 72	209 8	0 57	2	O Struve	1858 61	6 2	0 69	6	Madler

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1859 39	5 0	0 70	4	Madler	1870 38	43 6	1 04	8	Dembowski
1859 48	4 5	0 53	4	Secchi	1870 38	47 2	0 98	4-1	Pence
1859 61	5 9	0 79	4	O Struve	1870 44	44 6	1 1	2	Gledhill
1859 62	5 5	0 72	3	Dawes	1870 46	44 1	1 29	—	Leyton Obs
					1870 47	46 8	1 13	1	Knott
1860 35	8 4	0 87	2	Dawes	1870 51	43 7	0 98	7	Dunér
1861 58	15 8	0 90	3	O Struve	1870 54	47 2	0 97	3	O Struve
1861 58	16 5	0 94	6	Madler	1871 41	47 7	—	—	Leyton Obs
1862 54	16 4	1 27	3-2	Winnecke	1871 45	47 8	1 09	8	Dembowski
1862 56	16 9	0 71	11	Dembowski	1871 53	47 3	0 88	9	Dunér
1862 58	22 8	0 99	3	Madler	1871 54	45 7	1 00	5	Knott
1862 76	22 5	0 91	2	O Struve	1871 56	47 6	1 42	2	Seabroke
					1871 57	46 4	0 95	1	Gledhill
1863 43	20 8	0 81	13	Dembowski	1872 29	47 8	1 29	—	Leyton Obs
1863 54	23 6	1 10	4	O Struve	1872 43	51 3	1 03	9	Dembowski
1863 56	19 7	1 07	—	Leyton Obs	1872 48	51 7	0 92	7	Ferrari
1863 59	23 3	0 83	2	Secchi	1872 49	51 0	1 01	1	W & S
					1872 58	51 2	0 84	7	Dunér
1864 44	24 2	0 74	10	Dembowski	1872 59	55 4	0 91	5	O Struve
1864 46	28 3	1 09	2	Englemann	1873 40	57 1	1 11	3	W & S
1865 15	30 1	1 13	5	Englemann	1873 44	56 1	1 04	8	Dembowski
1865 35	29 7	1 14	3	O Struve	1873 47	56 0	—	1	Leyton Obs
1865 41	27 4	1 03	9	Dembowski	1873 53	58 0	—	1	Landemann
1865 44	27 3	1 07	3	Dawes	1873 53	59 0	—	3-0	Möller
1865 50	26 3	0 79	2	Secchi	1873 53	53 9	—	1-0	Romberg
1865 52	30 1	1 59	1	Leyton Obs	1873 53	57 4	—	1-0	Schwarz
					1873 53	50 3	—	1-0	Wagner
1866 38	32 3	1 40	2	Leyton Obs	1873 54	54 1	1 00	5-3	Gledhill
1866 44	30 1	1 04	9	Dembowski	1873 54	63 1	—	1-0	Brunnow
1866 54	33 1	1 12	3	Secchi	1873 54	57 4	0 81	4	O Struve
1866 61	31 4	1 47	4-3	Harvard	1873 72	55 0	1 08	2	Dunér
1866 66	35 5	1 13	4	O Struve					
1867 34	35 9	1 07	3	Knott	1874 39	58 6	0 99	3	Gledhill
1867 40	35 6	1 19	3-2	Harvard	1874 42	59 5	0 98	8	Dembowski
1867 47	32 6	1 24	2	O Struve	1874 43	61 2	0 62	2-1	Leyton Obs
1867 50	33 2	1 04	7	Dembowski	1874 46	58 2	0 93	2-1	W & S
1867 52	31 5	—	1	Leyton Obs	1874 61	64 7	0 83	4	O Struve
1867 62	30 8	0 96	1	Winnecke	1875 37	60 7	—	1	Leyton Obs
1867 69	29 2	1 12	1	Dunér	1875 41	66 7	0 86	8	Dembowski
					1875 42	66 1	0 91	4	Schiaparelli
1868 39	36 0	1 05	7	Dembowski	1875 48	62 5	0 74	1	O Struve
1868 55	41 3	1 05	5	O Struve	1875 55	68 7	0 70	11	Dunér
1868 61	36 0	—	2	Zollner	1876 38	70 3	0 79	8-2	Doberck
1868 65	37 0	1 15	4	Dunér	1876 44	70 5	0 77	4	Hall
1868 80	35 8	0 88	1	Peirce	1876 45	70 3	0 83	1	Leyton Obs
					1876 46	74 8	0 84	9	Dembowski
1869 53	40 1	1 03	9	Dunér	1876 51	72 3	0 79	5	Schiaparelli
1869 61	44 7	—	1	Leyton Obs	1876 61	73 6	0 66	4	O Struve

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1877 25	77 7	0 78	1	Copeland	1885 26	—	0 57	1	Copeland
1877 30	82 0	0 69	4-2	Doberck	1885 41	170 1	0 65	4	Hall
1877 36	70 3	—	6	W & S	1885 51	171 6	0 57 $\pm$	10	Schiaparelli
1877 42	79 6	0 75	5	Schiaparelli	1885 53	170 7	0 70	5-1	Sea & Smith
1877 48	81 1	0 78	9	Dembowski	1885 58	170 0	0 61	7	Englemann
1877 53	71 9	1 0 $\pm$	1	Plummer	1886 46	177 0	0 70	5	Hall
1877 56	77 9	0 58	4	O Struve	1886 49	180 8	0 72	4	Periotin
1878 41	90 8	0 62	1	Burnham	1886 51	178 6	0 63	3	Tarrant
1878 45	93 3	0 62	3	Doberck	1886 51	181 3	0 80 $\pm$	3-1	Smith
1878 50	91 0	0 60	8	Dembowski	1886 52	178 8	0 66	11	Schiaparelli
1878 53	88 3	0 75	9	Schiaparelli	1886 64	179 1	0 57	8	Englemann
1878 59	87 6	0 57	4	O Struve	1887 43	186 6	0 82	1	Hough
1878 80	84 4	0 67	1	Pritchett	1887 51	185 6	0 60	15	Schiaparelli
1879 52	102 4	0 62	7	Schiaparelli	1887 63	186 0	0 72	3	Tarrant
1879 54	98 7	0 48	4	Hall	1888 45	195 7	0 62	5	Hall
1880 45	111 9	—	2	Bigourdan	1888 53	199 0	—	1	Copeland
1880 50	116 7	0 52	3-2	Doberck	1888 55	194 8	0 60	14	Schiaparelli
1880 53	115 6	0 50	6	Schiaparelli	1888 63	193 9	0 74	3	O Struve
1880 59	114 2	oblong	5	Jedzejewicz	1889 42	182 0	—	1	Hodges
1880 62	114 3	0 46	5	Burnham	1889 50	202 3	0 63	4	Hall
1880 70	114 9	0 76	2	Copeland	1889 52	200 8	0 64	6	Schiaparelli
1881 26	121 3	—	2	Doberck	1889 58	202 1	0 72	1	O Struve
1881 40	124 9	0 46	4	Hall	1890 43	oblong	—	1	Glasenapp
1881 50	126 9	0 61 $\pm$	4	Schiaparelli	1890 50	210 1	0 64	6	Hall
1881 64	125 8	0 48	1	O Struve	1890 67	208 2	—	1	Bigou dan
1882 30	134 8	0 55	3-2	Doberck	1891 48	218 4	0 61	3	Hall
1882 45	138 4	0 51	4	Hall	1891 50	213 5	0 67 $\pm$	1	See
1882 50	135 4	0 59	8	Schiaparelli	1891 52	216 8	0 57	8	Schiaparelli
1882 55	141 7	0 50	2	O Struve	1891 54	222 0	0 75	3	Maw
1882 61	153 2	0 56	6-4	Englemann	1892 44	226 1	0 69	1	H C Wilson
1883 48	147 2	0 69	10	Schiaparelli	1892 45	230 1	0 72	2	Leavenworth
1883 51	152 5	0 57	6	Hall	1892 50	230 2	0 57	11	Bigou dan
1883 51	153 2	0 51	7	Englemann	1892 57	229 5	0 57	6	Schiaparelli
1883 56	156 0	0 61	7	Periotin	1892 65	229 8	0 48	3	Comstock
1883 59	151 6	0 58	3	O Struve	1893 48	244 7	0 63	1	Maw
1883 64	150 5	0 5 $\pm$	6-5	Jedzejewicz	1893 48	243 2	0 51	7	Schiaparelli
1884 43	159 4	—	6	Bigou dan	1893 50	242 8	0 50	3	Leavenworth
1884 48	160 1	0 57	3	Hall	1893 52	245 6	0 49	7-6	Bigou dan
1884 52	163 1	0 64	6	Periotin	1894 48	262 1	0 44	6	Schiaparelli
1884 52	162 0	0 54 $\pm$	6	Schiaparelli	1894 49	261 4	0 44	1	Bigou dan
1884 54	161 7	0 67	1	Pritchett	1895 30	285 0	0 45	3	See
1884 58	158 0	0 58	3	O Struve	1895 51	285 9	0 30 $\pm$	3	Comstock
1884 64	165 6	0 58	5	Englemann					
1884 66	172 4	—	3	Seabroke					

This beautiful pair proved to be one of the first objects which gave distinct evidence of orbital motion, and the binary character of the system was fully recognized by HERSCHEL in 1803. Since the time of STRUVE the measures are both numerous and satisfactory. The pair is always rather close, but as the components are nearly equal in magnitude, it is generally easy to separate. Numerous orbits have been published by previous computers; the following table of elements is fairly complete.

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
<sup>YRS</sup> 44 242	1806 20	0 26034	0 8325	220 6	37 4	358 63	Herschel, 1833	Mem R A S, VI, 156
43 246	1850 23	0 3376	1 0879	24 3	71 13	261 35	Madler, 1842	Doip Obs, IX, 195
43 310	1815 20	0 3537	1 1912	22 6	71 5	263 17	Madler, 1842	
42 500	1807 21	0 289	0 9024	20 1	59 47	215 2	Madler, 1847	Fixt Syp, I, p 243
42 501	1805 666	0 4743	1 0125	10 52	65 65	227 17	Villarcéau 1842	
66 257	1780 124	0 4695	1 1108	4 42	58 05	194 62	Villarcéau 1852	
67 309	1779 338	0 4043	1 2015	9 87	59 32	185 0	Villarcéau 1852	A N, 868
43 115	1850 329	0 2865	0 9567	22 3	60 67	215 48	Winnecke	
41 58	1850 26	0 2625	0 827	26 7	58 0	211 4	Wijkander	
41 576	1850 26	0 2625	0 827	26 7	58 0	215 6	Dunér, 1871	A N, 1868
40 17	1849 9	0 287	0 985	22 2	60 4	224 1	Flammann 1874	Cat ét Doub, p 88
41 562	1850 792	0 2667	0 892	25 72	59 68	218 6	Doberck, 1880	A N, 2338
41 6	1892 3	0 33	0 86	26 9	55 0	220 5	Comstock, 1893	Proc Am Assoc, 1894

Making use of all the measures up to 1895, we find the following elements of  $\eta$  *Coronae Borealis*\*:

$$\begin{aligned}
 P &= 41.60 \text{ years} & \Omega &= 27^\circ 10' \\
 T &= 1892.50 & i &= 58^\circ 50' \\
 e &= 0.267 & \lambda &= 217^\circ 57' \\
 a &= 0''.9165 & n &= +8^\circ 653846
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 1''.804 \\
 \text{Length of minor axis} &= 0''.934 \\
 \text{Angle of major axis} &= 28^\circ 7' \\
 \text{Angle of periastron} &= 229^\circ 0' \\
 \text{Distance of star from center} &= 0''.209
 \end{aligned}$$

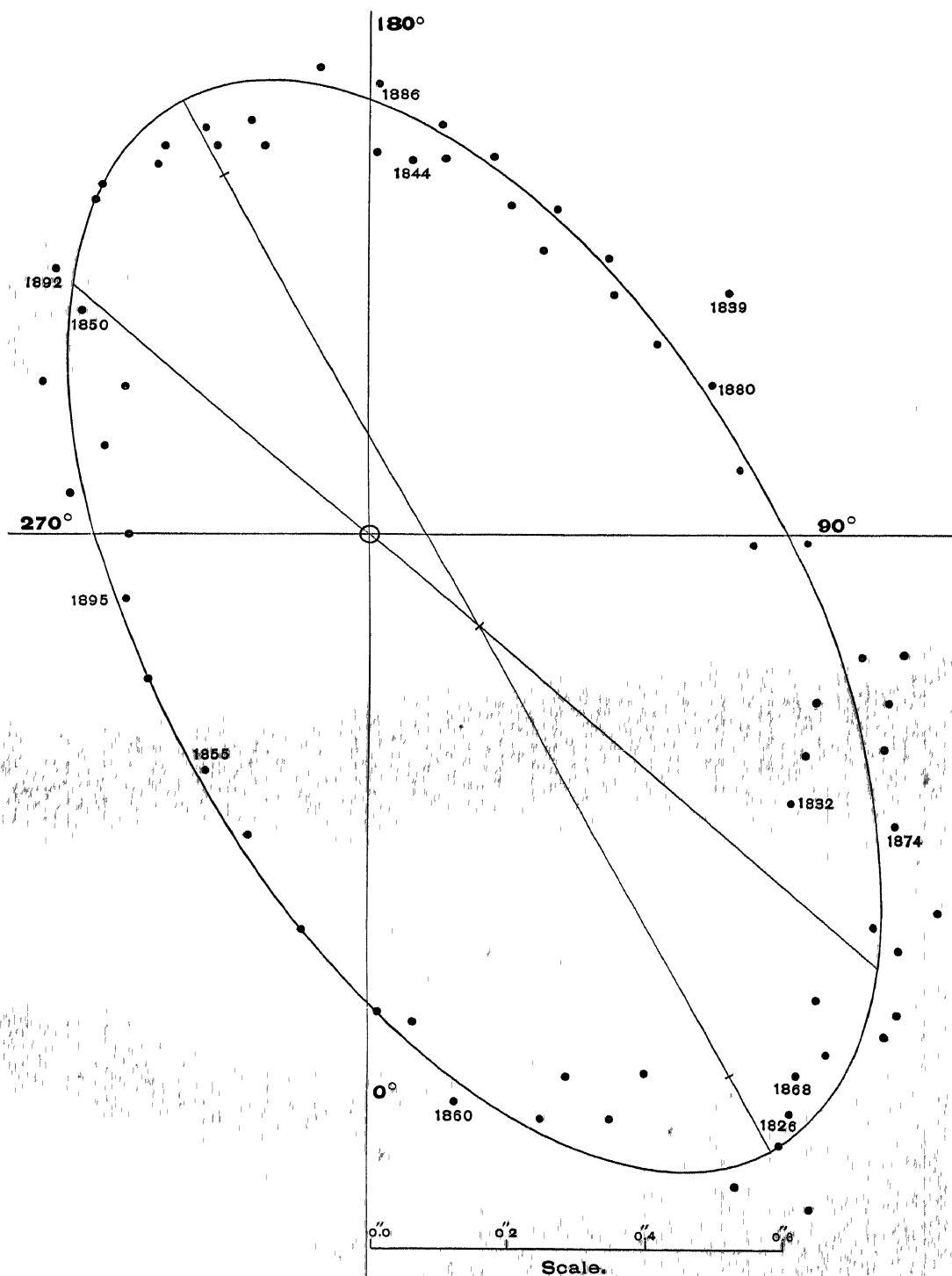
The accompanying table shows that the motion is well represented, and that the present elements will finally undergo but slight corrections.

\* *Astronomische Nachrichten*, 3361



## COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1781 69	30 7	27 4	—	1 08	+3 3	—	1	Heischel
1802 69	179 7	174 8	—	0 63	+4 9	—	1	Heischel
1823 27	25 9	27 3	1 58	1 08	-1 4	+0 50	2-1	Heischel and South
1826 77	35 3	37 9	1 07	1 09	-2 6	-0 02	4	Struve
1829 55	43 2	47 0	0 96	1 01	-3 8	-0 05	2	Struve
1831 48	51 4	54 5	0 95	0 92	-3 1	+0 03	15-4	Dawes 2-0, Heischel 10-1, $\Sigma$ 3
1832 60	56 9	59 5	0 74	0 86	-2 6	-0 12	13-5	Heischel 9-2, Dawes 1-0, $\geq$ 3
1833 33	62 7	63 4	0 72	0 82	-0 7	-0 10	11-2	Heischel 8-2, Dawes 3-0
1834 84	69 1	72 5	0 70	0 73	-3 4	-0 03	1	Struve
1835 41	75 7	76 6	0 74	0 70	-0 9	+0 04	5	Struve
1836 52	88 8	85 9	0 56	0 63	+2 9	-0 07	6	Struve
1839 70	125 9	122 2	0 63	0 53	+3 7	+0 10	4	Dawes 2, $O\Sigma$ 2
1840 57	136 0	133 4	0 51	0 53	+2 6	-0 02	7	$O\Sigma$ 5, Dawes 2
1841 52	149 8	146 0	0 50	0 54	+3 8	-0 04	16-11	Madler 5, $O\Sigma$ 5, Dawes 6-1
1842 48	157 8	157 5	0 54	0 57	+0 3	-0 03	9	Madler 5, Dawes 2, $O\Sigma$ 2
1843 50	169 2	168 2	0 58	0 60	+1 0	-0 02	13	Madler 6, Madler 7
1844 38	174 0	176 4	0 57	0 64	-2 4	-0 07	3	Madler
1845 46	179 3	184 8	0 58	0 68	-5 5	-0 10	6	O Struve
1846 61	195 7	194 1	0 61	0 71	+1 6	-0 10	3	O Struve
1847 42	201 0	200 0	0 63	0 71	+1 0	-0 08	24-21	Hind 3-0, Madler 11, $O\Sigma$ 5, Madler 5
1848 49	207 2	207 8	0 66	0 70	-0 6	-0 04	10	Madler 3, Dawes 2, Dawes 1, Bond 2, $O\Sigma$
1849 54	219 3	216 0	0 64	0 66	+3 3	-0 02	5-4	Dawes 2-1, $O\Sigma$ 3
1850 59	231 5	225 6	0 54	0 60	+5 9	-0 06	8	$O\Sigma$ 3, Fletcher 2, Madler 3
1851 53	237 8	235 9	0 42	0 53	+1 9	-0 11	22-19	Madler 3-2, Dawes 2, $O\Sigma$ 10, Madler 7-5
1852 60	250 8	253 5	0 41	0 44	-2 7	-0 03	21-19	Dawes 2, $O\Sigma$ 6, Madler 13-11
1853 51	270 3	272 9	0 35	0 40	-2 6	-0 05	17	Jacob 2, Madler 5, $O\Sigma$ 5, Dawes 4, Madler 1
1854 46	304 3	296 5	0 39	0 38	+7 8	+0 01	14-13	Jacob 3, Dawes 3, $O\Sigma$ 4, Madler 4-3
1855 56	326 6	321 6	0 43	0 43	+5 0	$\pm$ 0 00	19-13	Sec 2-0, Winn 10-6, Da 1-3, $O\Sigma$ 4, Ma 2-0
1856 47	339 1	337 7	0 49	0 50	+1 4	-0 01	30-25	Winn 9-6, Da 1-3, Ja 2, Winn 84, Sec 7, $O\Sigma$ 3
1857 57	351 3	350 6	0 61	0 61	+0 7	$\pm$ 0 00	18-16	Madler 2-0, Dawes 2, Secchi 7, $O\Sigma$ 4, Jacob 3
1858 54	1 3	359 0	0 73	0 70	+2 3	+0 03	24-11	Secchi 3-0, Dembowski 10-0, $O\Sigma$ 5, Madler 6
1859 52	5 2	5 6	0 74	0 79	-0 4	-0 05	15-11	Madler 4, Secchi 4-0, $O\Sigma$ 4, Dawes 3
1860 35	8 4	10 1	0 87	0 86	-1 7	+0 01	2	Dawes
1861 58	16 1	15 6	0 92	0 94	+0 5	-0 02	9	$O\Sigma$ 3, Madler 6
1862 61	19 6	19 7	0 87	1 00	-0 1	-0 13	19-16	Winn 3-0, Dembowski 11, Madler 3, $O\Sigma$ 2
1863 53	21 8	22 9	0 95	1 04	-1 1	-0 09	19+	Dem 13, $O\Sigma$ 4, Leyton Obs —, Secchi 2
1864 45	26 3	25 9	0 91	1 07	+0 4	-0 16	12	Dembowski 10, Englemann 2
1865 40	28 5	28 9	1 12	1 09	-0 4	+0 03	23	En 5, $O\Sigma$ 3, Dem 9, Da 3, Sec 2, Ley 1
1866 52	32 5	32 4	1 23	1 10	+0 1	+0 13	22-21	Leyton Obs 2, Dem 9, Sec 3, Hv 4-3, $O\Sigma$ 4
1867 50	33 0	35 3	1 10	1 10	-2 3	$\pm$ 0 00	18-16	Kn 3, Hv 3-2, $O\Sigma$ 2, Dem 7, Ley 1-0, Du 1,
1868 59	37 5	38 6	1 03	1 09	-1 1	-0 06	17	Dem 7, $O\Sigma$ 5, Dunér 4, Peirce 1 [Winn 1
1869 57	40 7	41 6	1 03	1 06	-0 9	-0 03	10-9	Dunér 9, Leyton Obs 1-0
1870 45	45 1	44 6	1 07	1 04	+0 5	+0 03	25-22	Dem 8, Pei 4-1, Gl 2, Ley —, Kn 1, Du 7, $O\Sigma$ 3
1871 51	47 1	48 3	1 06	1 00	-1 2	+0 06	25	Ley —, Dem 8, Du 9, Kn 9, Sea 2, Gl 1
1872 47	51 2	52 0	1 00	0 96	-0 8	+0 04	29+	Ley —, Dem 9, Fei 7, W & S 1, Du 7, $O\Sigma$ 5
1873 52	55 9	56 4	1 01	0 90	-0 5	+0 11	22-20	W & S 3, Dem 8, Ley —, Gl 5-3, $O\Sigma$ 4, Du 2
1874 47	60 5	61 0	0 89	0 85	-0 5	+0 03	19-17	Ley 2-1, Gl 3, Dem 8, W & S 2-1, $O\Sigma$ 4
1875 44	67 2	66 2	0 82	0 79	+1 0	+0 03	23	Dembowski 8, Schiaparelli 4, Dunér 11
1876 45	71 9	72 6	0 80	0 73	-0 7	+0 07	31-25	Dk 8-2, Hl 4, Ley 1, Dem 9, Sch 5, $O\Sigma$ 4
1877 41	77 2	79 4	0 80	0 68	-2 2	+0 12	30-22	Cop 1, Dk 4-2, W & S 6-0, Sch 5, Dem 9, Pl 1,
1878 55	89 2	89 7	0 64	0 61	-0 5	+0 03	26	$\beta$ 1, Dk 3, Dem 8, Sch 9, $O\Sigma$ 4, P1 1 [ $O\Sigma$ 4
1879 53	100 5	100 2	0 55	0 57	+0 3	-0 02	11	Schiaparelli 7, Hall 4
1880 56	114 5	112 5	0 54	0 54	+2 0	$\pm$ 0 00	23-20	Big 2-0, Dk 3-2, Sch 6, Jed 5, $\beta$ 5, Cop 2
1881 44	124 7	123 9	0 51	0 53	+0 8	-0 02	11-9	Doberck 2-0, Hall 4, Schiaparelli 4, $O\Sigma$ 1
1882 49	140 7	137 8	0 54	0 53	+2 9	+0 01	23-20	Doberck 3-2, Hall 4, Sch 8, $O\Sigma$ 2, En 6-4
1883 55	151 8	150 9	0 58	0 55	+0 9	+0 03	39-38	Sch 10, Hl 6, En 7, Per 7, $O\Sigma$ 3, Jed 6-5 [Sea 3-0
1884 54	163 5	162 5	0 60	0 58	+1 0	+0 02	33-24	Big 6-0, Hl 3, Per 6, Sch 6, P1 1, $O\Sigma$ 3, En 5,



$\eta$  Coronae Borealis =  $\Sigma$  1937.



$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1885 46	170 6	171 7	0 63	0 62	-1 1	+0 01	26-23	Cop 0-1, Hl 4, Sch 10, Sea & Sm 5-1, En 7
1886 52	179 3	181 1	0 68	0 66	-1 8	+0 02	34-32	Hall 5, Per 4, Tar 3, Sm 3-1, Sch 11, En 8
1887 51	186 1	189 0	0 71	0 69	-2 9	+0 02	19	Hough 1, Schiaparelli 15, Tarrant 3
1888 54	195 8	196 5	0 65	0 71	-0 7	-0 06	23-22	Hall 5, Copeland 1-0, Schiaparelli 14, $O\Sigma$ 3
1889 53	201 7	203 7	0 66	0 71	-2 0	-0 05	11	Hall, Schiaparelli 6, $O\Sigma$ 1
1890 53	209 1	211 4	0 64	0 69	-2 3	-0 05	7-6	Hall 6, Bigourdan 1-0
1891 51	217 6	219 1	0 65	0 64	-1 5	+0 01	15	Hall 3, See 1, Schiaparelli 8, Maw 3
1892 50	229 1	229 1	0 61	0 58	$\pm 0 0$	+0 03	23	H C W 1, Lv. 2, Big 11, Sch 6, Com 3
1893 49	244 1	241 8	0 53	0 50	+2 3	+0 03	18-17	Maw 1, Schiaparelli 7, Lv 3, Big 7-6
1894 49	261 8	259 3	0 44	0 43	+2 5	+0 01	7	Schiaparelli 6, Bigourdan 1
1895 51	285 9	282 7	0 37	0 38	+3 2	-0 01	3-6	See 0-3, Comstock 3

The uncertainty in the period does not surpass 0.1 year, and an alteration of the eccentricity amounting to  $\pm 0.01$  is not probable. It seems, however, that there are occasional systematic errors in the angles, and hence careful measurement should be continued. It will not be many years before a definitive determination of the elements of this interesting binary can be advantageously undertaken. The following is a short ephemeris for the use of observers

$t$	$\theta_o$	$\rho_o$	$t$	$\theta_o$	$\rho_o$
1896 50	306 9	0 39	1899 50	353 8	0 64
1897 50	327 7	0 45	1900 50	1 6	0 73
1898 50	342 9	0 54			

 $\mu^2$  BOOTIS =  $\Sigma$ 1938.

$\alpha = 15^h 20^m 7$ ,  $\delta = +37^\circ 43'$   
6 5, white, 8, white

Discovered by Sir William Herschel, September 10, 1781

## OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1782 68	357 2	—	1	Herschel	1833 02	319 3	1 00	3-1	Herschel
1802 86	346 2	—	—	Herschel	1833 39	319 8	1 15	1	Dawes
1822 21	330 7	—	2	Struve	1833 85	319 7	1 19	3	Struve
1823 41	333 7	1 65	3	H & So	1835 55	318 6	1 10	3	Struve
1825 46	333 53	1 43	5	South	1835 65	309 1	—	1	Madler
1826 77	327 0	1 38	2	Struve	1836 45	310 1	—	2	Madler
1829 73	324 0	1 24	2	Struve	1836 65	315 1	1 06	3	Struve
1830 24	324 1	0 85	2	Herschel	1837 37	314 9	1 0 $\pm$	1	Dawes
1831 36	321 7	1 14	1	Herschel	1837 70	315 0	0 9	—	Struve
					1839 83	310 4	—	—	W Struve

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1840 39	306 0	0 83	3	Dawes	1857 38	239 2	0 35	2	Madler
1840 46	313 8	0 98	3	O Struve	1857 52	231 7	0 55	1	Secchi
1841 47	308 7	0 82	2	Madler	1857 65	237 9	0 58	3	O Struve
1841 66	303 2	0 86	6 3	Dawes	1858 56	225 9	0 45	1	Secchi
1842 23	303 8	0 85	3	O Struve	1858 56	228 3	0 57	3	O Struve
1842 40	305 2	0 72	3	Madler	1858 57	236 0	0 32	4	Madler
1842 40	300 9	0 85 $\pm$	3	Dawes	1859 39	226 4	0 42	3-2	Madler
1842 66	304 9	0 78	2	Madler	1860 95	211 3	0 58	3	O Struve
1843 57	301 5	0 76	10	Madler	1861 58	215 1	0 42	2	Madler
1844 39	299 2	0 71	2	Madler	1862 56	202 9	0 3?	3	Dembowski
1845 54	295 8	0 64	10	Madler	1862 63	217 7	0 4 $\pm$	1	Madler
1846 40	291 8	0 64	12-11	Madler	1863 38	195 8	0 55	12	Dembowski
1846 68	287 1	0 57	4	O Struve	1863 63	195 8	0 75	-	Leyton Obs
1847 08	281 3	—	2	Hind	1864 41	193 0	0 51	4	Knott
1847 30	286 5	0 65 $\pm$	4	Dawes	1864 48	189.5	cuneo	5	Dembowski
1847 38	288 1	0 55	15-13	Madler	1865 45	184 8	0 53	10	Dembowski
1848 37	282 4	0 42	2	Madler	1865 46	190 1	0 48 $\pm$	3	Dawes
1848 52	280 0	0 65	4	Dawes	1865 72	197 9	—	1	Leyton Obs
1848 52	282 9	0 56	3-4	G P Bond	1865 78	187 5	0 57	5	Englemann
1849 44	276 2	0 68	2	Dawes	1866 40	179 2	0 60	3	O Struve
1850 46	272 7	0 53	2	O Struve	1866 41	196 4	0 85	3-2	Leyton Obs
1850 69	276 7	0 40	3-2	Madler	1866 48	181 2	0 50	7	Dembowski
1851 28	264 9	0 32	3	Madler	1866 54	180 3	in cont	1	Secchi
1851 42	266 6	0 52	2	Dawes	1867 48	175 8	0 60	6	Dembowski
1851 48	262 7	0 44	3	O Struve	1868 38	174 2	0 53	5	Dembowski
1851 77	263 4	0 31	4	Madler	1869 49	171 1	0 53	6	Dunér
1852 52	262 2	0 55 $\pm$	1	Dawes	1869 54	167 5	0 54	2	O Struve
1852 60	261 3	0 41	10	Madler	1870 39	165 8	0 62	7	Dembowski
1852 65	268 2	0 49	3	O Struve	1870 44	164 0	—	1	Gledhill
1853 23	265 1	0 45 $\pm$	2	Jacob	1870 52	163 9	0 59	4	Dunér
1853 34	256 2	0 33	4	Madler	1870 65	170 8	—	-	Leyton Obs
1853 71	254 6	0 5 $\pm$	1	Dawes	1871 43	161 2	0 61	7	Dembowski
1853 77	256 6	0 40	2	Madler	1871 54	160 8	0 67	5	Dunér
1854 05	253 7	0 5 $\pm$	2	Jacob	1871 57	167 9	0 76	1	Seabrooke
1854 41	249 3	0 47	3	Dawes	1871 65	158 4	0 5 $\pm$	1	Gledhill
1854 70	247 2	0 44	4	Madler	1872 29	167 5	—	-	Leyton Obs
1855 11	247 2	0 53	4	O Struve	1872 35	163 4	0 35 $\pm$	2	W & S
1855 52	256 9	0 42	2	Madler	1872 44	154 1	0 65	8	Dembowski
1856 42	236 5	0 45	1	Secchi	1872 46	152 0	0 6 $\pm$	4	Knott
1856 57	242 1	0 59	2	O Struve	1872 52	158 0	0 55	2	Dunér

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1873 09	158 2	0 63	4	O Struve	1883 50	115 0	0 70	2	Hall
1873 34	151 0	0 52 $\pm$	3-2	W & S	1883 57	117 5	0 76	6	Englemann
1873 41	151 0	0 67	7	Dembowski	1883 59	112 9	0 75	2	Peirrotin
1873 48	155 8	—	1	Leyton Obs	1883 63	110 2	0 64	1	O Struve
1873 47	152 3	0 48 $\pm$	2	Gledhill	1884 48	113 8	0 69	3	Hall
1874 22	150 7	0 58	2	Gledhill	1884 51	112 3	0 74 $\pm$	4	Schiaparelli
1874 44	149 1	0 7	1	W & S	1884 62	110 2	0 86	2	O Struve
1874 44	147 8	0 81	6	Dembowski	1884 67	119 9	—	4	Seabrooke
1874 54	155 4	—	1	Leyton Obs	1885 40	110 8	0 75	2	Peirrotin
1875 41	141 9	0 69	8	Dembowski	1885 49	105 8	1 00 $\pm$	3-1	Smith
1875 47	143 3	0 64 $\pm$	4	Schiaparelli	1885 49	110 1	0 79	3	Tarrant
1875 52	146 7	0 80	1	Dunér	1885 49	111 3	0 71	4	Hall
1876 35	143 6	—	2	Doberck	1885 50	109 4	0 89	4	Schiaparelli
1876 44	145 4	0 73	4	Hall	1885 63	116 9	0 85	7-6	Englemann
1876 46	138 2	0 70	8	Dembowski	1885 70	110 6	0 7 $\pm$	6	Jedrzejewicz
1877 24	138 5	0 75	5	Schiaparelli	1886 49	106 7	—	2	Smith
1877 38	131 6	0 56	4-2	Doberck	1886 51	107 3	0 65	3	Hall
1877 42	136 9	0 71	7	Dembowski	1886 51	106 0	0 72	2	Peirrotin
1877 49	145 3	0 73	4	W & S	1886 54	107 7	0 74	2	Schiaparelli
1877 62	143 0	0 67	1	O Struve	1886 78	106 2	0 7 $\pm$	5	Jedrzejewicz
1878 41	136 2	0 68	1	Burnham	1887 44	105 4	0 70	4	Hall
1878 49	137 6	0 62	4	Doberck	1887 55	99 0	—	1	Smith
1878 52	132 0	0 62	6	Dembowski	1887 56	103 0	0 74	6	Schiaparelli
1878 53	132 7	0 63 $\pm$	5	Schiaparelli	1888 45	100 0	0 60	4	Hall
1878 58	137 7	0 63	1	O Struve	1888 59	101 5	0 75	5-3	Schiaparelli
1879 51	128 6	0 79	4	Schiaparelli	1888 91	103 1	0 73	2	Tarrant
1879 54	133 3	0 73	4	Hall	1888 69	101 6	0 87	1	O Struve
1880 18	128 7	0 78	5	Burnham	1889 35	97 8	0 73	3	Maw
1880 40	129 6	0 64	1	Hall	1889 42	96 2	1 00	1	Hodges
1880 50	130 1	0 70	4	Doberck	1889 52	98 7	0 84	3	Schiaparelli
1880 53	126 7	0 79	4	Schiaparelli	1890 50	107 8	(0 85)	2	Glasenapp
1880 65	122 6	0 7 $\pm$	4	Jedrzejewicz	1891 49	95 4	0 80 $\pm$	2	Schiaparelli
1881 26	126 9	—	4	Doberck	1891 53	94 7	0 74 $\pm$	2	See
1881 38	126 0	0 63	4	Burnham	1892 42	92 6	0 82	1	Collins
1881 50	121 6	0 78	4	Schiaparelli	1892 58	89 1	0 74	4	Comstock
1881 50	123 7	0 62	6-4	Bigourdan	1893 47	88 0	0 98	4	Bigourdan
1881 50	121 9	0 62	3	Hall	1893 49	88 6	0 77	2	Maw
1881 63	122 4	0 72	1	O Struve	1894 48	85 6	1 19	1	Callandreaux
1882 32	125 0	0 75	2-1	Doberck	1894 50	86 0	1 05	5	Bigourdan
1882 43	121 7	0 64	3	Hall	1894 59	85 4	0 75	1	H C Wilson
1882 52	120 4	0 79	4	Schiaparelli	1895 31	83 5	0 84	3	See
1882 53	121 9	0 77	4	Englemann	1895 52	83 9	0 64	3	Comstock
1882 55	116 9	0 64	1	O Struve					
1883 47 *	114 3	0 87	4	Schiaparelli					

When the observations of 1782 were compared with those of 1802, the physical character of the system was fairly indicated\* Since the time of STRUVE it has been carefully followed by the best observers, and accordingly the material now available for an orbit is highly satisfactory. The companion is only slightly smaller than the principal star, and is therefore never very difficult to measure In all parts of the orbit the pair is sufficiently wide to be seen with a six-inch telescope, but as the minimum distance of 0".49 in angle 230° was passed in 1858, it is not surprising that the observers on either side of this epoch, with few exceptions, have made their observed distances too small Thus, although the measures of different observers are not infrequently affected by systematic errors of sensible magnitude, yet by combining the best measures into mean positions for each year, we obtain a set of places which give an orbit that seems likely to be very near the truth

Some of the elements hitherto published are as follows

<i>P</i>	<i>T</i>	<i>e</i>	<i>a</i>	$\Omega$	<i>i</i>	$\lambda$	Authority	Source
<sup>YRS</sup> 146 649	1851 57	0 8529	1 320	94 7	49 4	87 1	Madler, 1847	Fixt Syst, I, 252
182 6	1866 0	0 491	1 165	166 1	47 5	23 0	Winagr, 1872	
314 34	1860 88	0 5641	1 761	163 2	41 9	54 4	Hind, 1872	M N, vol XXXII, p 250
200 4	1865 2	0 51	—	172 0	45 0	20 1	Wilson, 1872	Handb D S, p 313
198 93	1865 5	0 4957	—	169 0	46 4	23 6	Klinkerfues	Handb D S, p 313
290 07	1863 51	0 6174	1 500	183 0	44 4	17 7	Doberck, 1875	A N, 2026
280 29	1860 51	0 5974	1 47	173 7	39 9	20 0	Doberck, 1878	A N, 2194
266 0	1862 55	0 5668	1 057	166 7	35 2	40 9	Pritchard, "	Ox Obs, No 1, p 64

From an investigation of all the observations which appear to be reliable, we find the following elements of  $\mu^2$  Bootis:

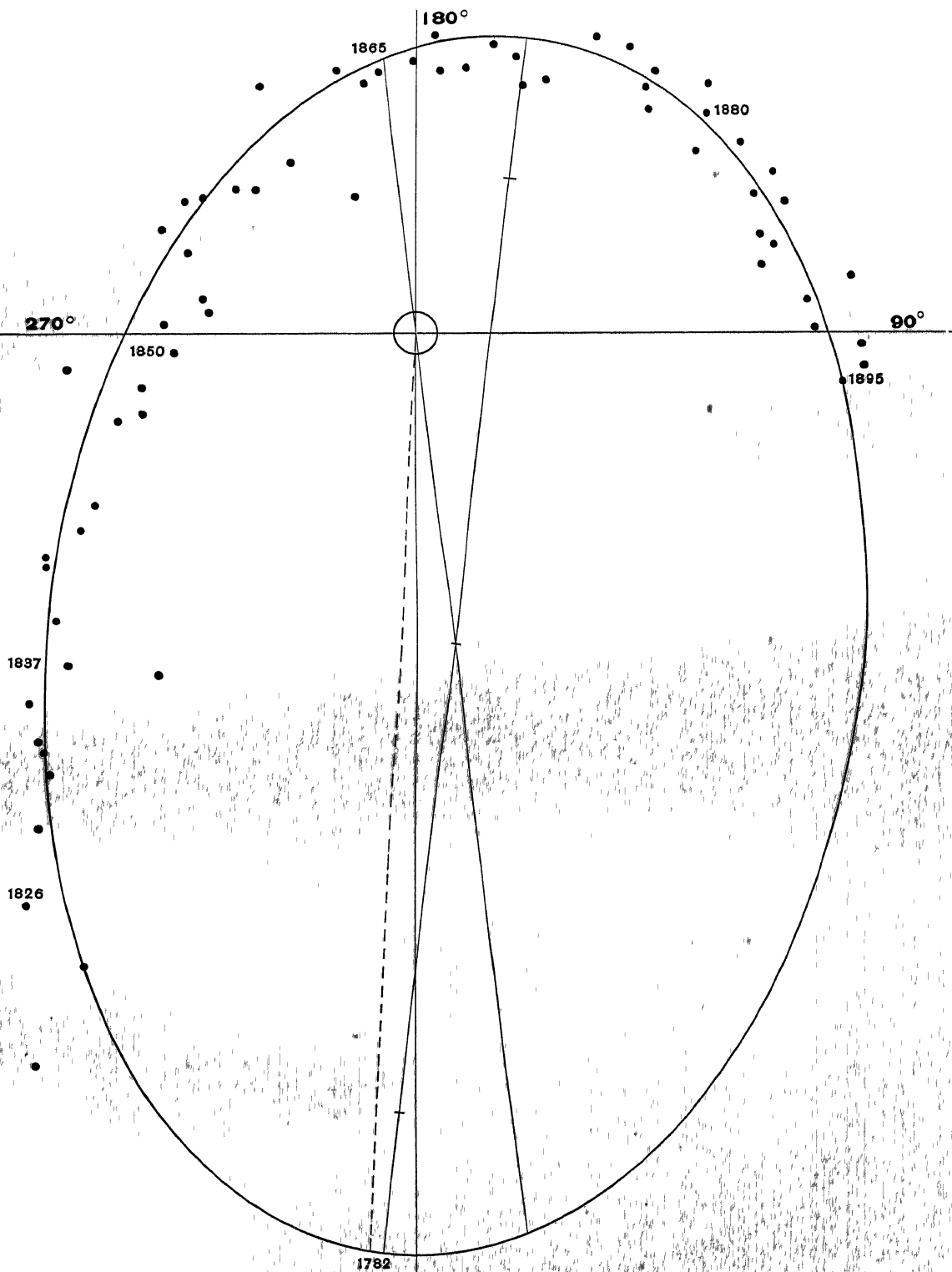
$P = 219\,42\text{ years}$  $\Omega = 163^{\circ}8$  $T = 1865\,30$  $i = 43^{\circ}9$  $e = 0\,537$  $\lambda = 329^{\circ}75$  $a = 1''\,2679$  $n = -1^{\circ}\,6407$

Apparent orbit:

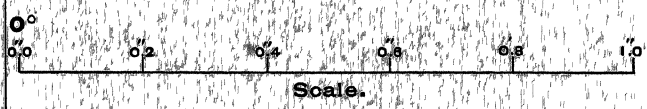
Length of major axis = 2" 656  
Length of minor axis = 1" 480  
Angle of major axis = 173° 5  
Angle of periastron = 186° 7  
Distance of star from centre = 0" 638

An examination of the computed and observed places, given in the following table, seems to justify the conclusion that the elements found above will

\* *Astronomische Nachrichten*, 3309



$\mu^{\circ}$  Boötis =  $\Sigma$  1938.







not be materially changed by future investigation. Thus, the period will hardly be varied by so much as ten years, while the resulting alterations in the eccentricity, inclination and other elements will be relatively inconsiderable.

TABLE OF COMPUTED AND OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1782 68	357 2	353 9	—	1 83	+3 3	—	1	Herschel
1802 86	346 2	343 6	—	1 68	+2 6	—	—	Herschel
1822 21	330 7	330 8	—	1 38	-0 1	—	2	Struve
1823 41	333 7	329 0	1 65	1 33	+4 7	+0 32	3	Herschel and South
1825 46	333 5	326 2	1 43	1 26	+6 3	+0 17	5	South
1826 77	327 0	325 9	1 38	1 25	+1 1	+0 13	2	Struve
1829 73	324 0	323 4	1 24	1 20	-0 9	+0 04	2	Struve
1830 24	324 1	322 1	0 85	1 17	+2 0	-0 32	2	Herschel
1831 36	321 7	320 9	1 14	1 14	+0 8	0 00	1	Herschel
1833 42	319 6	318 1	1 11	1 09	+1 5	+0 02	7-5	Herschel 3-1, Dawes 1, $\Sigma$ 3
1835 55	318 6	315 1	1 10	1 03	+3 5	+0 07	3	Struve
1836 65	315 1	313 4	1 06	1 00	+1 7	+0 06	3	Struve
1837 53	315 0	311 8	0 95	0 97	+3 2	-0 02	—	Dawes 1, $\Sigma$ —
1840 42	309 9	306 7	0 91	0 89	+3 2	+0 02	6	Dawes 3, $O\Sigma$ 3
1841 66	303 2	304 1	0 86	0 85	-0 9	+0 01	6-3	Dawes
1842 32	302 4	302 6	0 85	0 83	-0 2	+0 02	6	$O\Sigma$ 3, Dawes 3
1843 57	301 5	299 5	0 76	0 80	+2 0	-0 04	10	Mädler
1844 39	299 2	297 3	0 71	0 77	+1 9	-0 06	2	Mädler
1846 68	287 1	291 3	0 57	0 71	-3 2	-0 14	4	O Struve
1847 34	287 3	288 4	0 60	0 68	-1 1	-0 08	19-17	Dawes 4; Mädler 15-13
1848 47	281 7	284 3	0 54	0 65	-2 6	-0 11	9-10	Mädler 2, Dawes 4, Bond 3-4
1849 44	276 2	280 5	0 68	0 63	-4 3	+0 05	2	Dawes
1850 57	274 7	275 6	0 47	0 60	-0 9	-0 13	5-4	$O\Sigma$ 2, Mädler 3-2
1851 49	263 9	271 2	0 40	0 58	-7 3	-0 18	12	Mädler 7, Dawes 2, $O\Sigma$ 3
1852 55	268 2	265 8	0 49	0 55	+2 4	-0 06	3	O Struve
1853 50	260 9	260 1	0 42	0 53	+0 8	-0 11	4	Jacob 2, Mädler 2
1854 39	250 1	255 5	0 47	0 52	-5 4	-0 05	9	Jacob 2, Dawes 3, Mädler 4
1855 11	247 2	250 8	0 53	0 51	-2 6	+0 02	4	O Struve
1856 49	239 3	241 1	0 52	0 49	-1 8	+0 03	3	Secchi 1, $O\Sigma$ 2
1857 52	236 3	235 0	0 49	0 49	+1 3	0 00	6	Mädler 2, Secchi 1, $O\Sigma$ 3
1858 56	230 1	228 0	0 45	0 49	+2 1	-0 04	8	Secchi 1, $O\Sigma$ 3, Mädler 4
1859 39	226 4	223 7	0 42	0 49	+2 7	-0 07	3-2	Mädler
1860 95	211 3	212 4	0 58	0 50	-1 1	+0 08	3	O Struve
1861 58	215 1	207 9	0 42	0 50	-7 2	-0 08	2	Mädler
1862 56	202 9	202 2	0 32	0 52	+0 7	-0 20	3	Dembowski
1863 38	195 8	197 3	0 55	0 53	+1 5	+0 02	12	Dembowski
1864 44	191 2	191 2	0 51	0 54	0 0	-0 03	9	Knott 4, Dembowski 5
1865 56	187 5	184 7	0 53	0 56	+2 8	-0 03	18	Dem 10, Dawes 3, Englemann 5
1866 47	180 2	181 4	0 55	0 57	-1 2	-0 02	11	$O\Sigma$ 3, Dembowski 7, Secchi 1
1867 48	175 8	175 9	0 60	0 59	-0 1	+0 01	6	Dembowski
1868 38	174 2	171 8	0 53	0 60	+2 4	-0 07	5	Dembowski
1869 51	169 3	166 7	0 54	0 61	+2 6	-0 07	8	Dunér 6, $O\Sigma$ 2
1870 45	164 6	162 7	0 60	0 62	+1 9	-0 02	12-11	Dem 7, Gledhill 1-0, Dunér 4
1871 54	160 1	158 4	0 59	0 63	+1 7	-0 04	13	Dem 7, Dunér 5, Gledhill 1
1872 44	156 9	154 8	0 54	0 65	-1 8	-0 11	16	W & S, Dem 8, Kn 4, Du 2
1873 38	153 1	151 5	0 57	0 65	+1 6	-0 08	16-15	$O\Sigma$ 4, W & S 3-2, Dem 7, Gl 2
1874 37	149 2	147 6	0 69	0 66	+1 6	+0 03	9	Gledhill 2; W & S 1, Dem 6
1875 46	144 0	143 5	0 71	0 67	+0 5	+0 04	13	Dem 8, Schiaparelli 4, Dunér 1
1876 46	138 2	139 4	0 70	0 68	-1 2	+0 02	8	Dembowski
1877 38	137 6	136 5	0 67	0 68	+1 1	-0 01	20-18	Sch 5, Dk 4-2, Dem 7, W & S 4
1878 49	134 6	132 7	0 64	0 69	+1 9	-0 05	16	$\beta$ 1, Dk 4; Dem 6, Sch 5

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1879 52	131 0	129 3	0 76	0 69	+1 7	+0 07	8	Schiaparelli 4, Hall 4
1880 44	127 7	126 3	0 72	0 70	+1 4	+0 02	17	$\beta$ 5, Hl 1, Dk 4, Sch 4, Jed 4
1881 43	123 8	122 8	0 66	0 70	+1 0	-0 04	21-15	Dk 4-0, $\beta$ 4, Sch 4, Big 6-4, Hl 4
1882 45	121 3	119 6	0 74	0 71	+1 7	+0 03	13-12	Dk 0-1, Hall 3, Sch 4, En 4
1883 53	114 9	116 2	0 77	0 72	-1 3	+0 05	14	Sch 4, Hall 2, En 6, Per 2
1884 49	113 0	113 1	0 72	0 72	-0 1	0 00	7	Hall 3, Schiaparelli 4
1885 52	110 4	110 0	0 77	0 73	+0 4	+0 04	19	Per 2, Tai 3, Hl 4, Sch 4, Jed 6
1886 58	106 5	107 3	0 70	0 74	-0 8	-0 04	12	Hall 3, Per 2, Sch 2, Jed 5
1887 50	104 2	104 2	0 72	0 75	0 0	-0 03	10	Hall 4, Schiaparelli 6
1888 65	101 5	101 0	0 69	0 76	+0 5	-0 07	11-9	Hall 4, Schiaparelli 5-3, Tarrant 2
1889 43	97 6	98 7	0 86	0 77	-1 1	+0 09	7	Maw 3, Hodges 1, Schiaparelli 3
1891 51	95 0	93 2	0 77	0 79	+1 8	-0 02	4	Schiaparelli 2, See 2
1892 50	90 9	90 6	0 78	0 80	+0 3	-0 02	5-4	Collins 1, Comstock 4-3
1893 48	88 3	88 2	0 87	0 81	+0 1	+0 06	6	Bigourdan 4, Maw 2
1894 54	85 7	85 6	0 88	0 82	+0 1	+0 06	6	Bigourdan 5, H C Wilson 1
1895 31	83 5	83 8	0 84	0 84	-0 3	0 00	3	See

The following is a short ephemeris :

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 50	81 1	0 85	1899 50	74 8	0 89
1897 50	78 9	0 86	1900 50	72 6	0 90
1898 50	76 9	0 87			

O $\Sigma$ 298.

$\alpha = 15^h 32^m 4$  ,  $\delta = +40^\circ 9'$   
7, yellowish , 7 4, yellowish

Discovered by Otto Struve in 1845

## OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1845 50	180 5	1 25	2	O Struve	1865 53	210 2	1 0	1	Dembowski
1846 28	186 5	1 41	2	Madler	1866 29	207 0	0 8	1	Dembowski
1847 32	189 6	1 51	2-1	Madler	1867 61	209 5	0 99	1	Dembowski
1848 46	183 9	1 11	1	O Struve	1868 52	32 5	0 84	1	O Struve
1848 68	185 8	1 23	1	Dawes	1869 46	214 1	0 61	3	Dunér
1851 75	191 8	1 40	2	Madler	1870 26	225 8	separation doubtful	1	Dembowski
1856 58	193 1	1 21	1	O Struve	1871 63	226 6	contatto?	1	Dembowski
1857 68	196 8	1 24	1	O Struve	1872 58	235 8	0 58	1	O Struve
1859 62	197 4	1 13	1	O Struve	1875 52	84 2	0 53	1	O Struve
1861 44	13 5	1 16	1	O Struve	1875 65	265 5	0 37	2	Dembowski

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1876 47	280 8	0 3	cuneo 3	Dembowski	1887 50	142 0	0 39	3	Hall
1877 53	295 9	0 3	5	Dembowski	1887 56	143 0	0 33	6	Schiaparelli
1878 33	130 8	0 27	2	Burnham	1888 54	339 4	0 65	1	O Struve
1879 46	335 0	0 26	4	Hall	1888 59	153 4	0 42	5	Schiaparelli
1879 49	327 8	0 33	4	Schiaparelli	1889 52	158 1	0 55	3	Schiaparelli
1881 41	175 4	0 35	3	Hall	1891 48	167 3	0 68	3	Hall
1882 47	7 5	0 33	4	Schiaparelli	1891 49	347 5	0 63	1	Schiaparelli
1882 52	359 5	0 30	4-3	Englemann	1892 42	169 9	0 82	1	Collins
1882 55	358 0	0 32	1	O Struve	1892 47	169 3	0 88	2	Bigourdan
1883 52	22 4	0 31	6	Schiaparelli	1892 59	168 9	0 64	4	Comstock
1883 65	36 7	0 17	3	Englemann	1893 43	351 5	0 91	1	Bigourdan
1884 44	49 0	0 30	2	Perrotin	1893 71	173 6	0 64	1	Comstock
1884 51	57 3	0 31	5	Schiaparelli	1895 54	173 1	0 85	3	Comstock
1885 65	60 9	0 27	7-4	Englemann	1895 56	174 2	0 82	1	Schiaparelli
1886 67	133 7	0 29	2	Schiaparelli	1895 71	179 4	0 95	2	See
1886 68	104 9	0 29	7	Englemann	1895 74	177 2	1 05	1	Moulton

Since the discovery of this binary in 1845, the companion has described substantially an entire revolution. The period is therefore fixed with sufficient precision; indeed, the numerous and satisfactory measures of this pair secured during the last fifty years define the other elements in a manner almost equally satisfactory. The shape of the apparent orbit is such that the pair is never excessively difficult, and yet measurement near periastron, where the distance reduces to  $0'' 22$ , requires a good telescope. The components are of nearly equal brightness, and hence a number of the measures as recorded requires a correction of  $180^\circ$ .

The following orbits of this pair have been published by previous computers:

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
$^{yrs}$ 68 802	1812 96	0 4872	0 886	14 63	56 17	342 52	Doberck, 1879	A N, 2280
70 26	1882 22	0 51	0 83	12 29	50 63	346 15	Dolgorukow, 1883	A N, 2531
56 653	1882 857	0 5836	0 8835	2 13	65 85	21 9	Celonia, 1888	A N, 2843
51 0	1883 0	0 577	0 780	4 1	61 2	20 7	See, 1895	Unpublished

An investigation based on all the best observations leads to the following elements of O $\Sigma$  298.

$$\begin{aligned}
 P &= 52.0 \text{ years} \cdot & \Omega &= 1^\circ 9' \\
 T &= 1883.0 & i &= 60^\circ 9' \\
 e &= 0.581 & \lambda &= 26^\circ 1' \\
 a &= 0'' 7989 & n &= +6^\circ 9231
 \end{aligned}$$

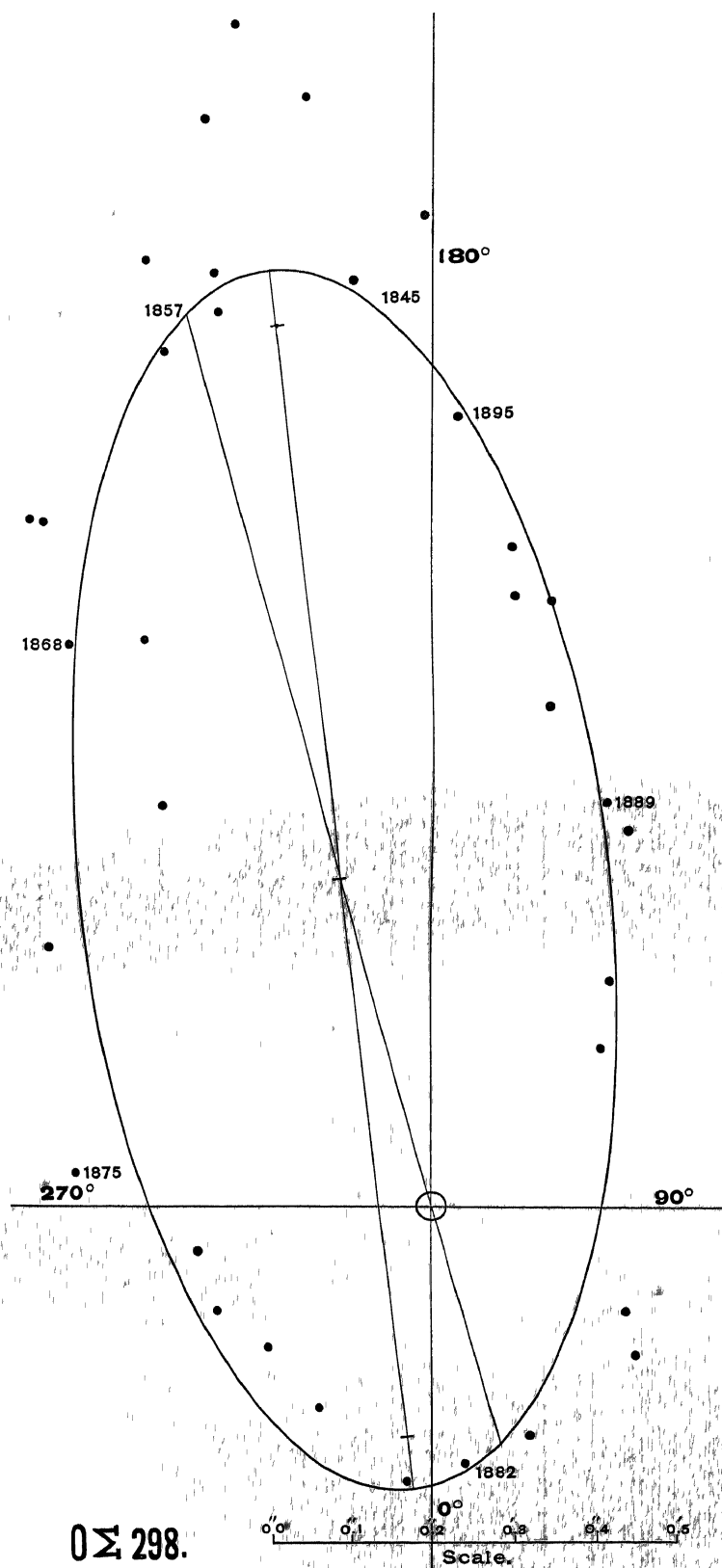
## Apparent orbit

Length of major axis	= 1" 546
Length of minor axis	= 0" 656
Angle of major axis	= 186° 9
Angle of periastron	= 15° 3
Distance of star from centre	= 0" 427

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1845 50	180 5	180 5	1 25	1 07	$\pm 0 0$	+0 18	2	O Struve
1846 28	186 5	181 6	1 41	1 09	+ 4 9	+0 32	2	Madler
1847 32	189 6	183 1	1 51	1 12	+ 6 5	+0 39	2-1	Madler
1848 57	184 9	184 8	1 17	1 16	+ 0 1	+0 01	2	O Struve 1, Dawes 1
1851 75	191 8	188 8	1 40	1 19	+ 3 0	+0 21	2	Madler
1856 58	193 1	190 2	1 21	1 20	+ 2 9	+0 01	1	O Struve
1857 68	196 8	196 2	1 24	1 15	+ 0 6	+0 09	1	O Struve
1859 62	197 4	198 9	1 13	1 11	- 1 5	+0 02	1	O Struve
1861 44	193 5	201 6	1 16	1 06	- 8 1	+0 10	1	O Struve
1865 53	210 2	209 1	1 0	0 90	+ 1 1	+0 10	1	Dembowski
1866 29	207 0	210 8	0 8	0 87	- 3 8	-0 07	1	Dembowski
1867 61	209 5	214 2	0 99	0 80	- 4 7	+0 19	1	Dembowski
1868 52	202 5	216 9	0 84	0 75	-14 4	+0 09	1	O Struve
1869 46	214 1	220 0	0 61	0 71	- 5 9	-0 10	3	Dunér
1870 26	225 8	222 7	—	0 67	+ 3 1	—	1	Dembowski
1871 63	226 6	229 6	—	0 58	- 3 0	—	1	Dembowski
1872 58	235 8	235 3	0 58	0 53	+ 0 5	+0 05	1	O Struve
1875 57	264 7	263 2	0 45	0 37	+ 1 5	+0 08	3	O Struve 1, Dembowski 2
1876 47	280 8	275 9	0 3	0 34	+ 4 9	-0 04	3	Dembowski
1877 53	295 9	292 8	0 3	0 33	+ 3 1	-0 03	5	Dembowski
1878 33	310 8	306 4	0 27	0 33	+ 4 4	-0 06	2	Burnham
1879 47	331 4	325 0	0 29	0 34	+ 6 4	-0 05	8	Hall 4, Schiaparelli 4
1881 41	355 4	352 1	0 35	0 36	+ 3 3	-0 01	3	Hall
1882 47	7 5	6 6	0 33	0 34	+ 0 9	-0 01	4	Schiaparelli
1883 57	22 4	26 7	0 31	0 28	- 4 3	+0 03	6	Schiaparelli
1884 47	53 1	53 7	0 31	0 22	- 0 6	+0 09	7	Perrotin 2, Schiaparelli 5
1885 65	60 9	102 4	0 27	0 22	-41 5	+0 05	7-4	Englemann
1886 68	133 7	130 6	0 29	0 29	+ 3 1	$\pm 0 00$	2	Schiaparelli
1887 53	142 5	144 1	0 36	0 38	- 1 6	-0 02	9	Hall 3, Schiaparelli 6
1888 56	153 4	153 6	0 53	0 48	- 0 2	+0 05	5-6	O Struve 0-1, Schiaparelli 5
1889 52	158 1	159 1	0 55	0 56	- 1 0	-0 01	3	Schiaparelli
1891 49	167 4	167 8	0 65	0 74	- 0 4	-0 09	4	Hall 3, Schiaparelli 1
1892 49	169 4	170 8	0 78	0 81	- 0 6	-0 03	7	Collins 1, Bigourdan 2, Com 4
1893 62	172 5	173 4	0 78	0 88	- 0 9	-0 10	2	Bigourdan 1, Comstock 1
1895 55	173 7	177 3	0 84	0 99	- 3 6	-0 15	4	Comstock 3, Schiaparelli 1
1895 74	178 3	177 6	1 00	1 00	+ 0 7	$\pm 0 00$	3	See 2, Moulton 1

The table of computed and observed places shows that these elements are extremely satisfactory. Future observations are not likely to vary the period given above by more than one year, while an error of  $\pm 0.02$  in the eccentricity is highly improbable. In spite of the accuracy of the present elements some improvement will ultimately be desirable, and hence astronomers should continue to give this interesting system regular attention. The star will be easy





for a number of years, and observers with small telescopes will find it an important object for measurement.

The following is a short ephemeris.

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 50	178° 9	1.03	1899 50	183° 3	1.13
1897 50	180 5	1.07	1900 50	184 7	1.15
1898 50	182 0	1.10			

### $\gamma$ CORONAE BOREALIS = $\Sigma$ 1967.

$$\alpha = 15^h 38^m 5, \quad \delta = +26^\circ 36'.$$

4, yellow, 7, blue

*Discovered by William Struve in 1826*

#### OBSERVATIONS

$t$	$\theta_c$	$\rho_c$	$n$	Observers	$t$	$\theta_c$	$\rho_c$	$n$	Observers
1826 75	110° 0	0.72	2	Struve	1848 39	297° 0	0.39	4	Madler
1828 98	110 7	0.54	3	Struve	1848 49	292 8	0.4 $\pm$	3	W. G. Bond
1832 21	102 7	0.4 $\pm$	3	Struve	1849 63	289 4	0.50	3	O. Struve
1833.34	105 8	0.4 $\pm$	2	Struve	1850 69	289 9	0.53	3	Madler
1835 46	simplex	—	3	Struve	1851 33	292 5	0.3 $\pm$	1	Madler
1836 52	338 ?	obl ?	4	Struve	1851 50	287 6	0.48	4	O. Struve
1840 51	252 cuneiforme		1	W. Struve	1852 07	285 1	0.57 $\pm$	4	Dawes
1840 78	255 cuneiforme		4	O. Struve	1852 58	296 4	0.46	7-6	Madler
1841 50	332 3	0.18	10-4	Madler	1853 01	287 9	0.46	5	O. Struve
1842 49	314 3	0.20	4-1	Madler	1853 20	294 3	0.5	2	Jacob
1842 80	272 0	0.47	2	Madler	1853 32	284 5	0.40	4-3	Madler
1843 30	292 5	0.41	3	O. Struve	1854 40	284 3	0.69	2	Dawes
1843 45	288 9	0.6 $\pm$	1	Dawes	1854 76	291 1	0.4 $\pm$	1	Madler
1843 48	276 6	0.39	9-2	Madler	1855 50	semplix	—	—	Secchi
1844 37	286 2 ?	—	1	Madler	1855 73	292 4	—	1	Madler
1845 37	292 1	0.45	9	Madler	1856 37	295 4	0.67	3	Winnecke
1845 61	296 0	0.44	5	O. Struve	1856 59	288 9	0.45	8-7	Secchi
1845 57	292 7	0.43	9-8	Madler	1856 62	283 8	0.47	6	O. Struve
1846 56	294 2	0.45	11	Madler	1857 39	286 5	0.32	2	Madler
1847 29	292 6	0.44	5	O. Struve	1857 52	281 0	0.5 $\pm$	1	Dawes
1847 43	295 1	0.36	11-9	Madler	1857 52	289 3	0.36	5	Secchi
					1858 51	281 0	cuneo	3	Dembowski
					1858 57	284 1	0.33	4-3	Madler
					1858 97	284 7	0.46	5	O. Struve



$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1859 36	282 6	0 45 $\pm$	1	Dawes	1883 53	142 6	0 16 $\pm$	3	Perrotin
1859 38	290 4	obl	3	Madler	1883 57	129 1	0 41	5	Schiaparelli
1861 59	287 7	0 42	3	O Struve	1883 60*	149 3	0 58	1	O Struve
1862 56	292 9	cuneo	3	Dembowski	1883 64	146 9	0 20	8	Englemann
1862 91	227 ?	doubtful	1	Madler	1884 52	125	cuneiforme	2	Perrotin
1863 25	semplce	—	1	Dembowski	1884 53	305 6	0 34	1	Perrotin
1863 64	290 5	0 41	3	O Struve	1884 53	132 4	0 34	6	Schiaparelli
1865 6	semplce	—	4	Secchi	1884 61	166 8	0 28	6	Englemann
1865 26	semplce	—	1	Dembowski	1885 48	round	—	1	Smith
1865 50	280	<0 5	1	Englemann	1885 54	134 3	0 35	3	Schiaparelli
1865 53	einfach	—	1	Englemann	1885 63	164 6	0 38	10-6	Englemann
1866 30	201 2	—	4	Halvaad	1886 51	129 1	0 38	6	Schiaparelli
1866 61	205 3	—	1	Winlock	1886 69	159 9 ?	0 93 ?	8	Englemann
1866 62	286 0	0 43	2	O Struve	1887 51	126 6	0 38	13	Schiaparelli
1867 75	simple	—	10	Dunér	1887 55	round	—	1	Smith
1868 02	260 2	0 36	2	O Struve	1888 55	124 3	0 40	16-15	Schiaparelli
1868 72	252	cuneiforme	—	O Struve	1888 61	132 0	0 85	2	O Struve
1869 36	280 4	—	1	Leyton Obs	1889 42	109 2	—	1	Hodges
1872 45	190 ?	—	1	W & S	1889 52	122 4	0 41	4	Schiaparelli
1873 38	195 ?	—	1	W & S	1890 68	124 1	0 51	1	Bigourdan
1874	simple	—	—	O Struve	1891 50	120 0	0 5 $\pm$	1	See
1874 56	166 9	—	1	Leyton Obs	1891 51	122 5	0 42	4	Schiaparelli
1875 40	single	—	1	Hall	1891 51	125 6	0 36	4	Hall
1875 41	165 4	—	1	Leyton Obs	1891 58	118 8	0 51	1	Bigourdan
1876 32	simple	—	1	Flammarion	1892 44	122 3	0 83 $\pm$	1	H C Wilson
1876	single	—	1	Doberck	1892 44	121 1	0 69	1	Bigourdan
1876 45	single	—	1	Hall	1892 60	122 8	0 47	7	Schiaparelli
1876 81	simple	—	—	Schiaparelli	1892 72	121 9	0 40	3	Comstock
1877 54	163 3	0 44	2	O Struve	1893 49	120 0	0 52	2	Schiaparelli
1878 60	150 7	0 56	2	O Struve	1893 50	118 4	0 65	2	Bigourdan
1879 56	single	—	2	Hall	1894 48	119 7	0 53	2	Schiaparelli
1879 81	single	—	5	Burnham	1894 60	121 3	0 60	5-4	Barnard
					1895 30	114 8	0 67	3	See
					1895 55	117 1	0 43	3	Comstock
					1895 61	123 7	0 64	4	Barnard

The components of this remarkable system are of the 4th and 7th magnitudes, and of yellow and bluish colors respectively, so that the object is generally very difficult. STRUVE happened to discover\* the companion near the time of its maximum elongation, when the polar coordinates were  $\theta = 111^\circ 0$ ,

\* *Astronomical Journal*, 376

$\rho = 0''.72$ . Measures in 1828, 1831 and 1833, showed that both angles and distances were steadily decreasing, and in 1835 the star appeared single under the best seeing. The companion was not again recognized with certainty until 1842, although STRUVE, O. STRUVE and MADLER searched for it repeatedly during the intervening period, and occasionally suspected an elongation. But the discordance in the angles of the supposed elongations justify the belief that the phenomena observed were probably nothing more than points of diffraction fringes, or some other kind of spurious images. MADLER's observation of  $332^\circ 3$  and  $0''.18$  at the epoch 1841.50 may be genuine, although at this time the star must have been excessively close. The binary character of the pair was early recognized by STRUVE, who pointed out the particular interest attaching to the system on account of its high inclination.  $\gamma$  *Coronae Borealis* has since been measured by many of the best observers, and yet the stars are so unequal and so close that the errors of observation assume formidable proportions, and render a satisfactory determination of the elements very difficult. The great inclination of the orbit throws nearly all the position-angles into small regions of about  $10^\circ$  on either side, and while the retrograde motion ought to make all angles steadily decrease, we are sometimes confounded by an appearance of direct motion (as from 1859 to 1863) which proves the existence of sensible systematic errors, probably due to the placing of the micrometer wires parallel to the edges of unequal images.

It is equally confusing to find that instead of a steady increase and decrease in the distance, nearly all of the distances are in the immediate neighborhood of  $0''.4$ , such measures are of course misleading, as the companion cannot be standing still at a constant angle and distance. While, therefore, it is clear that the elements can not lay claim to such accuracy as could be desired, it will yet appear that they are good and even excellent for observations which are so badly vitiated by accidental and systematic errors.

It is obvious that in case of a system whose orbit plane lies nearly in the line of vision, the angles will be practically useless unless measured with the greatest accuracy, yet, in this instance, even when the pair is fairly wide, we frequently find the angles of individual observers differing by so much as  $10^\circ$ , and when the stars are close the uncertainty in angle will amount to at least twice this quantity. On account of such conspicuous errors in angle we have based the present orbit largely upon the distances.

DOBERCK and CELORIA are the only astronomers who have previously attempted an orbit for this pair.

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
<sup>yrs</sup> 95 50	1843 7	0 387	0 75	111 0	83 0	239 0	Doberck, 1877	A N, Bd, 88
95 5	1843 70	0 350	0 70	110 4	85 2	233 5	Doberck, 1877	A N, 2123
85 276	1840 508	0 3483	0 631	113 47	81 67	250°7	Celoria, 1889	A N, 2904

From an investigation of the best observations we find the following elements .

$$\begin{aligned}
 P &= 730 \text{ years} & \Omega &= 110^\circ 7 \\
 T &= 1841.0 & i &= 82^\circ 63 \\
 e &= 0.482 & \lambda &= 97^\circ 95 \\
 a &= 0'' 7357 & n &= -4^\circ 9315
 \end{aligned}$$

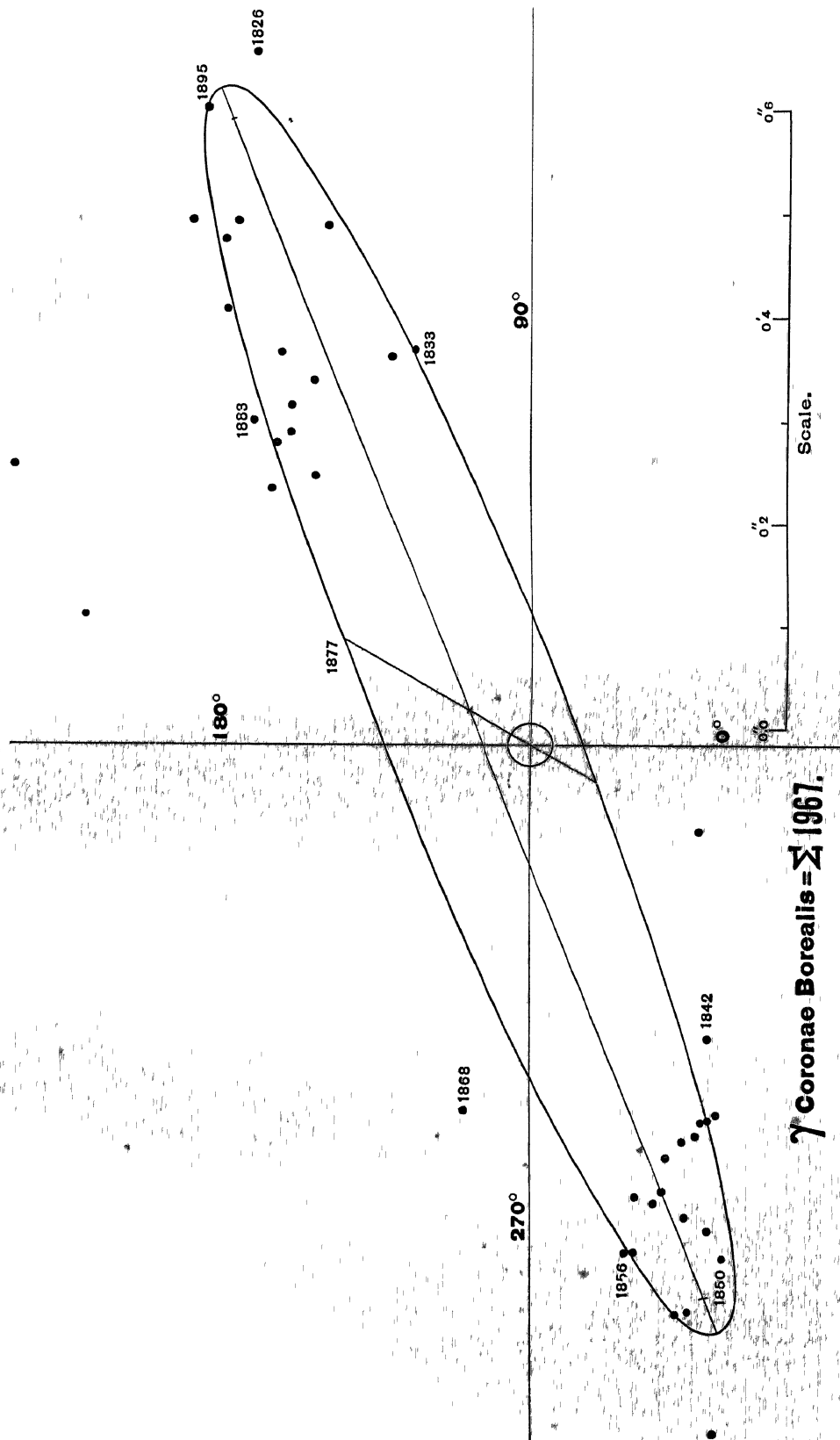
#### Apparent orbit

$$\begin{aligned}
 \text{Length of major axis} &= 1'' 30 \\
 \text{Length of minor axis} &= 0'' 175 \\
 \text{Angle of major axis} &= 111^\circ 3 \\
 \text{Angle of periastron} &= 329^\circ 6 \\
 \text{Distance of star from centre} &= 0'' 068
 \end{aligned}$$

The accompanying table shows the agreement of the above elements with the mean places.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1826 75	111 0	114 5	0 72	0 70	- 3 5	+0 02	2	Struve
1828 98	110 7	113 5	0 54	0 69	- 2 8	-0 15	3	Struve
1831 68	109 3	111 2	0 4±	0 63	- 1 9	-0 23±	1	Struve
1833 34	105 8	110 0	0 4±	0 57	- 4 2	-0 17±	2	Struve
1835 46	simplex	107 4	—	0 44	—	—	3	Struve
1836 52	oblong?	105 5	—	0 37	—	—	4	Struve
1840 78	75	95 1	cune	0 16	-20 1	—	4	O Struve
1841 50	332 3	314 8	0 18	0 10	+17 5	+0 08	10-4	Madler
1842 64	300 4	301 9	0 33	0 21	- 1 5	+0 12	6-3	Madler
1843 30	292 5	298 6	0 41	0 28	- 6 1	+0 13	3	O Struve
1844 37	286 2?	295 5	—	0 37	- 9 3	—	1	Madler
1845 61	296 0	293 1	0 44	0 45	+ 2 9	-0 01	5	O Struve
1847 36	293 8	290 8	0 40	0 54	+ 3 0	-0 14	16-14	O Struve 5, Madler 11-9
1848 44	294 9	289 7	0 4	0 57	+ 5 2	-0 17	7	Ma 4, W C & G P Bond 3
1849 63	289 9	288 6	0 50	0 58	+ 1 3	-0 08	3	O Struve
1850 69	289 9	287 7	0 53	0 59	+ 2 2	-0 06	3	Madler
1851 50	287 6	287 0	0 48	0 60	+ 0 6	-0 12	4	O Struve
1852 07	285 1	286 5	0 57±	0 60	- 1 4	-0 03±	4	Dawes
1853 17	286 2	285 6	0 45	0 59	+ 0 6	-0 14	9-10	OΣ 5, Ja 0-2, Ma 4-3
1854 40	284 3	284 4	0 69	0 58	- 0 1	+0 11	2	Dawes
1855 73	292 4	283 3	—	0 56	+ 9 1	—	1	Madler
1856 62	283 8	282 4	0 57	0 54	+ 1 4	+0 03	6-9	O Struve
1857 52	281 0	281 4	0 50	0 52	- 0 4	-0 02	1	Dawes
1858 97	284 7	279 8	0 46	0 48	+ 3 9	-0 02	5	O Struve
1859 36	282 6	279 4	0 45	0 47	+ 3 2	-0 02	1	Dawes
1861 59	287 7	276 2	0 42	0 41	+11 5	+0 01	3	O Struve
1862 73	260 0	274 0	cuneo	0 38	-14 0	—	4	Dembowski, Madler 1
1863 64	290 5	272 3	0 41	0 35	+18 0	+0 06	3	O Struve





$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1865 50	280°	267 7	<0 5	0 30	+12 3	+0 2—	1	Englemann
1866 62	286 0	260 0	0 43	0 24	+26 0	+0 19	2	O Struve
1868 02	260 2	257 9	0 36	0 22	+ 2 3	+0 14	2	O Struve
1872 91	192 5	209 0	—	0 13	—16 5	—	2	Wilson & Seabrooke
1874 56	166 9	184 8	—	0 14	—17 9	—	1	Leyton Observers
1875 41	165 4	175 3	—	0 14	— 9 9	—	1	Leyton Observers
1877 54	163 3	156 3	0 44	0 18	+ 7 0	+0 26	2	O Struve
1878 60	150 7	147 0	0 56	0 22	+ 3 7	+0 34	2	O Struve
1883 57	129 1	130 3	0 41	0 36	— 1 2	+0 05	5	Schiaparelli
1884 53	127 6	128 0	0 33	0 41	— 0 4	—0 08	9-13	Per 3-1, Sch 6, En 0-6
1885 54	134 3	126 8	0 35	0 43	+ 7 5	—0 08	3	Schiaparelli
1886 51	129 1	125 3	0 38	0 46	+ 3 8	—0 08	6	Schiaparelli
1887 51	126 6	124 2	0 38	0 48	+ 2 4	—0 10	13	Schiaparelli
1888 55	124 3	123 0	0 40	0 52	+ 1 3	—0 12	16-15	Schiaparelli
1889 50	119 8	122 0	0 41	0 54	— 2 2	—0 13	5-4	Hodges 1-0, Schiaparelli 4
1890 68	124 1	121 1	0 51	0 57	+ 3 0	—0 06	1	Bigourdan
1891 52	121 7	120 2	0 45	0 59	+ 1 5	—0 14	10	See 1, Sch 4, Hill 4, Big 1
1892 55	122 0	119 4	0 60	0 62	+ 2 6	—0 02	12	H C Wilson 1; Sch 7, Com 3
1893 50	118 4	118 7	0 58	0 64	— 0 3	—0 06	2-4	Bigourdan 2, Schiaparelli 0-2
1894 54	120 4	117 9	0 57	0 66	+ 2 5	—0 09	6	Schiaparelli 2, Barnard 4
1895 42	116 0	117 3	0 69	0 67	— 1 3	+0 02	6-3	See 3, Comstock 3-0

The following is a short ephemeris .

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 50	116 6	0 69	1899 50	115 3	0 70
1897 50	116 0	0 69	1900 50	114 1	0 70
1898 50	115 3	0 70			

According to this orbit previous investigators have materially overestimated the period. While the time of revolution must at present remain slightly uncertain, it does not seem at all probable that this element can surpass 75 years. It follows, therefore, that  $\gamma$  *Coronae Borealis* belongs to the class of unequal binaries with moderately short periods. The inclination and line of nodes here obtained will probably be nearly correct, while the eccentricity is not likely to be varied by so much as  $\pm 0.05$ .

Recent distances have been appreciably undermeasured by several observers; the separation of the components is now about 0".68, and will not change sensibly for several years.  $\gamma$  *Coronae Borealis* needs further observation, and astronomers should continue to give it regular attention; but owing to the peculiar shape of the apparent orbit great care must be exercised to avoid systematic errors, if the measures are to be of much value in effecting a further improvement of the elements.

$\xi$  SCORPII =  $\Sigma$ 1998.

$\alpha = 15^{\text{h}} 53^{\text{m}} 9$  ,  $\delta = -11^{\circ} 5'$   
5, yellow , 52, yellow

*Discovered by Sir William Herschel, September 9, 1781*

## OBSERVATIONS

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1782 36	188 0	—	1	Herschel	1846 17	23 2	1 00	3-1	Jacob
1825 47	355 3	1 15	3	Struve	1846 47	24 1	0 97	9-8	Mitchell
1828 48	1 0	—	1	Herschel	1847 58	26 0	1 71	1	Mitchell
1830 25	1 4	1 46	4-3	Herschel	1848 54	30 6	1 19	3	Dawes
1831 38	9 4	1 32	2-1	Herschel	1848 54	27 2	0 84	1	Mitchell
1831 48	3 5	1 21	1	Struve	1853 53	46 3	—	1	Dawes
1832 52	4 8	1 24	1	Struve	1855 36	48 2	—	4	Dembowski
1833 37	5 0	1 19	1	Struve	1855 53	53 1	0 46	3	Secchi
1833 39	6 2	1 15	1	Dawes	1856 20	65 5	0 63	3	Jacob
1834 45	8 3	1 24	2-1	Herschel	1856 41	58 1	—	4	Dembowski
1834 45	6 7	1 24	1	Struve	1856 49	70 3	0 36	10-8	Secchi
1834 50	7 1	1 17	4	Dawes	1856 58	69 8	0 47	1	O Struve
1834 51	14 6	—	3	Madler	1856 55	59 6	—	2	Winnecke
1835 39	10 6	1 58	5-1	Herschel	1857 68	81 4	0 50	1	Jacob
1835 48	11 0	—	4	Madler	1858 13	79 4	0 40	1	Jacob
1836 49	9 5	1 02	1	Dawes	1858 22	116 8	0 30	1	Jacob
1836 50	11 0	—	3	Madler	1862 56	137 9	—	3	Dembowski
1837 33	11 4	—	1	Herschel	1863 44	142 1	—	9	Dembowski
1839 61	16 7	1 28	2	Dawes	1864 45	147 8	0 21	4	Secchi
1840 56	18 6	1 19	3	Dawes	1864 51	150 9	—	10	Dembowski
1840 57	17 2	0 96	1	O Struve	1865 44	151 4	—	10	Dembowski
1841 48	16 7	1 28	4-3	Madler	1865 51	155 5	0 35	7	Secchi
1841 57	20 8	0 84	1	O Struve	1865 55	166 9	0 49	7	Englemann
1841 58	19 0	1 20	3-2	Dawes	1866 46	156 6	0 53	8-3	Dembowski
1841 61	17 7	1 30	2-1	Kaiser	1866 52	161 0	0 40	2-1	Secchi
1842 42	20 4	1 05	4-2	Madler	1867 45	160 7	0 83	7-4	Dembowski
1842 46	21 6	—	2	Dawes	1868 40	165 0	0 90	7-4	Dembowski
1842 53	21 0	—	1	Kaiser	1868 48	166 5	0 99	1	Knott
1843 40	23 5	1 09	2	Dawes	1869 51	172 5	0 83	6	Dunér
1843 40	23 8	1 16	6-4	Madler	1869 52	168 2	0 88	5	Dembowski
1843 62	20 8	1 20	11-1	Kaiser	1870 21	168 2	—	1	Gledhill
1844 40	23 7	1 82	3	Madler	1870 39	169 8	0 89	7-5	Dembowski
					1870 54	173 3	0 88	2	Dunér

<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1871 41	173 1	1 06	7-5	Dembowski	1882 27	193 6	1 19	1	Dobereck
1871 49	174 0	1 00	1	Gledhill	1882 43	196 7	1 44	1	Englemann
1871 60	174 8	0 88	5	Dunér	1882 46	192 7	1 12	3	Hall
1872 45	177 3	0 95	1	W & S	1882 54	191 8	1 31	5	Schiaparelli
1872 46	176 9	1 12	1	Knott	1882 59	192 1	1 35	3	Frisby
1872 46	173 8	1 12	8-5	Dembowski	1883 45	191 0	1 33	4	Frisby
1872 50	175 8	1 10	2	Ferrari	1883 51	193 9	1 38	2	Hall
1872 53	177 4	0 96	3	Dunér	1883 49	195 3	—	1	Küstner
1873 36	180 4	1 04	1	W & S	1883 49	195 5	1 20	3	Englemann
1873 36	176 8	1 19	5-3	Dembowski	1883 52	191 5	1 16	3	Perrotin
1873 68	176 5	1 10	1	Gledhill	1883 55	193 5	1 24	12	Schiaparelli
1874 44	183 1	1 19	1	W & S	1884 38	195 8	1 34	4-3	H C Wilson
1874 49	178 7	1 05	5	Dembowski	1884 44	195 6	1 46	3	Englemann
1875 44	180 5	1 10	5	Dembowski	1884 50	194 6	1 28	5	Hall
1875 51	182 0	1 18	5	Schiaparelli	1884 53	195 1	1 27	3	Perrotin
1875 51	180 0	1 33	1	W & S	1884 54	195 6	1 41	1	H S Pr
1875 56	180 9	0 96	4	Dunér	1884 54	195 0	1 26	9	Schiaparelli
1876 44	185 6	1 04	1	Howe	1885 53	196 2	1 34	8	Schiaparelli
1876 45	181 8	1 21	6	Dembowski	1885 57	198 1	1 38	5	Englemann
1876 52	183 9	1 14	3	Hall	1886 35	197 4	1 19	1	H.C Wilson
1876 52	183 6	1 18	4	Schiaparelli	1886 46	197 5	1 24	2	Perrotin
1876 54	182 5	1 00 ±	1	Plummer	1886 49	198.6	1.54	2-1	Smith
1876 61	186 5	—	3	Dobereck	1886.51	198.1	1 29	3	Tarrant
1877 43	179 5	0 97	2-1	Dobereck	1886 56	198.0	1 07	3	Hall
1877 43	183 3	1 20	5	Dembowski	1886 63	198 9	1 07	7	Englemann
1877.43	184 1	1 61	1	Upton	1886 55	197 2	1 19	3	Schiaparelli
1877 46	184 9	1 27	1	W & S	1887 54	199 6	1 16	9	Schiaparelli
1877 47	187 0	1 12	4-1	Howe	1888 50	200 4	1 24	2	Lv
1877 55	184 0	1 25	9	Schiaparelli	1888 56	200 6	0 96	2	Hall
1877 55	182 5	1 27	3	Jedrzejewicz	1888 57	201 9	1 14	7	Schiaparelli
1878 46	186 2	1 22	5-4	Dembowski	1889 43	197 5	1 20	2	Hodges
1878 54	186 1	1 31	6	Schiaparelli	1890 39	205 2	—	2	Glasenapp
1879 41	189.2	1 22	5	Howe	1891 46	200 6	1 27	2	Collins
1879 42	186 7	1 44	3	Stone	1891 48	208 7	2 87	1	See
1879 47	187 6	1 45	3	Egbert	1892 53	208 0	1 23	3	Maw
1879 54	189 8	1 07	3	Hall	1892 58	206 5	0 82	4	Comstock
1879 56	186 8	1 29	7	Schiaparelli	1893 46	211 1	1 01	2	Burnham
1879 58	185 6	1 47	2	C W Pr	1893 49	209 5	1 10	1	Schiaparelli
1879 60	188 8	1 16	3	Burnham	1893 51	210 9	0 89	2	Lv
1879 67	194 5	0 70	3-1	Sea & Smith	1893 60	209 7	1 07	5	Bigourdan
1880 36	188 8	1 12	2	Egbert	1894.59	207 5	1 0 ±	2-1	Glasenapp
1880 40	189 7	1 17	4-2	Dobereck	1895 31	210 3	1 04	3	See
1880 52	185 7	1 13	1	Frisby	1895 41	213 9	0 91	2	Schiaparelli
1880 54	189 0	1 24	6	Schiaparelli	1895 53	213 4	0 81	3	Comstock
1880 87	189 6	1 10	3	H S Pr.					
1881 24	191 3	1 03	1	Dobereck					
1881 40	190 8	1 21	2-1	Bigourdan					



This bright star has been observed with considerable regularity since the time of STRUVE, and much material is now available for the investigation of its orbit. But while the measures are numerous, the considerable southern declination of the object renders them rather difficult, especially for European observers, and hence there is reason to suppose that the results are not free from systematic errors. In the investigation of the orbit we have adopted the usual method, depending on both angles and distances, and, as in case of  $\zeta$  *Cancer*, have neglected the influence of the third star. This procedure has been adopted by DR. SCHORR in his *Dissertation* on the motion of this system, and is fully justified by the rough and somewhat unsatisfactory state of the measures, which will not yet permit any very fine determination of the elements. Several computers have previously worked on the motion of this system; the following list of orbits is believed to be fairly complete.

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
<sup>YRS</sup> 105 522	1832 611	—	1 287	4 75	70 22	—	Madler, 1846	
49 048	1860 59	—	1 749	112 7	70 02	78 57	Thiele, 1859	A N, 1199
95 90	1859 62	0 0768	1 26	12 25	68 7	89 27	Doberck, 1877	A N, 2121
105 195	1862 32	0 122	1 3093	10 45	67 64	102 63	Schorr 1889	Dissertation, Munich

We find the following elements

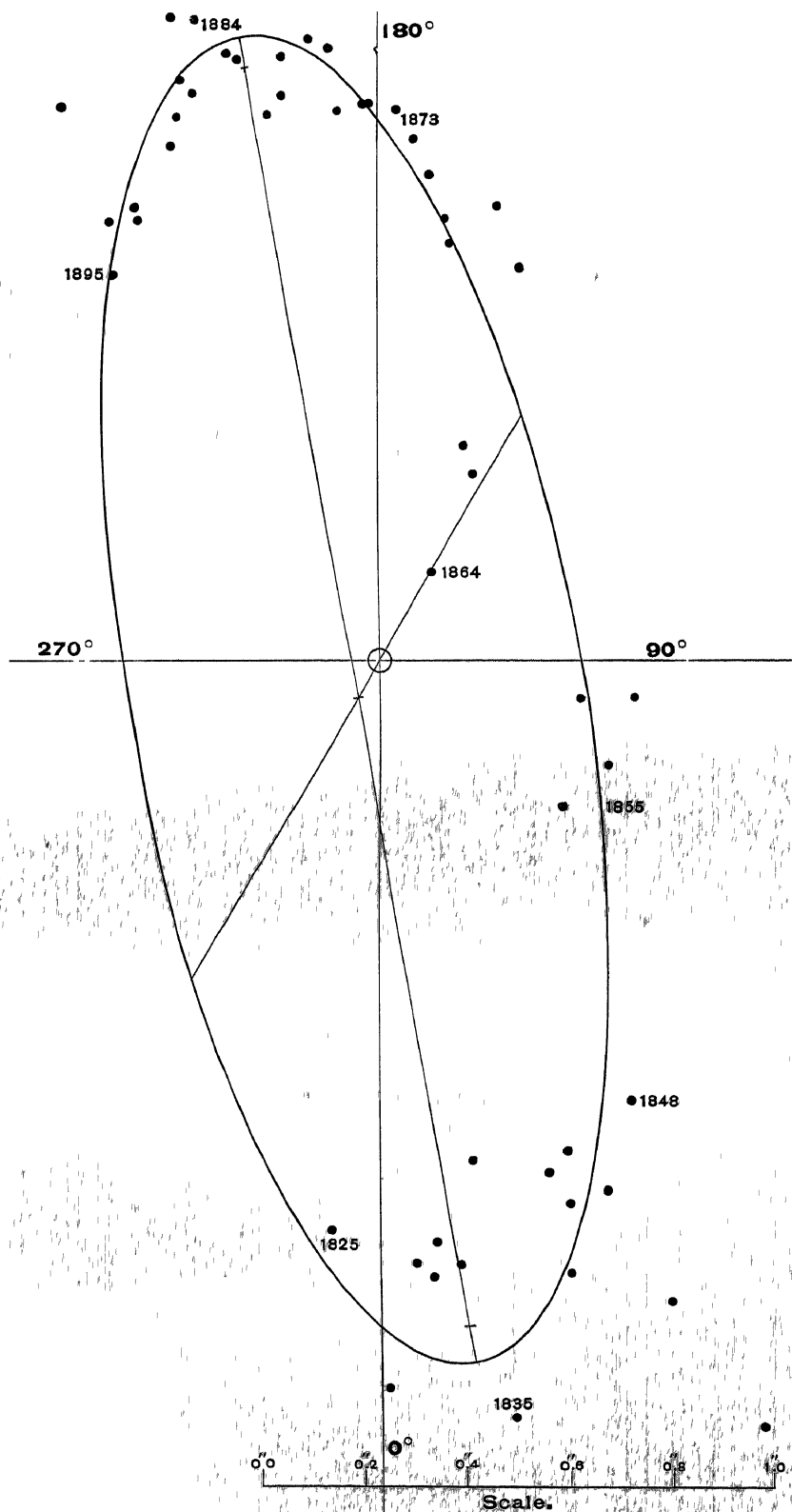
$$\begin{aligned}
 P &= 1040 \text{ years} & \Omega &= 9^\circ 5' \\
 T &= 1864 60 & i &= 70^\circ 3' \\
 e &= 0 131 & \lambda &= 111^\circ 6' \\
 a &= 1'' 3612 & n &= +3^\circ 4616
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 2'' 696 \\
 \text{Length of minor axis} &= 0'' 884 \\
 \text{Angle of major axis} &= 9^\circ 6' \\
 \text{Angle of periastron} &= 150^\circ 2' \\
 \text{Distance of star from centre} &= 0'' 085
 \end{aligned}$$

The table of computed and observed places shows a very satisfactory agreement, and we may conclude that no very considerable alteration is likely to be made in these elements. But the orbit is so nearly circular and so highly inclined that the definition of  $\lambda$  is not very exact, and in case of this element a larger alteration may be found necessary, when the material shall be sufficient for a definitive determination.

The small eccentricity of this orbit is rather remarkable. Among known binaries there are very few which have such circular orbits,  $\delta$  *Equulei*,  $\Sigma$  2173 and  $\mu$  *Herculis* being the principal objects of this kind, and as most of these orbits are highly inclined, there is still some uncertainty attaching to the eccentricity. It will be necessary to have more exact observations of these stars in



$\xi$  Scorpii AB =  $\Sigma$  1998.



critical parts of their orbits before this element can be defined with the desired precision

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1782 36	188 0	197 2	—	—	—9 2	—	1	Herschel
1825 47	355 3	357 2	1 15	1 28	-1 9	-0 13	3	Struve
1830 25	1 4	2 7	1 46	1 39	-1 3	+0 07	4-3	Herschel
1831 48	3 5	4 0	1 21	1 40	-0 5	-0 19	1	Struve
1832 52	4 8	5 2	1 24	1 41	-0 4	-0 17	1	Struve
1833 38	5 6	6 1	1 17	1 41	-0 5	-0 24	2	Struve 1, Dawes 1
1834 47	7 4	7 3	1 22	1 42	+0 1	-0 20	7-6	Herschel 2-1, Σ 1, Dawes 4
1835 39	10 6	8 3	1 58	1 42	+2 3	+0 16	5-1	Herschel
1836 50	10 3	9 7	1 02	1 40	+0 6	-0 38	4-1	Dawes 1, Mädler 3-0
1839 61	16 7	13 0	1 28	1 35	+3 7	-0 07	2	Dawes
1840 57	17 9	14 0	1 08	1 33	+3 9	-0 25	4	Dawes 3, OΣ 1
1841 56	18 5	15 7	1 15	1 29	+2 8	-0 14	10-7	Mädler 4-3, OΣ 1, Dawes 3-2, Kaiser 2-1
1842 42	20 4	16 1	1 05	1 28	+4 3	-0 23	4-2	Mädler 4-2
1843 47	22 7	18 0	1 15	1 22	+4 7	-0 07	19-7	Dawes 2, Mädler 6-4, Kaiser 11-1
1845 36	23 7	20 7	1 40	1 14	+3 0	+0 26	15-12	Mädler 3, Jacob 3-1, Mitchell 9-8
1847 58	26 0	24 7	1 71	1 02	+1 3	+0 69	1	Mitchell
1848 54	28 9	26 6	1 01	0 96	+2 3	+0 05	4	Dawes 3, Mitchell 1
1855 44	50 6	53 5	0 46	0 54	-2 9	-0 08	7-3	Dembowski 4-0, Secchi 3
1856 45	64 7	61 2	0 49	0 50	+3 5	-0 01	20-12	Jacob 3, Dem 4-0; Sec 10-8, OΣ 1, Winn 2-0
1857 68	81 4	72 8	0 50	0 43	+8 6	+0 07	1	Jacob
1858 13	79 4	78 5	0 40	0 42	+0 9	-0 02	1	Jacob
1864 48	149 3	144 8	0 21	0 52	+4 5	-0 31	14-4	Secchi 4, Dembowski 10-0
1865 50	153 4	154 4	0 42	0 60	-0 1	-0 18	17-14	Dembowski 10-0, Secchi 7; Englemann 7
1866 49	158 8	159 3	0 46	0 66	-0 5	-0 20	10-4	Dembowski 8-3; Secchi 2-1
1867 45	160 7	163 4	0 83	0 72	-2 7	+0 11	7-4	Dembowski
1868 44	165 7	166 9	0 94	0 78	-1 2	+0 16	8-5	Dembowski 7-4, Knott 1
1869 51	170 3	170 2	0 85	0 83	+0 1	+0 02	11	Dunér 6, Dembowski 5
1870 46	171 6	172 6	0 89	0 90	-1 0	-0 01	9-7	Dembowski 7-5, Dunér 2
1871 50	174 0	174 9	0 98	0 96	-0 9	+0 02	13-11	Dembowski 7-5, Gledhill 1, Dunér 5
1872 48	176 2	177 1	1 05	0 02	-0 9	+0 03	15-12	W & S 1, Kn 1, Dem 8-5, Fer 2; Du 3
1873 47	177 9	179 0	1 11	1 06	-1 1	+0 05	7-5	W & S 1, Dembowski 5-3, Gledhill 1
1874 46	180 9	180 8	1 12	1 11	+0 1	+0 01	6	W & S 1, Dembowski 5
1875 50	181 1	182 4	1 12	1 15	-1 3	-0 03	15	Dembowski 5, Schiaparelli 5, W & S 1, Dunér 4
1876 51	184 0	184 0	1 11	1 19	±0 0	-0 08	18-15	Howe 1, Dem 6, Hl 3, Sch 4, Pl 1, Dk 3 [Jed 3
1877 47	184 3	185 4	1 24	1 22	-1 1	+0 02	23-21	Dk 2-1, Dem 5, Upton 1, W & S 1, Howe 4-1, Sch 9,
1878 50	186 1	186 8	1 26	1 24	-0 7	+0 02	11-10	Dem 5-4, Sch 6 [β 3, Sea & S 3-1
1879 53	188 6	188 2	1 23	1 26	+0 4	-0 03	29-27	Howe 5, Stone 3, Egbert 3, Hl 3, Sch 7, Pr. 2,
1880 54	189 3	189 6	1 15	1 27	-0 3	-0 12	15-14	Egbert 2, Dk 4-2, Frisby 1, Sch 6, Pr 3
1881 32	191 0	190 3	1 12	1 28	+0 7	-0 16	3-2	Doberck 1, Bigourdan 2-1
1882 46	192 5	192 1	1 24	1 28	+0 4	-0 04	12	Doberck 1, Hall 3, Schiaparelli 5, Frisby 3
1883 50	193 4	193 4	1 26	1 29	±0 0	-0 03	25-24	Frisby 4, Hl 2, Kü 1-0; En 3, Per 3, Sch 12
1884 49	195 3	194 7	1 35	1 28	+0 6	+0 07	25-24	H C W 4-3, En 3, Hl 5; Per 3, Pr 1, Sch 9
1885 55	197 1	196 0	1 36	1 27	+1 1	+0 09	13	Schiaparelli 8, Englemann 5 [Sch 3
1886 51	198 0	197 4	1 23	1 25	+0 6	-0 02	21-20	H C W 1, Per 2, Sm 2-1, Tar 3, Hall 3, En 7,
1887 54	199 6	198 8	1 16	1 23	+0 8	-0 07	9	Schiaparelli
1888 54	201 0	200 3	1 11	1 21	+0 7	-0 10	11	Leavenworth 2; Hall 2, Schiaparelli 7
1889 43	197 5	201 6	1 20	1 18	-4 1	+0 02	2	Hodges
1890 39	205 2	203 1	—	1 15	+2 1	—	2	Glasenapp
1891 47	208 7	204 9	1 27	1 11	+3 8	+0 16	1-2	Collins 0-2, See 1-0
1892 55	207 3	206 8	1 02	1 07	+0 5	-0 05	7	Maw 3, Comstock 4
1893 51	210 3	208 7	1 02	1 03	+1 6	-0 01	10	β 2, Schiaparelli 1; Leavenworth 2, Bigourdan 5
1894 59	207 5	210 9	1 0 ±	0 99	-3 4	+0 01	2-1	Glasenapp 2-1
1895 42	213 3	212 8	0 93	0 95	+0 5	-0 02	4-6	See 1-3, Comstock 3

The following ephemeris will be useful to observers

	$\theta_0$ °	$\rho_0$ "		$\theta_0$ °	$\rho_0$ "
1896 50	216 3	0 88	1899 50	225 6	0 74
1897 50	219 3	0 84	1900 50	229 6	0 70
1898 50	222 4	0 79			

The motion will be rather slow for a good many years, but as the object becomes closer, about 1910, it will deserve the most careful attention

### $\sigma$ CORONAE BOREALIS = $\Sigma 2032$ .

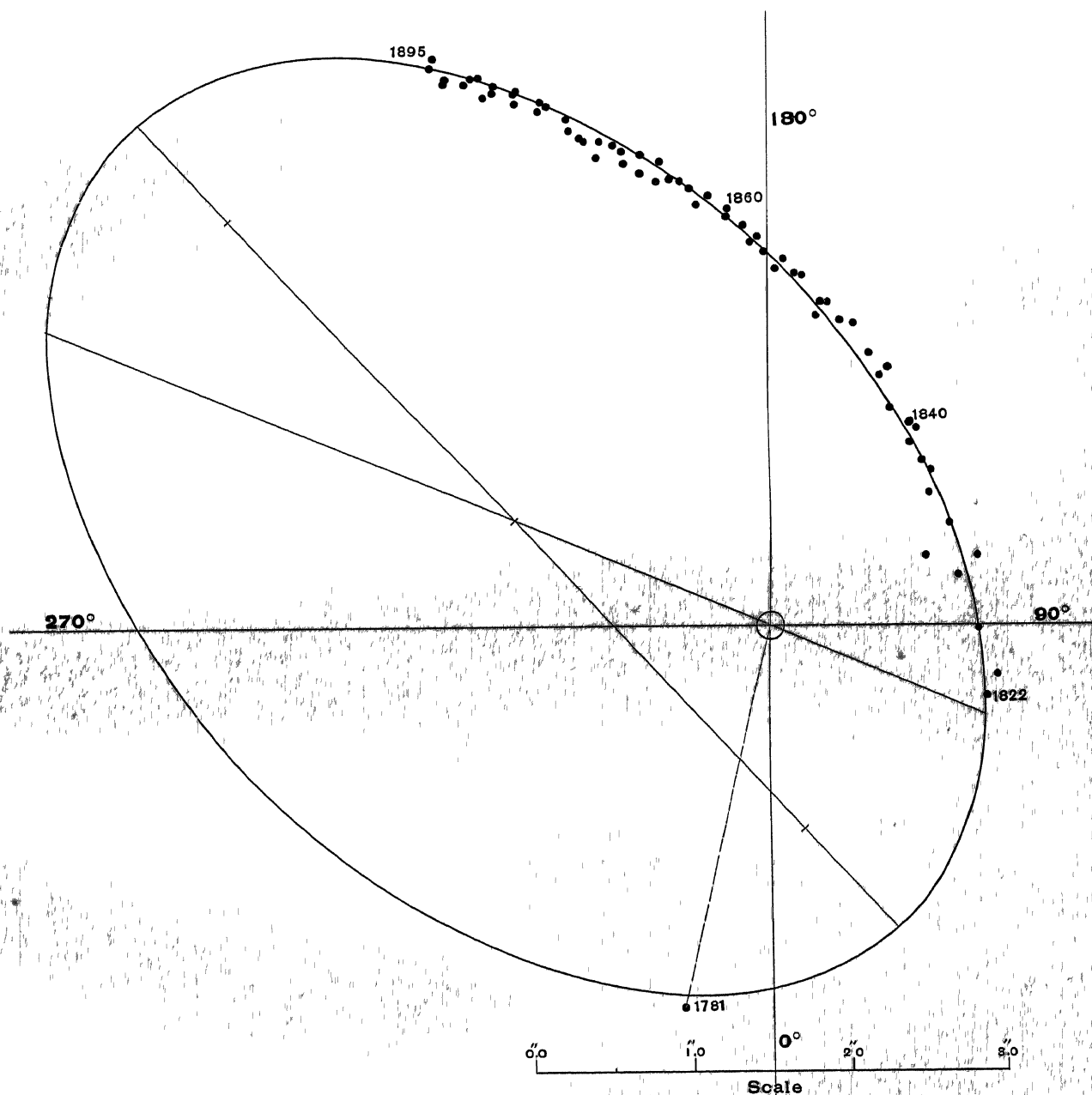
$\alpha = 16^h 11^m$  ,  $\delta = +34^\circ 7'$   
6, yellow , 7, bluish

*Discovered by Sir William Herschel, August 7, 1780*

OBSERVATIONS								
$t$	$\theta_0$ °	$\rho_0$ "	$n$	Observers	$t$	$\theta_0$ °	$\rho_0$ "	$n$ Observers
1781 79	347 5	—	1	Herschel	1836 47	138 5	—	5-0 Madler
1802 59	348 6?	—	1	Herschel	1836 59	134 7	1 43	6 Struve
1804 74	11 4	—	1	Herschel	1837 47	136 8	—	1 Dawes
1819 62	48 0	—	—	Struve	1837 55	139 9	1 42	5 Struve
1821 30	65 2	—	—	Herschel	1838 45	143 4	1 48	7 Struve
1822 83	71 5	1 44	2-1	H & So	1839 52	147 8	1 55	— Galle
1823 47	72 9	—	—	Herschel	1839 53	144 3	1 60	1 Dawes
1825 44	77 5	1 48	6-3	South	1840 57	147 8	1 66	3 Dawes
1827 02	89 3	1 31	4	Struve	1840 63	149 3	1 54	4 O Struve
1828 50	92 1	—	6	Herschel	1840 68	145 2	1 53	1 Struve
1830 11	104 9	1 22	3	Struve	1841 48	150 3	1 66	3 Dawes
1830 28	105 1	1 22	9-5	Herschel	1841 56	148 8	1 57	— Kaiser
1831 36	108 8	1 38	3-2	Herschel	1841 56	152 3	1 60	7 Madler
1832 52	113 6	1 07	6-1	Herschel	1841 60	153 7	1 56	1 O Struve
1832 55	115 4	—	3	Dawes	1842 31	156 4	1 81	4 Madler
1833 26	120 0	1 29	3-2	Herschel	1842 37	153 3	—	1 Dawes
1833 36	120 6	1 30	4	Dawes	1842 73	157 5	1 86	4 Madler
1834 55	125 6	—	3	Dawes	1843 45	156 8	1 85	6 Madler
1835 40	134 9	1 3±	4-1	Madler	1843 47	156 5	1 77	1 Dawes
1835 50	130 5	1 31	5	Struve	1843 68	156 3	1 66	— Kaiser
					1844 40	160 6	2 05	4 Madler
					1844 44	157 2	1 53	1 Greenwich
					1845 51	163 1	2 03	20-19 Madler

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1846 32	162 8	—	—	Hind	1856 39	182 8	2 52	1	Winnecke
1846 36	162 4	2 25	—	Jacob	1856 42	181 9	2 68	6	Dembowski
1846 46	165 1	2 07	11	Madler	1856 43	182 4	2 45	2	Secchi
1846 68	168 3	1 76	2	O Struve	1856 57	179 9	2 46	4	O Struve
1847 44	166 6	2 16	14	Madler	1856 73	181 2	2 52	3	Jacob
1847 44	166 0	1 88	2	Dawes	1857 39	183 3	2 46	2	Madler
1847 69	169 6	1 69	1	O Struve	1857 61	183 6	2 43	2	Secchi
1847 70	166 7	1 33	1	Mitchell	1857 66	183 1	2 53	3	Jacob
1848.41	168 4	2 39	2-1	Madler	1857 66	180 0	2 52	2	Dembowski
1848 42	171 9	2 2	1	W <sup>c</sup> Bond	1858 01	181 9	2 51	5	O Struve
1848 53	168 6	1 99	3	Dawes	1858 20	184 0	2 57	3	Jacob
1848 74	170 8	1 91	1	O Struve	1858 50	184 7	2 69	6	Dembowski
1849 45	170 1	2 09	1	Dawes	1858 54	183 6	2 64	7	Madler
1849 74	172 3	1 96	3	O Struve	1859 34	184 9	2 70	20 obs	Morton
1850 52	168 9	1 99	3	O Struve	1859 49	185 8	2 69	8-6	Madler
1850 70	173 0	2 23	2	Madler	1859 94	186 1	2 62	4	O Struve
1851 22	174 4	2 32	43 obs	Fletcher	1860 36	185 5	2 71	2	Dawes
1851 25	174 5	2 34	6	Madler	1861 55	188 4	2 95	5-3	Madler
1851 42	173 8	2 26	1	Dawes	1861 58	187 4	2 69	5	O Struve
1851 63	173 4	2 06	6	O Struve	1862 71	190 5	3 01	6	Madler
1851 76	176 2	2 43	9	Madler	1862 76	189 1	2 77	2	O Struve
1852 31	176 4	2 38	24-38 obs	Miller	1862 79	189.3	2 87	1	Scheumann
1852.60	177.5	2.39	12-11	Madler	1863 09	190 1	2 76	14	Dembowski
1852 63	173 3	2 06	4	O Struve	1863.60	188 2	2 77	4	O Struve
1853 14	177 9	2 18	2	Jacob	1864 45	190 5	3 09	2	Englemann
1853 38	177 7	2 46	6	Madler	1864 95	191 2	2 79	12	Dembowski
1853 63	177 9	2 39	4-3	Dawes	1865 36	191 9	2 94	3	O Struve
1853 64	—	2 56	1	Argelander	1865 38	191 5	3 08	1	Dawes
1853 64	—	2 47	1	W Struve	1865 64	194 3	2 93	4	Englemann
1853 66	175 6	2 17	4	O Struve	1865 72	189.1	—	1	Leyton Obs
1853 77	178 7	2 65	2	Madler	1865 74	192 5	30 3	1	v Fuss
1854 05	177 9	2 25	3	Jacob	1865 81	192 3	2 98	4	Secchi
1854 56	178 5	2 26	3	Dawes	1866 31	190 5	2 82	1	Englemann
1854 66	179 0	2 24	2	O Struve	1866 42	189 2	3 73	2	Leyton Obs
1854 67	178 6	2 22	20 obs	Morton	1866 49	189 6	—	2	Wagner
1854 67	179 8	2 36	5	Dembowski	1866 49	191 3	—	2	Gylden
1854 70	179 4	2 51	5	Madler	1866 49	192 3	—	2	Smysloff
1855 19	179 9	2 39	3	Dembowski	1866 49	193 2	—	2	Kortazzi
1855 48	180 1	2 43	1	Dawes	1866 55	194 6	3 23	3	Winlock
1855 54	181 6	2 49	6-5	Winnecke	1866 59	193.5	3.22	3-2	Searle
1855 61	180 8	2 31	4	Secchi	1866 63	193 0	3 00	6	O Struve
1855 61	179 1	2 29	4	O Struve	1866 68	193 9	2 86	—	Kaiser
1855 78	181 8	2 64	2-1	Madler	1866 92	193 2	2 88	11	Dembowski

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1867 30	190 2	3 15	1	Searle	1877 46	202 2	3 68	—	W & S
1867 31	195 0	2 95	1	Winlock	1877 49	200 1	3 49	7	Schiaparelli
1867 34	194 7	3 0	—	Knott	1877 53	201 6	3 61	5	Jedrzejewicz
1867 37	192 1	3 0	1	Main	1877 58	200 1	3 50	3	O Struve
1867 72	195 5	2 79	1	Dunér	1878 39	202 3	3 51	2-1	Burnham
1868 29	193 8	3 62	1	Leyton Obs	1878 50	202 0	3 51	5	Dembowski
1868 58	194 7	2 98	2	O Struve	1878 51	201 1	3 39	3-2	Doberck
1868 60	194 7	3 14	4	Dunér	1878 53	201 2	3 53	6	Schiaparelli
1868 61	195 5	—	2	Zollner	1878 57	199 1	3 52	3	O Struve
1868 88	195 3	2 99	9	Dembowski	1879 45	202 5	3 66	4	Hall
1869 57	195 2	3 60	1	Leyton Obs	1879 54	202 1	3 68	6	Schiaparelli
1869 63	195 1	3 05	5	Dunér	1880 39	203 0	3 61	1	Burnham
1870 56	196 6	3 18	1	Dunér	1880 55	203 4	3 71	9	Schiaparelli
1870 97	196 8	3 10	12	Dembowski	1881 05	200 6	3 94	5	Hough
1871 41	197 9	3 23	2-3	C S Peirce	1881 46	203 0	3 64	3	Hall
1871 42	196 7	3 30	—	Leyton Obs	1881 70	204 3	3 56	6	Seabroke
1871 54	195 4	3 23	—	Knott	1882 43	202 6	3 75	4	Hall
1871 61	196 5	3 14	3	Dunér	1882 51	203 8	3 79	6	Schiaparelli
1872 29	198 0	3 34	—	Leyton Obs	1882 52	204 1	3 90	3	O Struve
1872 57	195 3	3 26	3	O Struve	1882 65	204 9	—	1	Seabroke
1872 96	198 1	3 20	12	Dembowski	1882 71	205 7	3 92	4	Jedrzejewicz
1873 42	198 4	3 14	—	W & S	1883 26	205 4	3 77	6	Englemann
1873 55	200 6	3 64	1	Leyton Obs	1883 47	204 5	3 77	3	Hall
1873 56	197 6	3 14	2	O Struve	1883 49	203 2	3 79	4	Perrotin
1873 68	198 9	3 4	—	Gledhill	1883 56	204 6	3 74	12	Schiaparelli
1873 54	197 3	—	1	Muller	1883 63	206 0	3 99	2	Jedrzejewicz
1873 54	201 6	—	1	H Bruns	1884 48	206 0	3 80	3	Hall
1873 57	199 6	—	1	H Struve	1884 53	205 8	3 86	3	Perrotin
1874 44	200 5	3 55	1	Main	1884 53	202 4	3 63	2	O Struve
1874 46	199 2	2 67	2	Leyton Obs	1884 54	205 4	3 76	11	Schiaparelli
1874 61	199 8	3 41	4	O Struve	1885 43	205 4	3 88	4	deBall
1874 90	199 1	3 28	11	Dembowski	1885 43	205 7	3 89	3	Hall
1875 42	199 8	2 56	1	Leyton Obs	1885 54	204 9	3 94	2	Perrotin
1875 46	198 6	3 34	4	Schiaparelli	1885 55	205 8	3 86	9	Schiaparelli
1875 50	200 6	3 47	—	W & S	1885 66	206 8	3 93	3	Jedrzejewicz
1875 54	199 6	3 28	5	Dunér	1885 74	207 3	4 09	6	Englemann
1875 65	200 6	3 74	—	Nobile	1886 47	205 6	3 99	5	Perrotin
1876 29	199 3	—	—	Doberck	1886 48	206 9	3 96	6	Hall
1876 45	200 0	3 50	3	Hall	1886 49	208 0	4 01	4	Tarrant
1876 48	200 6	3 28	—	Gledhill	1887 44	205 5	3 99	4	Hall
1876 61	196 3	3 34	3	O Struve	1887 53	207 1	3 78	7	Schiaparelli
1876 61	200 7	3 45	1	Leyton Obs	1888 44	206 6	3 92	4	Hall
1877 03	201 0	3 40	11	Dembowski	1888 57	207 4	3 92	8-7	Schiaparelli
1877 33	199 6	3 58	—	Doberck	1888 62	207 8	3 82	3	Maw



\*  $\sigma$  Coronae Borealis =  $\Sigma$  2032.





$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1889 14	207 7	4 08	2	O Struve	1893 55	209 3	4 28	4	Bigourdan
1889 52	208 3	4 05	2	Glasenapp	1893 64	209 8	4 24	2	Maw
1889 52	208 8	3 94	1	Schiaparelli	1894 56	209 8	4 09	2	Glasenapp
1890 33	207 8	4 08	3	Burnham	1895 49	210 8	4 28	3	Comstock
1890 69	207 3	4 00	1	Bigourdan	1895 54	210 7	4 16	10	Schiaparelli
1891 49	208.5	—	1	Schiaparelli	1895 59	210 3	4 23	2	Collins
1892 61	209 9	4 06	3	Comstock	1895 59	209 9	4 25	4	Schwarzschild
1892 64	209 4	4 05	2	Schiaparelli	1895 72	208 9	4 26	3	See
1892 64	209 3	4 21	1	Bigourdan					

Since HERSCHEL'S discovery of this star the companion has described an arc\* of  $223^\circ$ . The shape of this arc is such that it fixes the apparent ellipse with considerable precision, and enables us to obtain a set of elements which will never be radically changed. It is singular, however, that the periods heretofore obtained for this star are very discordant, and in several instances more than double that found below. Such extraordinary divergence of results may be explained by the lack of sufficient curvature in the arc swept over by the companion at the time the earlier elements were derived, and by the use of injudicious methods in the determination of the orbit.

In this as in most other cases the graphical method based on both angles and distances is superior to analytical methods, and at once enables us to trace the apparent ellipse with the necessary precision. The following table gives a complete summary of the elements found by previous computers who have worked on the motion of this interesting binary.

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
286 60	1835 60	0 6112	3 68	138 0	41 25	7 3	Herschel, 1833	Mem R A S., V, p 205
608 45	1826 60	0 6998	3 92	25 12	29 48	64 63	Madler	Dorp Obs, IX, p 182
478 04	1829 44	0 6406	3 90	0 5	38 93	96 73	Madler, 1847	Fixt-Syst, I, p 240
736 88	1826 48	0 7256	5 194	21 05	25 65	69 4	Hind, 1845	A.N., 551
195 12	1831 17	0 3088	2 72	1 95	46 78	101 95	Jacob, 1855	M.N., XV, p 180
240 0	1829 7	0 3887	2 94	3 13	45 1	96 88	Powell, 1855	M.N., XV, p 91
420 24	1825 32	0 5899	2 385	20 73	40 87	65 9	Klinkerfues, 1856	A.N., 990
843 2	1828 91	0.7502	6 001	6 72	29 67	89 3	Doberck, 1875	A.N., 2037
845 86	1826 93	0 7515	5 885	18 35	31 37	71 6	Doberck, 1876	A.N., 2103

Making use of all the observations up to 1895 we find the following elements :

$$\begin{aligned}
 P &= 370.0 \text{ years} & \Omega &= 30^\circ 5' \\
 T &= 1821.80 & i &= 47^\circ 48' \\
 e &= 0.540 & \lambda &= 47^\circ 7' \\
 a &= 3'' 8187 & n &= +9^\circ 7297
 \end{aligned}$$

\* *Astronomische Nachrichten*, 3389

## Apparent orbit

Length of major axis	= 7" 08
Length of minor axis	= 4" 71
Angle of major axis	= 42° 4
Angle of periastron	= 66° 9
Distance of star from centre	= 1" 735

There is of course some uncertainty attaching to a period of such great length, but careful consideration of all possible variations of the apparent ellipse convinces me that the value given above is not likely to be varied by more than 25 years, and a change of twice this amount is apparently impossible. The eccentricity is very well determined, and a change of  $\pm 0.04$  in the above value is not to be expected.

The distance of the components of  $\sigma$  *Coronae Borealis* is now so great that the companion will move very slowly for the next two centuries. Therefore, so far as the orbit is concerned observations of the pair will be of small value, as very little improvement can be effected for a great many years, but it may still be worth while to secure careful measures of the system, with a view of establishing the regularity of the elliptical motion, and the absence of sensible disturbing influences. There are no irregularities in the measures heretofore secured which are not attributable to errors of observation. The table of computed and observed places shows an agreement which is extremely satisfactory.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1781.79	347.5	348.5	—	2.44	-1.0	—	1	Herschel
1804.74	11.4	23.9	—	2.08	-12.5	—	1	Herschel
1819.62	48.0	59.1	—	1.57	-11.1	—	—	Struve
1821.30	65.2	65.2	—	1.50	0.0	—	—	Herschel
1822.83	71.5	71.0	1.44	1.45	+0.5	-0.01	2-1	Herschel and South
1823.47	72.9	73.3	—	1.43	-0.4	—	—	Herschel
1825.44	77.5	81.6	1.48	1.36	-4.1	+0.12	6-3	South
1827.02	89.3	88.6	1.31	1.33	+0.7	-0.02	4	Struve
1828.50	92.1	95.3	—	1.31	-3.2	—	6	Herschel
1830.20	105.0	103.8	1.22	1.30	+1.2	-0.08	12-8	Struve 3, Herschel 9-5
1831.36	108.8	109.1	1.38	1.30	-0.3	+0.08	3-2	Herschel
1832.54	114.5	111.7	1.07	1.30	+2.8	-0.23	9-1	Herschel 6-1, Dawes 3-0
1833.31	120.3	118.7	1.30	1.31	+1.6	-0.01	7-6	Herschel 3-2, Dawes 4
1834.55	125.6	124.3	—	1.34	+1.3	—	3	Dawes
1835.50	130.5	128.5	1.31	1.36	+2.0	-0.05	5	Struve
1836.59	134.7	133.5	1.43	1.40	+1.2	+0.03	6	Struve
1837.51	138.3	137.0	1.42	1.43	+1.3	-0.01	6-5	Dawes 1-0, Struve 5
1838.45	143.4	140.7	1.48	1.47	+2.7	+0.01	7	Struve
1839.52	146.0	144.5	1.57	1.51	+1.5	+0.06	2+	Galle —, Dawes 1
1840.63	147.4	148.3	1.58	1.56	-0.9	+0.02	8	Dawes 3, $\Sigma$ 4, Struve 1
1841.55	151.3	151.5	1.60	1.61	-0.2	-0.01	12+	Dawes 3, Kaiser —, Madler 7, $\Sigma$ 1
1842.47	155.7	154.1	1.83	1.66	+1.6	+0.17	9-8	Madler 4, Dawes 1-0, Madler 4

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1843 53	156 5	157 2	1 76	1 72	- 0 7	+0 04	8+	Dawes 1, Madler 6, Kaisei —
1844 45	160 3	159 9	1 87	1 78	+ 0 4	+0 09	6+	Madler 4, Greenwich 1, Madler —
1846 45	164 6	164 7	2 02	1 90	- 0 1	+0 12	15+	Hind —, Jacob —, Madler 11, $O\Sigma$ 2
1847 57	167 2	167 3	2 02	1 96	- 0 1	+0 06	18-16	Madler 14, Dawes 2, $O\Sigma$ 1, Mitchell 1
1848 52	169 9	169 0	2 12	2 01	+ 0 9	+0 11	7-6	Madler 2-1, Bond 1, Dawes 3, $O\Sigma$ 1
1849 60	171 2	170 5	2 03	2 05	+ 0 7	-0 02	4	Dawes 1, $O\Sigma$ 3
1850 61	171 0	173 0	2 11	2 14	- 2 0	-0 03	5	$O\Sigma$ 3, Madler 2
1851.46	174.5	174 5	2 28	2 19	0 0	+0 09	24+	Flt 43 obs, Ma 6, Da 1, $O\Sigma$ 6, Ma 9
1852 51	175 7	176 2	2 28	2 26	- 0 5	+0 02	18+	Miller 24-38 obs, Madler 11, $O\Sigma$ 4
1853.52	177.6	177 7	2 37	2 31	- 0 1	+0 06	18-17	Jacob 2, Ma 6, Dawes 4-3, $O\Sigma$ 4, Ma 2
1854 55	178 9	179 2	2 31	2 37	- 0 3	-0 06	20+	Ja 3, Da 3, $O\Sigma$ 2, Mo 20 obs, Ma 5, Dem 5
1855 53	180 5	180 7	2 42	2 43	- 0 2	-0 01	20-18	Dem 3, Da 1, Winn 6-5, Sec 4, $O\Sigma$ 4, Ma 2-1
1856 50	181 6	182 0	2 52	2 49	- 0 4	+0 03	16	Winn 1, Dem 6, Sec 2, $O\Sigma$ 4, Ja 3
1857 58	182 5	183 3	2 49	2 55	- 0 8	-0 06	9	Madler 2, Secchi 2, Jacob 3, Dembowski 2
1858 31	183 5	184 2	2 60	2 60	- 0 7	0 00	21	$O\Sigma$ 5, Jacob 3, Dembowski 6, Madler 7
1859 59	185 6	185 9	2 67	2 68	- 0 3	-0 01	14-12+	Mo 20 obs, Madler 8-6, $O\Sigma$ 4
1860 36	185 5	186 6	2 71	2 71	- 1 1	0 00	2	Dawes
1861 57	187 7	187 7	2 82	2 77	0 0	+0 05	10-8	Madler 5-3; $O\Sigma$ 5
1862 73	189 8	189 0	2 89	2 84	+ 0 8	+0 05	8	Madler 6, $O\Sigma$ 2
1863 34	189 2	189 7	2 77	2 89	- 0 5	-0 12	18	Dembowski 14 $O\Sigma$ 4
1864 70	190 8	190 9	2 94	2 95	- 0 1	-0 01	14	Englemann 2, Dembowski 12
1865 72	191 8	191 9	2 98	3 01	- 0 1	-0 03	13	$O\Sigma$ 3, Da 1, En 4, Ley 1, Sec 4, Dem 4
1866 59	192 6	192 7	3 10	3 05	- 0 1	+0 05	26-25	En 1, Ley 2, Wk 3, Si 3-2, $O\Sigma$ 6, Ka —
1867 41	193 5	193 4	2 98	3 09	+ 0 1	-0 11	5+	Si 1, Wk 1, Kn —, Ma 1, Dunér 1
1868 59	194 6	194 3	3 18	3 14	+ 0 3	+0 04	16	Ley 1, $O\Sigma$ 2, Dunér 4, Dembowski 9
1869 60	195 2	195 1	3 05	3 19	+ 0 1	-0 14	6-5	Ley 1-0, Dunér 5
1870 77	196 7	196 1	3 14	3 26	+ 0 6	-0 12	13	Dunér 1, Dembowski 12
1871 49	196 6	196 8	3 22	3 30	- 0 2	-0 08	7+	Pierce 2-3, Ley —, Knott —, Dunér 3
1872 61	197 1	197 4	3 27	3 34	- 0 3	-0 07	16+	Ley —, $O\Sigma$ 3 Dembowski 12
1873.55	198.3	198 0	3 33	3 38	+ 0 3	-0 05	5+	Ley 1, $O\Sigma$ 2, Gledhill —
1874.60	199.4	198 9	3 23	3 44	+ 0 5	-0 21	17-18	Main 0-1; Ley 2, $O\Sigma$ 4, Dembowski 11
1875.51	200 0	199.4	3 36	3 47	+ 0 6	-0 11	12-10+	Ley 1-0, Sch 4, W & S —, Du 5, Nobile —
1876 47	200 2	200 1	3 39	3 52	+ 0 1	-0 13	9+	Dk —, Hall 3, Gl —, $O\Sigma$ 3, Ley 1
1877 40	200 8	200 6	3 54	3 55	+ 0 2	-0 01	28+	Dem 11, Dob —, W & S —, Sch 7, Jed 5, $O\Sigma$ 3
1878 50	201 1	201 3	3 47	3 60	- 0 2	-0 13	19-17	$\beta$ 2-1, Dem 5, Dk 3-2, Sch 6, $O\Sigma$ 3
1879 49	202 3	201 9	3 67	3 64	+ 0 4	+0 03	10	Hall 4, Schiaparelli 6
1880 47	203 2	202 6	3 66	3 69	+ 0 6	- 0 03	10	$\beta$ 1, Schiaparelli 9
1881 40	202 6	203 1	3 71	3 73	- 0 5	-0 02	14	Hough 5, Hall 3, Seabroke 6
1882 56	204 2	203 7	3 84	3 77	+ 0 5	+0 07	18-17	Hall 4, Sch 6, $O\Sigma$ 3, Sea 1-0 Jed 4
1883 48	204 7	204 3	3 83	3 80	+ 0 4	+0 03	27	En 6; Hall 3, Per 4, Sch 12, Jed 2
1884 52	204 9	204 8	3 76	3 84	+ 0 1	-0 08	19	Hall 3, Perrotin 3, $O\Sigma$ 2, Schiaparelli 11
1885 56	206 0	205 4	3 93	3 89	+ 0 6	+0 04	27	de Ball 4, Hl 3, Per 2, Sch 9, Jed 3, En 6
1886 48	206 8	205 9	4 02	3 92	+ 0 9	+0 10	15	Perrotin 5, Hall 6, Tarrant 4
1887 48	206 3	206 5	3 89	3 97	- 0 2	-0 08	11	Hall 4, Schiaparelli 7
1888 54	207 3	206 9	3 89	4 00	+ 0 4	-0 19	15-14	Hall 4, Schiaparelli 8-7, Maw 3
1889 39	208 3	207 4	4 02	4 03	+ 0 9	-0 01	5	$O\Sigma$ 2, Glasenapp 2, Schiaparelli 1
1890 51	207 5	208 0	4 04	4 07	- 0 5	-0 03	4	$\beta$ 3, Bigourdan 1
1891 49	208 5	208 5	—	4 11	0 0	—	1	Schiaparelli
1892 63	209 5	208 9	4 11	4 14	+ 0 6	-0 03	6	Comstock 3, Schiaparelli 2, Bigourdan 1
1893 60	209 6	209 4	4 26	4 17	+ 0 2	+0 09	6	Bigourdan 4, Maw 2
1894 56	209 8	209 8	4 09	4 19	0 0	-0 10	2	Glasenapp
1895 58	210 2	210 3	4 23	4 23	- 0 1	0 00	18	Comstock 3, Schiaparelli 10, Collins 2, See 3

## ζ HERCULIS    Σ2081.

$\alpha$      $10^{\text{h}} 37^{\text{m}} 6^{\text{s}}$  ;     $\delta$      $+31^{\circ} 17'$   
*a*, yellow ;    *b*, bluish.

*Discovered by Sir William Herschel, July 18, 1782*

## OBSERVATIONS.

<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1783.55	69.3			Herschel	1847.45	104.1	1.23	18-17	Madler
1826.63	23.4	0.91	5	Struve	1847.53	108.0	1.63	1	Dawes
1828.77	simplex		1	Struve	1847.68	111.3	1.42	2	O. Struve
1829.67	simplex		2	Struve	1848.40	98.8	1.08	3	Madler
1831.65	simplex		1	Struve	1848.61	102.4	1.51	3	Dawes
1832.75	220.5	0.81	1	Struve	1848.76	104.2	1.53	2	O. Struve
1834.45	203.5	0.91	2	Struve	1849.48	99.2	1.71	1	Dawes
1835.45	196.9	1.09	5	Struve	1850.00	96.9	1.50	3	O. Struve
1836.57	188.0		3	Madler	1850.54	91.7	1.44	2	Fletcher
1836.60	186.2	1.09	5	Struve	1850.55	91.3	1.27	3-1	Madler
1838.70	168.5	1.35	3-1	Galle	1851.23	84.9	1.29	3	Madler
1839.67	159.7	1.15	1	W. Struve	1851.51	89.3	1.34	6	Fletcher
1839.76	161.9	1.22	4	Dawes	1851.62	88.4	1.47	5	O. Struve
1840.58	161.7	1.49	1	W. Struve	1851.65	89.1		2	Miller
1840.66	157.1	1.25	5	O. Struve	1852.63	84.2	1.52	5	O. Struve
1840.66	161.9	1.22	4	Dawes	1852.63	82.8	1.21	8-7	Madler
1841.44	149.3	1.12	9-8	Madler	1852.64	84.0	1.24	5-2	Fletcher
1841.60	147.0	1.23	3	O. Struve	1852.77	84.1		2	Miller
1841.65	143.0	1.24	4-3	Dawes	1853.15	81.2	1.58	2	Jacob
1842.40	141.6	0.92	3	Madler	1853.33	78.6	1.40	6-3	Miller
1842.58	138.5	1.07	3-1	Dawes	1853.39	77.3	1.23	8	Madler
1842.64	146.0	1.21	3	O. Struve	1853.59	80.0	1.48	4	O. Struve
1843.60	130.5	0.90	8-7	Madler	1853.83	74.7	1.19	3	Madler
1843.64	129.9	1.30	3-2	Dawes	1854.06	78.0	1.52	3	Jacob
1843.71	130.0	0.94	9-8	Madler	1854.66	76.8	1.56	3	O. Struve
1844.29	124.0	1.05	5-4	Madler	1854.67	72.3	1.33	5	Madler
1844.71	125.4	1.12	2	O. Struve	1855.05	69.6	1.4	13	Dembowski
1845.43	119.4	1.01	11	Madler	1855.41	68.0	1.56	4-2	Winnecke
1845.64	121.3	1.24	3	O. Struve	1855.53	69.7	1.41	3	Secchi
1846.54	111.5	1.18	16	Madler	1855.62	70.8	1.55	4	O. Struve
1846.69	110.5	1.31	2	O. Struve	1855.66	73.3	1.45	4	Morton
1846.89	112.2	—	5	Dawes	1856.25	66.2	1.60	3	Jacob
					1856.43	62.6	1.43	6-3	Winnecke
					1856.52	64.1	1.2	15	Dembowski
					1856.52	64.1	1.41	6	Secchi
					1856.62	64.7	1.49	3	O. Struve

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1857 38	60 0	1 07	4	Mädler	1870 49	190 3	1 10	11	Dembowski
1857 46	60 4	1 60	2	Morton	1870 59	193 6	1 21	6	Dunér
1857 59	59 5	1 29	6	Secchi	1871 42	181 1	1 27	14	Dembowski
1857 63	58 4	1 49	4	O Struve	1871 52	179 6	1 34	1	O Struve
1857 75	58 9	1 2	5	Dembowski	1871 54	183 3	1 02	5	Knott
1857 87	57 0	1 46	3	Jacob	1871 60	183 7	1 19	12	Dunér
1858 48	54 6	1 06	2	Secchi	1872 48	173 9	1 34	12	Dembowski
1858 56	49 9	1 0	8	Dembowski	1872 58	177 2	1 19	12	Dunér
1858 62	51 0	1 48	4	O Struve	1872 60	168 8	1 14	3	O Struve
1858 65	48 6	1 20	8-7	Mädler	1873 50	165 9	—	1	Mädler
1859 49	40 3	1 13	6	Mädler	1873 50	164 7	—	1	Romberg
1859 59	43 8	0 86	3	Secchi	1873 52	169 5	—	2	H Bruhns
1859 61	45 8	1 34	6-5	Dawes	1873 46	166 7	0 98	3-2	W & S
1859 63	42 3	1 29	4	O Struve	1873 52	162 4	1 39	11	Dembowski
1860 67	31 5	0 72	3	Secchi	1873 52	169 9	1 23	2	O Struve
1860 74	32 5	1 38	1	O Struve	1873 54	rotunda	—	1	Ferrari
1861 44	20 0	0 8±	2	Mädler	1873 70	166 3	1 40	4	Dunér
1861 57	17 1	1 05	4	O Struve	1874 53	157 0	1 36	10	Dembowski
1862 54	361 8	cuneo	8	Dembowski	1874 57	155 5	0 78	2	Gledhill
1862 55	unsichtbar	—	1	Winnecke	1874 57	156 4	1 08	2	W & S
1862 74	341	1 00	1	O Struve	1874 62	162 9	1 40	4	O Struve
1862 91	50.9	0 82	2	Mädler	1874 65	154 9	1 35	1	Dunér
1863 49	343.0	cuneo	4	Dembowski	1874 66	156 5	1 22	2	Newcomb
1864 43	—	semplice	3	Dembowski	1875 52	149 1	1 41	8	Dembowski
1865 32	—	semplice	2	Dembowski	1875 55	147 2	1 21	7	Schiaparelli
1865 54	rotunda	—	3	Secchi	1875 57	150 3	—	2	W & S
1865 55	250 0	<0 5	3	Englemann	1875 61	147 4	1 28	12	Dunér
1866 45	244 7	0 6	5	Dembowski	1876 52	143 1	1 32	2	Hall
1866 60	142 3	—	1	Searle	1876 54	138 1	1 17	7	Schiaparelli
1866 70	235 1	0 86	3	Dawes	1876 56	139 6	1 37	7	Dembowski
1866 74	228 6	0 97	2	O Struve	1876 61	140 1	1 2±	1	Plummer
1866 81	229 2	0 83	2	Dawes	1876 62	148 8	1 24	4	O Struve
1866 99	225.1	0 98	2	Dawes	1877 53	133 8	1 36	8	Dembowski
1867 52	225 6	0 80	7	Dembowski	1877 58	130 3	1 27	8	Schiaparelli
1867 59	227 6	—	1	Winlock	1877 58	141 2	1 60	1	Pritchett
1867 72	221 4	1 03	2	Dunér	1877 58	135 1	1 16	3	O Struve
1868 44	210 1	0 94	6	Dembowski	1877 59	134 0	1 24	2	Hall
1868 48	206 1	0 99	4	Knott	1878 41	127 0	1 51	1	Burnham
1868 58	203 6	1 23	2	O Struve	1878 53	124 0	1 43	4	Schiaparelli
1868 61	199 9	—	1	Zollner	1878 53	127 0	1 29	2-1	Doberck
1868 67	213 3	1 05	5	Dunér	1878 58	126 7	1 38	7	Dembowski
1869 58	200 6	1 09	8	Dembowski	1878 59	128 7	1 23	3	O Struve
1869 62	203 1	1 06	11	Dunér	1879 45	122 0	1 52	3	Burnham
1869 74	196 1	—	1	Peirce	1879 46	120 7	1 50	4	Hall
					1879 58	117 2	1 38	8	Schiaparelli
					1879 67	124 9	1 56	1	Pritchett

<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1880 41	118 4	1 29	2-1	Doberck	1886 63	85 8	1 45	9	Schiaparelli
1880 48	115 0	—	3	Bigourdan	1886 73	88 0	1 42	9-7	Jedrzejewicz
1880 49	114 1	1 34	5	Burnham	1886 75	89 9	1 78	7	Englemann
1880 58	112 5	1 38	9	Schiaparelli	1887 55	83 6	1 59	6	Hall
1881 23	112 9	1 43	2	Doberck	1887 65	79 4	1 55	18	Schiaparelli
1881 45	110 6	1 53	5	Burnham	1888 51	78 7	1 52	6	Hall
1881 51	109 2	1 41	4	Hough	1888 57	74 3	1 88	3	Comstock
1881 51	110 6	1 43	5	Hall	1888 61	76 5	1 56	9-8	Schiaparelli
1881 64	101 8	1 41	2	O Struve	1888 65	74 9	1 71	3	Maw
1881 74	108 9	1 47	1	Bigourdan	1888 69	70 9	1 74	1	O Struve
1882 47	105 0	1 48	2-1	H Struve	1889 45	77 0	1 00	1	Hodges
1882 47	104 3	1 67	2-1	Doberck	1889 52	72 6	1 67	3	Schiaparelli
1882 52	106 3	1 44	5	Hall	1889 52	76 2	1 2±	2-1	Glasenapp
1882 52	98 7	1 49	4	O Struve	1889 53	72 4	1 49	6	Hall
1882 60	101 5	1 47	11	Schiaparelli	1889 56	72 0	1 67	4	Comstock
1882 66	104 9	1 48	4-3	Jedrzejewicz	1889 66	70 2	1 73	3	Maw
1882 76	107 0	1 75	6	Englemann	1890 42	68 6	1 5	2	Glasenapp
1883 52	99 5	1 50	4	Perrotin	1890 51	68 5	1 49	6	Hall
1883 55	102 4	1 51	5	Hall	1890 70	65 8	1 68	3	Maw
1883 60	96 6	1 52	15	Schiaparelli	1890 77	64 2	1 46	5-4	Schiaparelli
1883 65	96 4	1 38	2	O Struve	1891 52	64 3	1 35	6	Hall
1883 72	102 5	1 65	5	Englemann	1891 54	60 4	1 45	7-4	See
1884 45	94 9	—	2	Bigourdan	1891 55	63 3	1 50	2	Schiaparelli
1884 52	94 7	1 63	4	Hall	1891 62	62 7	1 45	5	Bigourdan
1884 55	94 1	1 47	3	Perrotin	1891 63	60 1	1 40	3	Maw
1884 55	90 9	1 32	1	Pritchett	1891 64	63 7	1 38	4	Tarrant
1884 58	90 8	1 64	9	Schiaparelli	1892 57	55 5	1 51	5	Comstock
1884 68	88 4	1 57	2	O Struve	1892 63	56 0	1 37	8	Schiaparelli
1884 70	94 8	1 95	6-2	Seabroke	1893 68	47 6	1 42	3-2	Schiaparelli
1884 71	98 8	1 89	3	Englemann	1893 80	47 6	1 27	5	Bigourdan
1885 47	88 6	1 50	6	Perrotin	1894 51	43 8	1 24	3	Barnard
1885 52	89 4	1 70	4	Tarrant	1894 52	42 1	0 85	2	Glasenapp
1885 52	92 0	1 61	7	Hall	1894 54	40 4	1 23	2	Lewis
1885 62	86 3	1 57	5	Schiaparelli	1894 73	39 6	1 28	9-8	Collandreaux
1885 64	92 1	1 59	4	Jedrzejewicz	1894 74	37 4	1 12	16-14	Bigourdan
1885 71	98 0	1 82	6-5	Englemann	1895 32	36 7	1 17	3	See
1885 69	90 5	—	3	Seabroke	1895 57	30 2	1 00	4	Comstock
1886 54	88 8	1 50	6	Hall					
1886 55	84 5	1 56	1	Perrotin					
1886 58	85 0	—	3	Seabroke					

SIR WILLIAM HERSCHEL made his first measure of this star, July 21, 1782, and found the position-angle to be 69°.3\*

In 1795 he again examined the object, and noted that the distance had

\* *Astronomical Journal*, 357

decreased, but that it was in the same quadrant as before; this appears, however, to be a mistake, as the companion at that time must have been in the opposite quadrant. It is remarkable that HERSCHEL could not separate the companion in 1802, as the angle was then  $174^{\circ}5$ , and the distance  $1''.24$ .

Beginning with STRUVE's observation in 1826 the record is practically continuous, and we have measures for each year, except when the companion was so close as to be lost in the rays of the larger star

The periastron is so near the central star, on account of the considerable eccentricity and the position of the node, that the companion has never been seen in this part of the orbit. According to the elements found below, the minimum distance is about  $0''.45$ . Therefore, in spite of the comparative faintness of the companion, whose magnitude is 6.5, while that of the central star is 3.0, this object ought to be constantly within the reach of our great refractors. In previous revolutions, however, the star has been lost, and it will therefore be a matter of great interest to follow it during the next periastron passage in 1899. Good observations in this part of the orbit are needed, and the rare phenomenon which will be presented by  $\zeta$  *Herculis* about the end of this century will be worthy of the attention of observers with large telescopes.

Notwithstanding the three revolutions which have been completed since HERSCHEL's discovery in 1782, our knowledge of the orbit of this pair has remained somewhat unsatisfactory; the elements heretofore obtained are by no means accordant. This divergency of results may be attributed partly to errors of observation incident to the inequality of the components, and partly to a sensible mistake in the old position-angle of HERSCHEL, which ought to have been about  $80^{\circ}$ . Indeed, HERSCHEL's observation does not seem to lay claim to much accuracy, for on August 30, 1782, he says. "Saw it better than I ever did,"—implying that on the previous occasions the companion was not very well defined. The following table gives the elements published by previous investigators:

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
$P$ 31 4678	1829 50	0 4545	1 189	39 43	50 9	262 1	Madler, 1842	Dorp Obs, IX, p 192
30 216	1830 42	0 432	1 208	19 4	44 1	276 65	Madler, 1847	Fixt Syst, I, p 269
36 357	1830 481	0 4482	—	214 35	43 7	284 9	Villarcéau	A N., vol XXVI, p 305
37 21	1830 56	0 4381	—	37 23	39 35	266 9	Fletcher, 1853	M.N., XIII, p 258
36 715	1830 237	0 4831	1 350	41.9	49 1	290 6	Villarcéau, 1854	C R., XXXVIII, p 871
34 221	1830 01	0 4239	1 223	45 93	34 87	209 5	Dunér, 1871	A N., 1868
36 606	1829 635	0 5511	1 374	27 0	50 23	266 7	Plummer, 1871	M N., XXXI, 195
34 58	1864 90	0 405	1 36	26 13	51 11	260 97	Flammarion, '74	Catal Ét Doub., p 101
34.4	1864 8	0 463	1 284	41 73	43 23	252 75	Doberck, 1880	A N., 2332
34 411	1864 78	0 4666	1 345	44 1	44 53	251 8	Doberck	



After an examination of all the observations we formed mean positions for each year, and from these mean places deduced the following elements.

$$\begin{array}{ll} P = 35.00 \text{ years} & \Omega = 37^\circ 5' \\ T = 1864.80 & i = 51^\circ 77' \\ e = 0.497 & \lambda = 101^\circ 7' \\ a = 1''.4321 & n = -10^\circ 2843' \end{array}$$

#### Apparent orbit

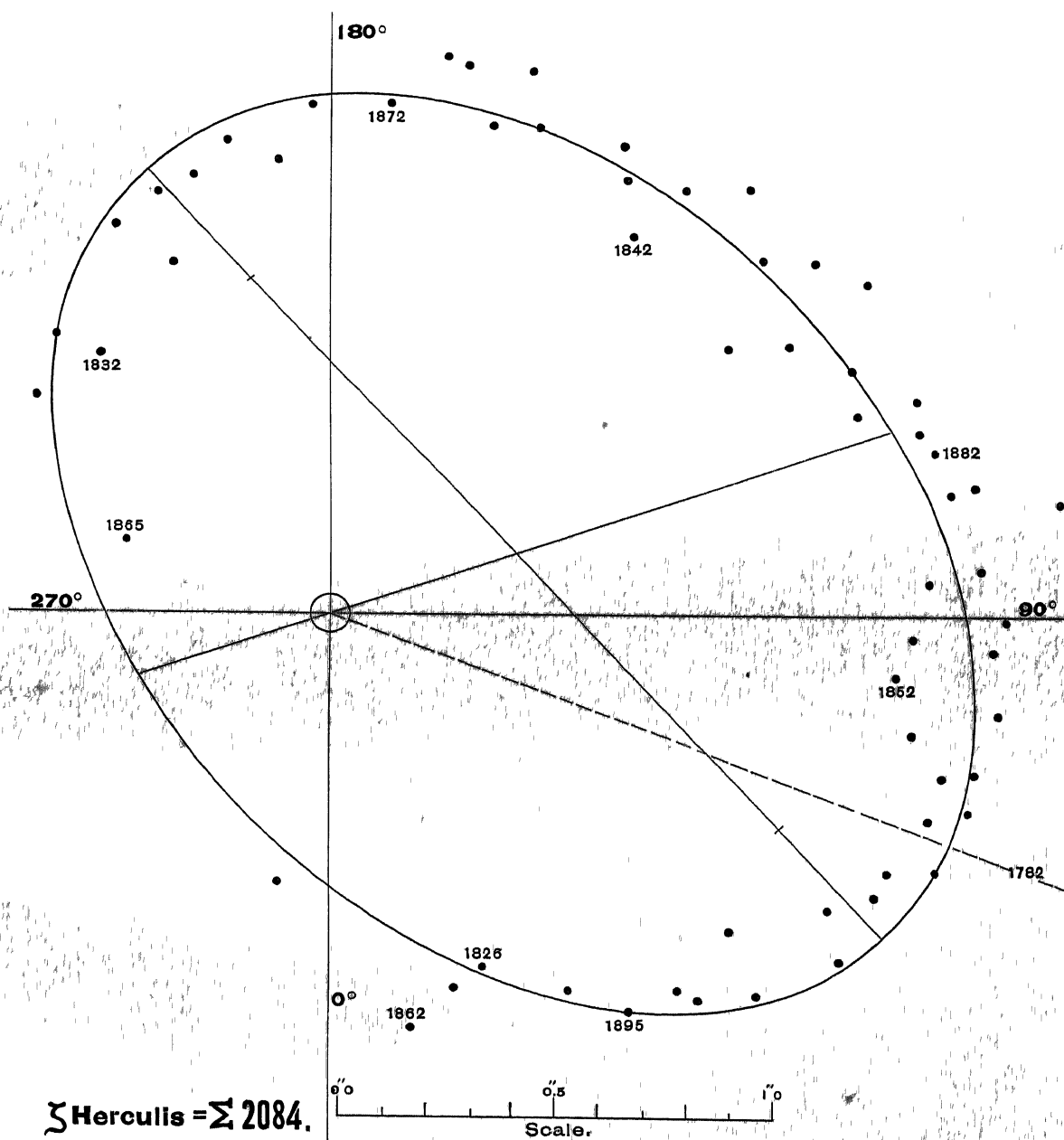
$$\begin{array}{ll} \text{Length of major axis} & = 2''.492 \\ \text{Length of minor axis} & = 1''.752 \\ \text{Angle of major axis} & = 43^\circ 1' \\ \text{Angle of periastron} & = 289^\circ 0' \\ \text{Distance of star from center} & = 0''.455 \end{array}$$

The following table of computed and observed places shows that the elements give a good representation of the observations, and render it probable that the present orbit is very near the truth. There are some errors in the position-angles which appear to be systematic, and we have not been able to improve the representation, for whatever would improve the agreement in one place would injure it in another, or in the same place during the next revolution.

It will be seen that this orbit is slightly more eccentric than most of those heretofore deduced, but it is not probable that the eccentricity will prove to be too large. If any change should be required in this element, it is likely to increase rather than diminish the value given above. The eccentricity of the orbit of  $\zeta$  *Herculis* is near the mean value of this element among double stars.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1782.55	69.3	80.5	—	1.51	-11.2	—	1	Herschel
1795.80	—	248.9	—	0.65	—	—	1	Herschel
1802.74	—	174.5	—	1.24	—	—	1	Herschel
1826.63	23.4	27.1	0.91	1.00	-3.7	-0.09	5	Struve
1828.71	349.5	344.0	0.65	0.54	+5.5	+0.11	1	Struve
1832.72	220.5	216.0	0.81	0.97	+4.5	-0.16	1	Struve
1834.45	203.5	201.5	0.91	1.14	+2.0	-0.23	2	Struve
1835.45	196.9	191.6	1.09	1.20	+5.3	-0.11	5	Struve
1836.60	186.2	182.8	1.09	1.23	+3.4	-0.14	5	Struve
1838.70	168.5	167.5	1.35	1.24	+1.0	+0.11	3±	Galle
1839.76	161.9	159.9	1.22	1.25	+2.0	-0.03	4	Dawes
1840.66	157.1	153.7	1.25	1.25	+3.4	±0.00	5	O Struve
1841.56	146.4	147.2	1.24	1.25	-0.8	-0.01	16-6	Madler 9-0, $O\Sigma$ 3, Dawes 4-3
1842.54	142.0	140.3	1.14	1.26	+1.7	-0.12	9-4	Madler 3-0, Dawes 3-1, $O\Sigma$ 3
1843.65	130.1	132.1	1.30	1.28	-2.0	+0.02	20-2	Madler 8-0, Dawes 3-2, Madler 9-0
1844.50	124.7	127.1	1.12	1.30	-2.4	-0.18	7-2	Madler 5-0, $O\Sigma$ 2
1845.64	121.3	119.3	1.24	1.32	+2.0	-0.08	3	O Struve
1846.79	111.3	112.1	1.31	1.35	-0.8	-0.04	7-2	$O\Sigma$ 2, Dawes 5-0





$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1847 55	107 9	107 4	1 43	1 38	+0 5	+0 05	21-20	Madler 18-17, Dawes 1, $O\Sigma$ 2
1848 59	101 8	101 4	1 52	1 41	+0 4	+0 11	8-5	Madler 3-0, Dawes 3, $O\Sigma$ 2
1849 48	99 2	96 5	1 71	1 44	+2 7	+0 27	1	Dawes
1850 36	93 3	91 7	1 39	1 46	+1 6	-0 07	8-6	$O\Sigma$ 3, Fletcher 2, Madler 3-1
1851 50	87 9	85 7	1 35	1 49	+2 2	-0 14	16-14	Madler 3, Fletcher 6, $O\Sigma$ 5, Miller 2-0
1852 67	83 8	79 8	1 32	1 52	+4 0	-0 20	20-14	$O\Sigma$ 5, Madler 8-7, Fletcher 5-2, Miller 2-0
1853 46	78 3	76 1	1 38	1 53	+2 2	-0 15	23-20	Jacob 2, Miller 6-3, Ma 8, $O\Sigma$ 4, Ma 3
1854 46	75 0	71 2	1 47	1 53	+3 8	-0 06	11	Jacob 3, $O\Sigma$ 3, Madler 5
1855 46	70 8	66 4	1 47	1 53	+4 4	-0 06	24-11	Dem 13-0, Secchi 3, $O\Sigma$ 4, Morton 4
1856 48	64 8	62 2	1 43	1 51	+2 6	-0 08	27	Jacob 3, Dembowski 15, Secchi 6, $O\Sigma$ 3
1857 61	59 0	55 5	1 35	1 46	+3 5	-0 11	24	Ma 4, Mo 2, Sec 6, $O\Sigma$ 4, Dem 5, Ja 3
1858 58	51 0	50 5	1 19	1 40	+0 5	-0 21	22-21	Secchi 2, Dembowski 8, $O\Sigma$ 4, Madler 8-7
1859 58	43 1	44 1	1 25	1 30	-1 0	-0 05	19-15	Madler 6, Secchi 3-0, Dawes 6-5, $O\Sigma$ 4
1860 70	32 0	36 5	1 05	1 16	-4 5	-0 11	4	Secchi 3, $O\Sigma$ 1
1861 50	18 6	28 5	0 93	1 02	-9 9	-0 09	6	Madler 2, $O\Sigma$ 4
1862 73	11 2	12 9	1 00	0 78	-1 7	+0 22	9-1	Dembowski 8, $O\Sigma$ 1, Madler 2
1863 49	34 3	35 2	cuneo	0 59	-9 2	—	4	Dembowski
1865 55	250 0	256 6	<0 5	0 59	-6 6	-0 09	3	Englemann
1866 74	232 5	229 0	0 85	0 85	+3 5	$\pm 0 00$	14	Dem 5, Dawes 3, $O\Sigma$ 2, Dawes 2, Dawes 2
1867 62	223 5	217 2	0 91	0 99	+6 3	-0 08	9	Dembowski 7, Dunér 2
1868 54	208 3	207 7	1 05	1 09	+0 6	-0 04	17	Dembowski 6, Knott 4, $O\Sigma$ 2, Dunér 5
1869 60	201 8	198 2	1 08	1 16	+3 6	-0 08	19	Dembowski 8, Dunér 11
1870 54	192 0	190 9	1 15	1 20	+1 1	-0 05	17	Dembowski 11, Dunér 6
1871 52	181 9	183 5	1 21	1 23	-1 6	-0 02	32	Dembowski 14, $O\Sigma$ 1, Knott 5, Dunér 12
1872 55	173 3	176 0	1 22	1 24	-2 7	-0 02	27	Dembowski 12, Dunér 12, $O\Sigma$ 3
1873 60	166 1	168 3	1 22	1 24	-2 2	-0 02	17	Dembowski 11, $O\Sigma$ 2, Dunér 4
1874 60	160 0	161 1	1 37	1 24	-1 1	+0 13	14-15	Dembowski 10, $O\Sigma$ 4; Dunér 0-1
1875 56	148 5	154 5	1 30	1 25	-6 0	+0 05	29-27	Dem 8; Sch 7, W. & S 2-0; Dunér 12
1876 56	141 0	148 6	1 30	1 25	-7 6	+0 05	10	Hall 2, Dembowski 7, Plummer 1
1877 57	136 3	140 1	1 40	1 26	-3 8	+0 14	11	Dembowski 8, Fritchett 1, Hall 2
1878 51	126 9	138 6	1 40	1 28	-6 7	+0 12	10-13	$\beta$ 1, Sch 4, Doberck 2-1, Dembowski 7
1879 54	122 5	126 6	1 47	1 30	-4 1	+0 17	8-16	$\beta$ 3; Hall 4, Schiaparelli 0-8; Fritchett 1
1880 49	115 8	120 4	1 34	1 32	-4 8	+0 02	10-15	Doberck 2-1, Big 3-0, $\beta$ 5, Sch 0-9
1881 49	110 6	113 9	1 45	1 35	-3 3	+0 10	17	Doberck 2, $\beta$ 5, Hough 4, Hall 5, Big 1
1882 60	105 6	107 2	1 46	1 38	-1 6	+0 08	17-19	Dk 2-0, Hl 5, Sch 0-11, Jed 4-3, En 6-0
1883 60	101 5	101 3	1 54	1 41	+0 2	+0 13	14-29	Per 4, Hall 5, Sch. 0-15, En 5 [En 3-0
1884 58	94 1	96 6	1 51	1 44	-2 5	+0 07	28-17	Big 2-0, Hl 4, Per 3, Prit 1, Sch 9, Sea 6-0
1885 58	89 8	90 5	1 57	1 47	-0 7	+0 10	29-22	Per 6, Tar 4-0, Hl 7, Sch 5, Jed 4, Sea 3-0
1886 63	87 0	85 0	1 54	1 50	+2 0	+0 04	35-32	Hl 6, Per 1, Sea 3-0, Sch 9, Jed 9-7, En 7
1887 60	81 5	80 2	1 57	1 52	+1 3	+0 05	24	Hall 6, Schiaparelli 18
1888 58	76 1	76 6	1 54	1 53	-0 5	+0 01	21-14	Hall 6, Comstock 3-0, Sch 9-8, Maw 3-0
1889 56	72 7	70 6	1 55	1 54	+2 1	+0 01	18-17	Sch 3, Glas 2-1, Hall 6, Com 4, Maw 3
1890 60	66 8	65 6	1 53	1 53	+0 2	$\pm 0 00$	16-15	Glas 2, Hall 6, Maw 3, Schiaparelli 5-4
1891 57	62 2	60 9	1 43	1 50	+1 3	-0 07	23-20	Hall 6, See 7-4, Sch 2, Big 5, Maw 3
1892 60	55 3	55 5	1 44	1 46	-0 2	-0 02	13	Comstock 5, Schiaparelli 8
1893 74	47 6	48 2	1 34	1 37	-0 6	-0 03	8-7	Schiaparelli 3-2, Bigourdan 5
1894 58	42 1	44 1	1 20	1 31	-2 0	-0 11	7-19	Barnard 3, Glas 2-0, Lewis 2, Big 0-14
1895 32	36 7	38 8	1 17	1 21	-2 1	-0 04	3	See

The companion is worthy of regular attention in the part of the orbit now being described, but observation will become more urgent as the star approaches periastron in 1899. If good observations can be secured they will enable us to give the highest precision to the theory of the motion of this star; but if the measures in so delicate a case are affected by sensible systematic errors they

will prove to be of little value. The phenomena of the approaching appulse of  $\zeta$  *Herculis* will therefore be difficult to observe, and results of importance can only be obtained by skillful treatment. It is hardly necessary to add that this phenomenon will not again be witnessed for more than a third of a century.

It seems worthy of remark that STRUVE, who devoted so much attention to the colors of double stars, noted the color of the companion as reddish, while it is now distinctly bluish, and although a change of color does not seem probable, this has been suspected as well as variability.

In order that astronomers may be able to compare the present theory with observations during the rapid motion of the companion in passing periastron, we give an ephemeris for the next ten years

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 50	28 5	1 02	1901 50	233 0	0 80
1897 50	15 5	0 82	1902 50	218 4	0 97
1898 50	351 9	0 56	1903 50	207 8	1 09
1899 50	289 7	0 47	1904 50	198 9	1 16
1900 50	258 4	0 58	1905 50	191 0	1 20

### $\beta$ 416 = LACAILLE 7215.

$\alpha = 17^h 12^m 1$  ,  $\delta = -34^\circ 52'$   
6 4, yellowish , 7 8, yellowish

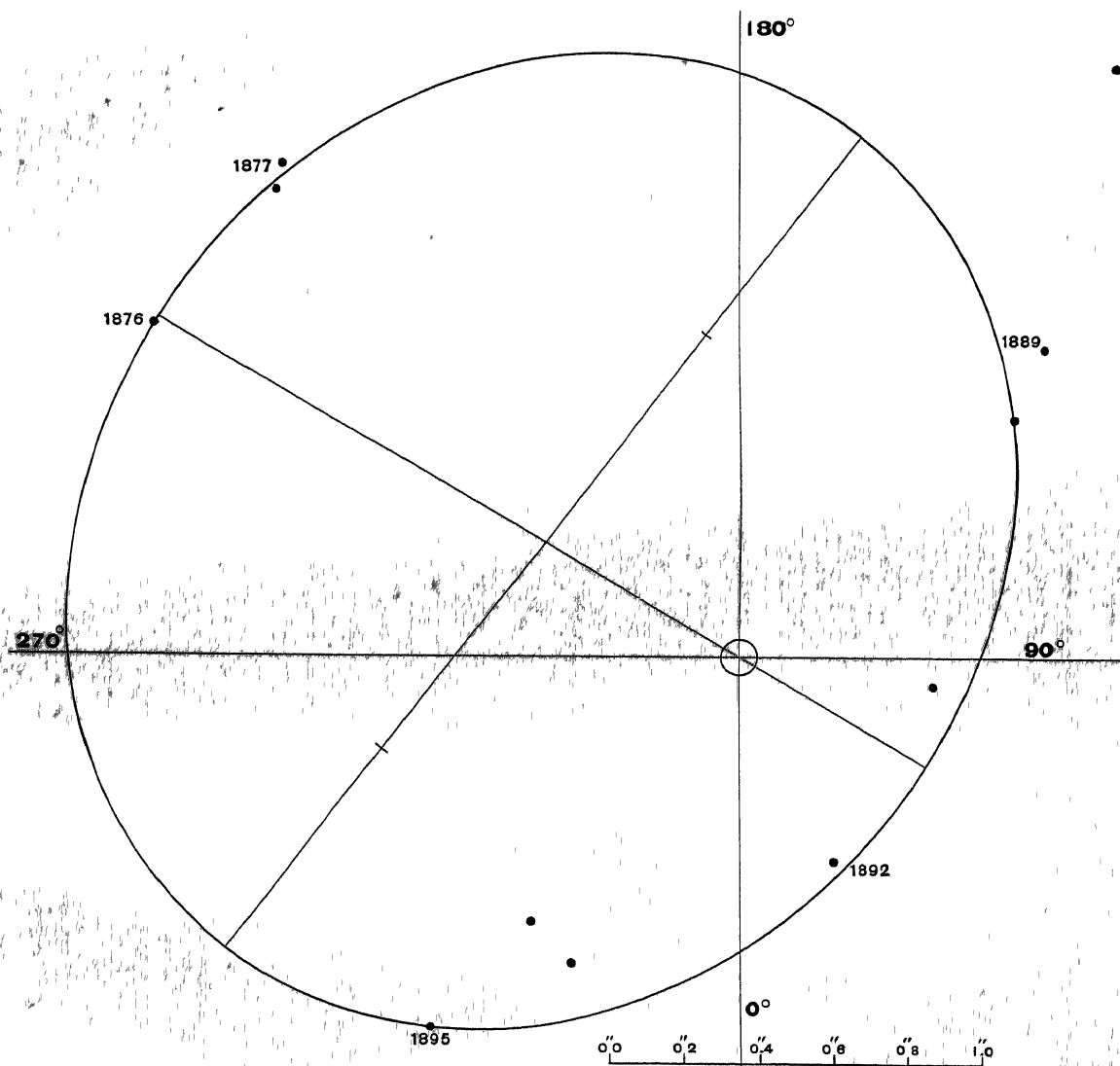
*Discovered by Burnham in 1876*

#### OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1876 52	240 $\pm$	1 8 $\pm$	1	Burnham	1891 53	81 2	0 53	3-2	Burnham
1877 53	222 6	1 80	1	Cincinnati	1892 38	24 4	0 61	4-3	Burnham
1877 64	224 4	1 77	1	Russell	1894 57	330 8	0 94	7-2	Sellois
1888 72	147 5	1 88	1	Burnham	1894 63	334 7	1 27	3	Barnard
1889 43	135 2	1 17	2-1	Burnham	1895 60	321 7	0 91	2-1	Comstock
1889 63	131 9	0 97	1	Pollock	1895 74	320 0	1 30 $\pm$	1	See

Since the discovery of this rapid binary the companion has described an arc of  $280^\circ$ . The magnitudes of the components are 6 4 and 7 8 respectively, and as the pair is never closer than  $0'' 58$  the object is difficult only on account of its southern declination.\* The period is surprisingly short for a system of

\* *Astronomical Journal*, 872



$\beta 416 = \text{Lac. } 7215.$



such considerable separation, and this circumstance lends decided probability to the view that the parallax is sensible. Provisional elements for this system have been computed by GLASENAPP, GORE and BURNHAM. Their results are as follows:

$P$ yrs	$T$	$e$	$a$ "	$\Omega$ °	$i$ °	$\lambda$ °	Authority	Source
34 85	1892 00	0 65	1 52	104 3	45 4	300 7	Glasenapp, 1893	Astron and Astroph, May, 1893
34 48	1891 85	0 556	2 13	139 4	56 7	278 2	Gore, 1893	Monthly Notices, March, 1893
24 7	1892 26	0 56	1 46	122 0	44 4	93 5	Burnham, 1893	Publ Lick Obs, vol II, p 247

The observations which I secured recently at the Washburn Observatory have enabled me to redetermine the orbit. Using all available measures, we find the following elements of  $\beta$  416.

$$\begin{aligned}
 P &= 330 \text{ years} & \Omega &= 144^\circ 6 \\
 T &= 1891.85 & i &= 37^\circ 35 \\
 e &= 0.512 & \lambda &= 86^\circ 1 \\
 a &= 1''.2212 & n &= -9^\circ 0908
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 2''.76 \\
 \text{Length of minor axis} &= 2''.38 \\
 \text{Angle of major axis} &= 142^\circ.5 \\
 \text{Angle of periastron} &= 59^\circ.5 \\
 \text{Distance of star from centre} &= 0''.61
 \end{aligned}$$

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$ °	$\theta_c$ °	$\rho_o$ "	$\rho_c$ "	$\theta_o - \theta_c$ °	$\rho_o - \rho_c$ "	$n$	Observers
1876 52	240 ±	233 4	1 8 ±	1 79	+6 6	+0 01	1	Burnham
1877 53	222 6	228 2	1 80	1 79	-5 4	+0 01	1	Cincinnati
1877 64	224 4	227 6	1 77	1 78	-3 2	-0 01	1	Russell
1888 72	147 5	147 7	1 88	1 19	-0 2	+0 69	1	Burnham
1889 43	135 2	136 7	1 17	1 04	-1.5	+0 13	2-1	Burnham
1889 63	131 9	133 1	0 97	1 00	-1 2	-0 03	1	Pollock
1891 53	81 2	75 2	0 53	0 60	+6 1	-0 07	3-2	Burnham
1892 38	24 4	34 0	0 61	0 61	-9 6	0 00	4-3	Burnham
1894 57	330 8	333 6	0 94	1 10	-2 8	-0.16	7-2	Sellers
1895 60	321 7	319 9	0 91	1 30	+1 8	-0 39	2-1	Comstock
1895 74	320 0	318 4	1 30 ±	1 32	+1 6	-0 02	1	See

The angular motion during the last three years has not been very rapid, and the constancy of areas shows that the distances have been somewhat under-measured. It is now apparent that the period will be sensibly longer than BURNHAM supposed. The value found above is not likely to be in error by more than one year, while the correction of the eccentricity will hardly exceed  $\pm 0.03$ . Considering the small number of observations on which this orbit is based, the elements may be regarded as highly satisfactory. As this system is



visible in the United States, it is worthy of particular attention from American observers.

The following ephemeris gives the place of the companion for five years

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 50	310 6	1 43	1899 50	287 7	1 69
1897 50	302 1	1 54	1900 50	281 5	1 72
1898 50	294 6	1 62			

### $\Sigma 2173.$

$\alpha = 17^h 25^m 3$  ,  $\delta = -0^\circ 59'$   
6, yellow , 6, yellow

*Discovered by William Struve in July, 1829*

#### OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1829 56	147 2	0 62	2	Struve	1851 32	334 1	1 27	4	Madler
1831 69	141 5	0 62	3	Struve	1851 74	335 6	1 18	2	Madler
1836 69	single	—	4	Struve	1852 72	334 1	1 23	2	Madler
1837 70	353	obl ?	1	Struve	1853 12	331 2	1 04	4	O Struve
1840 47	347 1	0 5 $\pm$	1	Dawes	1854 66	330 5	1 37	3	Madler
1840 64	358 8	0 61	3	O Struve	1856 53	153 2	0 9 $\pm$	1	Winnecke
1841 36	352 3	0 67	6-2	Madler	1856 53	329 1	1 $\pm$	4	Dembowski
1841 61	352 2	0 67	3	O Struve	1856 53	329 8	0 97	1	Secchi
1841 64	347 4	0 71	2-1	Dawes	1856 90	326 0	0 94	4	O Struve
1842 45	354 9	—	5	Kaiser	1857 43	326 9	0 88	1	Secchi
1842 51	349 9	0 75	3	Madler	1858 56	325 9	0 84	2	Secchi
1842 67	343 3	0 7 $\pm$	3	Dawes	1858 61	328 3	0 88	4-2	Madler
1843 30	343 1	0 74	3	O Struve	1858 61	325 0	0 25 $\pm$	1	Moitron
1843 50	346 2	0 78	8-5	Madler	1859 33	324 2	0 71	3	O Struve
1843 54	341 2	0 9 $\pm$	6	Dawes	1861 57	324 0	—	3	Madler
1843 65	345 1	0 68	10-2	Kaiser	1861 63	315 2	0 48	1	O Struve
1844 36	345 0	0 8 $\pm$	3	Madler	1864 45	160 ?	0 6 ?	2	Englemann
1845 55	342 1	0 97	1	Madler	1864 53	single		1	Dembowski
1846 46	339 4	1 07	6-5	Madler	1865 51	182 2	—	1	Leyton Obs
1846 47	336 1	0 85	5	O Struve	1866 32	360 7	0 47	3	O Struve
1847 47	339 2	1 16	2	Madler	1866 43	181 3	—	1	Leyton Obs
1848 44	339 2	1 15	1	Madler	1866 59	107 7	—	1	Winlock
1848 45	339 4	1 10	1	Dawes	1866 62	139 4	1 60	5-1	Searle
1848 58	340 4	1 23	1	Mitchell	1866 69	167 7	—	1	Winlock

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1867 79	174 5	0 68	1	Dunér	1881 74	elong ?	"	1	Bigourdan
1868 18	161 3	0 65	3	O Struve	1882 57	109 9	0 3	7	Schiaparelli
1868 60	160 6	0 5±	2	Dembowski	1882 62	349 0	oblong	1	O Struve
1868 66	169 3	0 68	3	Dunér	1883 50	elong	20°-45°	4	Perrotin
1869 58	157 1	0 58	5-1	Dembowski	1883.60	190 0	oblong	1	O Struve
1869 93	161 1	0 68	6	Dunér	1883 60	single		7	Schiaparelli
1870 35	156 4	0 8±	2	Gledhill	1883 88	24 8	0 22	9	Englemann
1870 45	156 8	0 81	6-4	Dembowski	1884 56	17 4	0 38	3-2	Perrotin
1870 67	159 7	0 80	4	Dunér	1884 59	99 ?	—	1	Bigourdan
1871 44	155 0	0 99	4-2	Dembowski	1884 60	single		7	Schiaparelli
1871 64	156 5	0 87	6	Dunér	1884 61	42 7?	0 25±?	3	Schiaparelli
1872 15	155 7	0 89	5	O Struve	1884 62	9 9	0 32	3	Hall
1872 55	152 3	0 95	5-3	Dembowski	1885 66	21 9	0 30	8-6	Englemann
1873 50	154 1	1 00	2	W & S	1886 55	356 6	0 56	3	Perrotin
1873 51	150 8	0 77	4-1	Dembowski	1886 56	355 1	0 41	7	Schiaparelli
1873 67	152 6	1 10	1	Dunér	1886 56	353 0	0 42	3	Hall
1874 46	150 0	0 91	4-3	Dembowski	1886 64	365 6	0 30	8	Englemann
1874 57	151 1	0 99	2-1	Gledhill	1887 40	350 5	0 46	4	Tarrant
1874 59	151 2	0 90	2-1	W & S	1887 56	348 5	0 53	7	Schiaparelli
1874.62	149 3	0 77	2	O Struve	1888 49	347 8	0.68	3	Leavenworth
1874 66	148 8	1 09	2	Newcomb	1888 55	344 4	0 53	3	Hall
1875 53	147 5	0 74	4	Dembowski	1888 60	346 9	0 58	8	Schiaparelli
1875.57	146.5	0 83	7	Schiaparelli	1888 69	342 3	0 81	1	O Struve
1875 57	147.8	1 ±	1	W & S	1889 46	345 0	0 66	5	Tarrant
1875 67	148 7	0 90	5	Dunér	1889 63	345 5	0 70	7	Schiaparelli
1876 52	149 3	0 77	3	Hall	1890 26	341 5	in cont	10	Giacomelli
1876 55	144 8	0 69	5	Dembowski	1890 49	340 9	0 8±	2	Glasenapp
1876 59	143 8	0 83	4	Schiaparelli	1890 69	343 1	0 84	3	Maw
1876 65	144 0	0 61	2	O Struve	1890 71	334 6	0 76	2	Bigourdan
1876 66	149 9	—	4	Doberck	1890 74	341 7	0 70	7-5	Schiaparelli
1877 49	141 6	—	2	Cincinnati	1891 51	340 1	0 97	3	Hall
1877 53	142 5	0 62	5-4	Dembowski	1891 53	340 0	0 81	4	Schiaparelli
1877 59	141 4	0 65	2	O. Struve	1891 58	339 7	0 93	3	Burnham
1877 59	142 0	0 72	8	Schiaparelli	1891 69	340 3	0 91	3	Bigourdan
1877 68	153 5	0 67	2	Doberck	1892 54	341 8	0 90	4	Comstock
1878 40	142 5	0 52	1	Doberck	1892 61	339 1	1 10	1	Bigourdan
1878 48	139 4	0 60	4	Dembowski	1892 62	339 3	0 88	7	Schiaparelli
1879 22	137 0	0 69	7-3	Cincinnati	1892 72	340 7	0 91	3	Maw
1879 58	136 0	0 5±	8	Schiaparelli	1893 68	338 0	1 08	3	Schiaparelli
1879 72	152 2	0 7±	3	Seabroke	1893 87	340 6	1 11	3	H C Wilson
1880 47	131 3	0 36	1	Burnham	1894 55	336 8	1 15	2	Lewis
1880 65	133 9	0 4±	9	Schiaparelli	1894 74	159 9	1 27	1	Callandreau
1881 51	114 9	0 24	3	Burnham	1895 30	337 3	1 19	3	See
1881.52	121 5?	0 27?	1	Hall	1895 57	337 7	1 13	3	Comstock

When this interesting double star in the constellation *Ophiuchus* was discovered by WILLIAM STRUVE, the companion was measured on two nights,\* and again observed in 1831, but in 1836 it had disappeared, so that under the best seeing the star appeared absolutely round. STRUVE therefore surmised (*Mensurae Micrometricae*, p 294) that this is a case of occultation similar to those of  $\gamma$  *Coronae Borealis* and  $\omega$  *Leonis*, "*summa attentione digna*" The companion came out on the opposite side in 1840, and was subsequently followed systematically by the best observers, so that at the present time a large amount of good material is available for the investigation of its orbit. The components are so nearly equal in brightness that the angles frequently require a correction of  $180^\circ$ , and for a time it remained uncertain whether the period would be 46 or 23 years. Prof DUNÉR was the first astronomer who attempted to investigate the orbit of this pair, using measures up to 1876, the illustrious Director of the Observatory of Upsala arrived at the following results:

$$\begin{array}{ll} P = 45.43 \text{ years} & \Omega = 152^\circ 65' \\ T = 1872.91 & i = 80^\circ 53' \\ e = 0.1349 & \lambda = 7^\circ 26' \\ a = 1''.009 \end{array}$$

From an investigation of all the observations, including the measures recently secured at the Leander McCormick Observatory in Virginia, we find the following elements of Σ2173.

$$\begin{array}{ll} P = 46.0 \text{ years} & \Omega = 153^\circ 7' \\ T = 1869.50 & i = 80^\circ 75' \\ e = 0.20 & \lambda = 322^\circ 2' \\ a = 1''.1428 & n = -7^\circ 8261 \end{array}$$

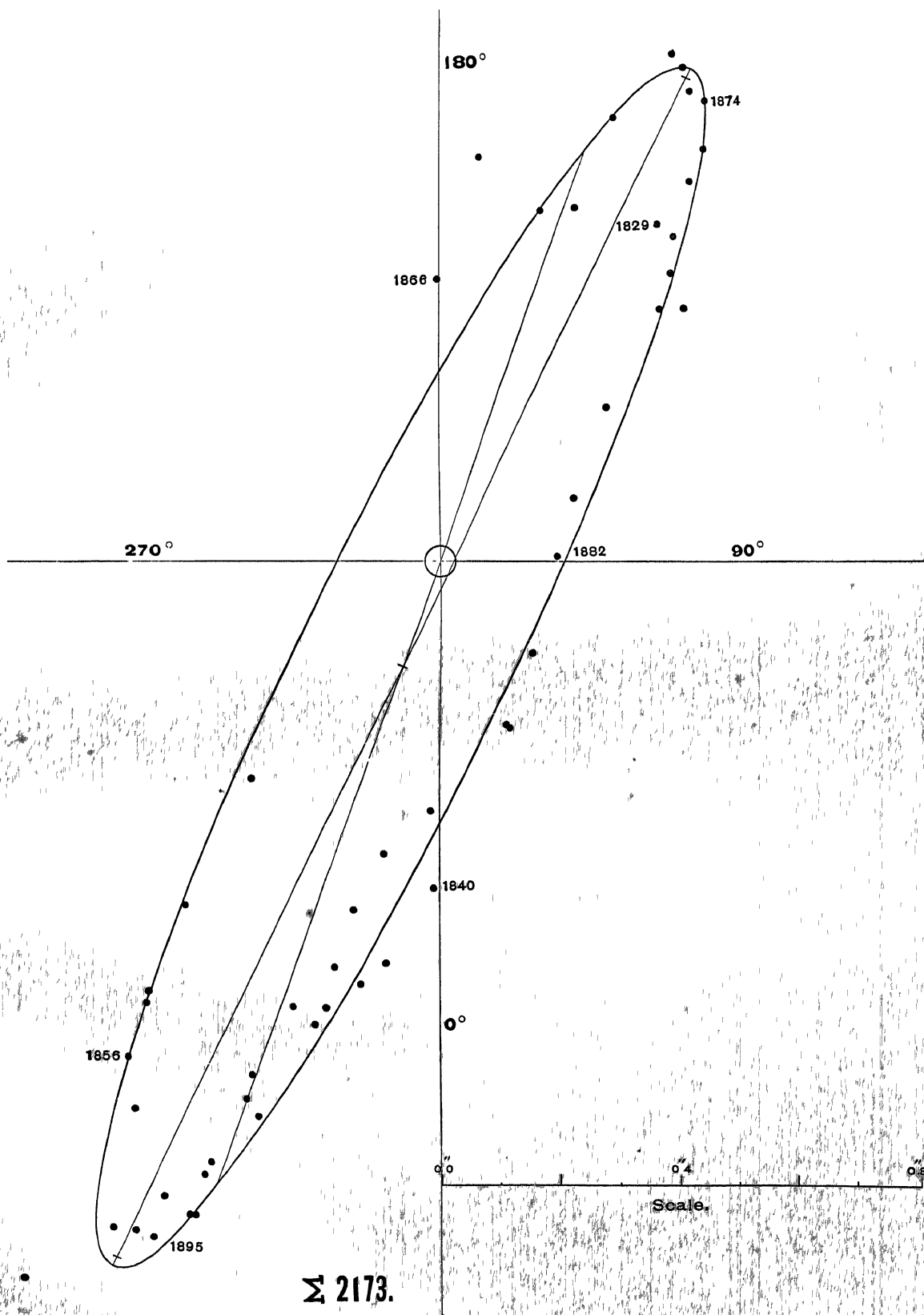
#### Apparent orbit

$$\begin{array}{ll} \text{Length of major axis} & = 2''.22 \\ \text{Length of minor axis} & = 0''.35 \\ \text{Angle of major axis} & = 154^\circ 5' \\ \text{Angle of periastron} & = 160^\circ 8' \\ \text{Distance of star from centre} & = 0''.18 \end{array}$$

The accompanying table of computed and observed places shows that these elements are very satisfactory.

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\* *Astronomische Nachrichten*, 3811





## COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1829 56	147 2	148 5	0 67	0 84	-1 3	-0 17	2-1	Struve
1831 69	141 5	143 4	0 62	0 69	-1 9	-0 07	3	Struve
1840 64	358 8	358 0	0 55	0 47	+0 8	+0 08	3-4	OΣ 3, Dawes 0-1
1841 48	352 3	352 5	0 67	0 59	-0 2	+0 08	9-5	Madler 6-2, OΣ 3
1842 54	349 3	348 8	0 72	0 69	+0 5	+0 03	11-6	Kaiser 5-0, Madler 3, Dawes 3
1843 57	345 7	345 9	0 77	0 82	-0 2	-0 05	18-16	Ma 8-5, Ka 10-2, OΣ 0-3, Da 6
1844 36	345 0	344 3	0 8 ±	0 90	+0 7	-0 10	3	Madler
1845 55	342 1	342 2	0 97	0 95	-1 1	+0 02	1	Madler
1846 46	339 4	340 8	1 07	1 06	-1 4	+0 01	6-5	Madler
1847 47	339 2	339 6	1 16	1 14	-0 4	+0 02	2	Madler
1848 49	339 6	338 4	1 16	1 21	+1 2	+0 05	3	Madler 1, Dawes 1; Mitchell 1
1851 74	335 6	335 0	1 22	1 29	+0 6	-0 07	2-6	Madler
1852 72	334 1	334 3	1 23	1 28	-0 2	-0 05	2	Madler
1853 12	331 2	333 9	1 04	1 27	-2 7	-0 23	4	O Struve
1854 66	330 5	332 7	1 37	1 24	-2 2	+0 13	3	Madler
1856 65	328 3	330 2	0 97	1 10	-1 9	-0 13	9	Dem 4, Se 1, OΣ 4
1857 43	326 9	329 3	0 88	1 04	-2 4	-0 16	1	Secchi
1858 59	326 4	327 5	0 86	0 93	-1 1	-0 07	7-4	Se 2, Ma 4-2, Mo 1-0
1859 33	324 2	326 1	0 71	0 85	-1 9	-0 14	3	O Struve
1861 60	319 6	319 7	0 48	0 57	-0 1	-0 09	4-1	Madler 3-0, OΣ 1
1866 32	180 7	184 8	0 47	0 28	-4 1	+0 19	3	O Struve
1867 79	174 5	168 3	0 68	0 51	+6 2	+0 17	1	Dunér
1868 48	163 8	164 5	0 61	0 59	-0 7	+0 02	8	OΣ 3, Dembowski 2, Dunér 3
1869 76	159 1	160 0	0 63	0 75	-0 9	-0 12	11-7	Dembowski 5-1, Dunér 6
1870 56	158 3	158 1	0 80	0 83	+0 2	-0 03	10-8	Dembowski 6-4, Dunér 4
1871 54	155 8	155 9	0 93	0 89	-0 1	+0 04	10-8	Dembowski 4-2, Dunér 6
1872 35	154 0	154 4	0 92	0 92	-0 4	0 00	10-8	OΣ 5; Dembowski 5-3
1873 56	152 5	152 2	0 89	0 92	+0 3	-0 03	7-3	W & S 2; Dem 4-1, Du 1-0
1874 56	150 4	150 4	0 89	0 89	0 0	0 00	10-7	Dem 4-3, Gl 2-1, W & S 2-1, OΣ 2
1875 58	147 6	148 4	0 82	0 84	-0 8	-0 02	17-16	Dem 4, Sch. 7, W & S 1-0, Du 5
1876 58	146 9	146 2	0 76	0 78	+0 7	-0 02	16-12	Hl 3, Dem 5, Sch 4, Dk 4-0
1877 57	144 4	143 7	0 67	0 70	+0 7	-0 03	17-14	Cin 2-0, Dem 5-4, Sch 8, Dk 2
1878 48	139 4	140 6	0 56	0 61	-1 2	-0 05	5	Doberck 1, Dembowski 4
1879 40	136 5	136 4	0 59	0 52	+0 1	+0 07	15-11	Cincinnati 7-3, Schiaparelli 8
1880 56	132 6	128 0	0 38	0 40	+4 6	-0 02	10	β 1, Schiaparelli 9
1881 51	114 9	114 9	0 24	0 29	0 0	-0 05	3	Burnham
1882 61	91 6	90 5	0 2	0 21	+1 1	-0 01	1	Schiaparelli
1883 69	45 0	48 9	0 22	0 20	-3 9	+0 02	4-9	Perrotin 4-0, Englemann 0-9
1884 59	23 3	21 8	0 31	0 27	+1 5	+0 04	9-8	Perrotin 3-2, Sch 3, Hall 3
1885 66	21 9	5 2	0 30	0 37	+16 7	-0 07	8-6	Englemann
1886 58	357 6	358 0	0 42	0 47	-0 4	-0 05	21	Per 3, Sch 7, Hall 3, En 8
1887 48	349 5	352 7	0 50	0 59	-3 2	-0 09	11	Tarrant 4, Schiaparelli 7
1888 55	346 3	348 1	0 60	0 72	-1 8	-0 12	14	Lv 3, Hall 3, Schiaparelli 8
1889 63	345 5	345 8	0 70	0 82	-0 3	-0 12	7	Schiaparelli [Big 0-2; Sch 7-5
1890 58	341 8	343 8	0 78	0 92	-2 0	-0 14	24-12	Gia 10-0, Glasenapp 2, Maw 3,
1891 58	340 0	342 1	0 91	1 01	-2 1	-0 10	13	Hall 3, Sch 4, β 3, Big 3
1892 62	340 2	340 6	0 95	1 09	-0 4	-0 14	15	Com 4, Big 1, Sch 7, Maw 3
1893 77	339 3	338 9	1 09	1 19	+0 4	-0 10	6	Schiaparelli 3, H C W 3
1894 55	336 8	338 3	1 15	1 22	-1 5	-0 07	2	Lewis
1895 30	337 3	337 9	1 22	1 24	-0 6	-0 02	3-1	See

Owing to the high inclination of the orbit, it is clear that a small error in angle would very sensibly alter the apparent radius vector of the companion, and for this reason good measures of distance are more trustworthy than

angles. Therefore, while the present orbit is based on both coordinates, unusual weight has been given to the observed distances

The residuals in angle are very small, except in the case of ENGLEMANN'S measure of 1885, when the components were so close as to render all observations with a small telescope very uncertain. It should be remarked that the position for 1882 is based on a measure which was rejected by SCHIAPARELLI on account of its discordance, but as the other six measures by that distinguished astronomer give

$$\theta_o = 109^\circ 9 \quad \rho_o = 0'' 30,$$

which cannot well be reconciled with the theory of the star's motion, it appears probable that the single outstanding observation is nearer the truth, and it is therefore adopted in the above table.

The most remarkable characteristic of  $\Sigma 2173$  is the relatively small eccentricity of its orbit. Although this element is not so well defined as might be desired, yet the value given above seems to be fairly indicated by the best observations, and is not likely to need any large correction. Good measures of distance about the time of maximum elongation, in 1898 and 1899, would fix the eccentricity more accurately, and accordingly for the next five years this system will deserve the particular attention of astronomers

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### $\mu^1$ HERCULIS BC = A.C. 7.

$$\alpha = 17^h 42^m 6 \quad , \quad \delta = +27^\circ 47'$$

9 4, bluish white , 10, bluish

*Discovered by Alvan Clark in July, 1856*

OBSERVATIONS									
$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1857 47	$63^\circ \pm$	—	1	Dawes	1865 43	$80^\circ 5$	1 84	2-1	Knott
1857 50	59 3	1 82	2	Dawes	1865 44	82 0	1 27	5	Dembowski
1857 85	71 7	1 74	1	Secchi	1866 59	86 3	—	1	Winlock
1859 70	60 4	2 05	3	Dawes	1866 56	86 3	—	1	Searle
1860 30	67 7	1 64	1	O Struve	1866 68	89 5	1 10	2	O Struve
1862 83	78 5	1 50	1	O Struve	1867 58	97 9	—	3	Searle
1864 43	77 6	1 81	1	Dawes	1867 59	93 0	—	1	Winlock
1864 49	67 5	1 70	1	Englemann	1868 50	97 7	0 88	1	O Struve
1864 76	78 8	1 76	1	Winnecke	1868 61	106 4	—	1	Winlock

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1869 73	130 9	—	1	Winlock	1883 53	262 1	0 74	3	Burnham
1869 73	111 7	—	2	Pearce	1883 57	261 1	0 84	3	Hough
1871 51	100 $\pm$	0 6 $\pm$	2	W & S	1883 58	262 9	0 66	3	Hall
1871 52	156 8	0 62	1	O Struve	1883 58	261 4	0 86	2	Frisby
1873 50	180 5	—	1	Romberg	1883 63	274 8	—	5	Schiaparelli
1873 50	174 5	—	1	H Bruns	1883 96	80 6	0 62	8-6	Englemann
1873 50	175 4	—	1	Müller	1884 64	273 4	0 65	3	Hall
1873.50	185 5	0 63	1	O Struve	1884 68	272 7	0 77	1	O Struve
1873 50	90 $\pm$	0 6 $\pm$	1	W & S	1885.51	268 1	1 15	2-1	Holetschek
1873 67	semplíce	—	1	Dembowski	1885 56	288 0	0 61	3	Hall
1874 48	202 4	0 76	4-2	Newcomb	1885 62	245 2	—	2	Smith
1874 65	100 5	0 4 $\pm$	2	Gledhill	1886 60	302 1	0 39	5	Hall
1875 58	215 2	—	6	Schiaparelli	1887 54	318 3	0 49	6-5	Schiaparelli
1875 69	225 9	—	1	Newcomb	1887 58	321 5	0 42	3	Hall
1875 69	220 6	1 18	5-3	Hall	1888 47	330 7	0 45	3-2	Tarrant
1875 70	217 6	—	1	Holden	1888 62	343 1	0 43	11-9	Schiaparelli
1876 59	223 4	0 72	4	Hall	1888 63	341 4	0 39	4	Hall
1876 60	228 7	1 01	4	O Struve	1889 51	357 9	0.55	4	Burnham
1876 68	216 0	0 83	4	Dembowski	1889 58	354 4	0 58	3	Schiaparelli
1877 47	236 0	—	1	Seabroke	1889 65	0 6	0.34	4	Hall
1877 56	234 3	1 10	2	O Struve	1890.38	9 4	0.66	4	Burnham
1877 59	227 9	0 8 ?	5	Schiaparelli	1890 55	13.2	0 51	4	Hall
1877.59	232.8	0 85	2	Hall	1890 78	15 0	0 57	3	Schiaparelli
1877.62	229 9	0.92	4	Dembowski	1891 55	21 4	0 6	2	Schiaparelli
1878 45	234 9	1 05	6	Burnham	1891 57	24 8	0 54	4	Hall
1878 50	233 8	0 88	2	Hall	1891 60	23 6	0 90	3	Bigourdan
1878 64	238 2	1 17	1	O Struve	1892 58	29 1	0 83	4	Comstock
1879 45	242 7	0 90	5	Burnham	1892 62	30 3	0 87	5-4	Schiaparelli
1879 55	239 5	0 97	3	Hall	1892 63	30 5	0 90	1	Bigourdan
1879.75	234 8	—	11	Seabroke	1892 65	31 6	0 84	4	Hall
1880 46	230 2	0 7 ?	5	Schiaparelli	1893 62	36 0	0 90	1	Bigourdan
1880 47	245 9	0 96	7	Burnham	1894 43	41 1	1 19	7	Barnard
1880 65	246 3	1 00	4	Hall	1894 46	38 0	0.95	4	Hough
1880 78	246 5	1 18	3	Frisby	1894 54	38 7	1 17	3	Stone
1881 41	252 1	0 92	5	Burnham	1894 77	41 6	1 16	3	Comstock
1881 52	254 2	0 87	3	Hough	1895 34	41 2	0 86	1	See
1881 55	249 1	1 01	5	Hall	1895 54	44 0	1 3 ?	2-1	Schiaparelli
1882 52	259 1	0 70	4	Hall	1895 60	44 4	1 16	3	Comstock
1882 53	255 4	—	1	H Struve	1895 73	43 7	1 13	2	See
1882 53	261 7	0 90	3	Hough	1895 73	43 4	1 34	1	See
1882 56	263 2	1 03	3	O Struve	1895 73	44 8	1 10	2-1	Moulton
1882 60	266 8	—	7	Schiaparelli					

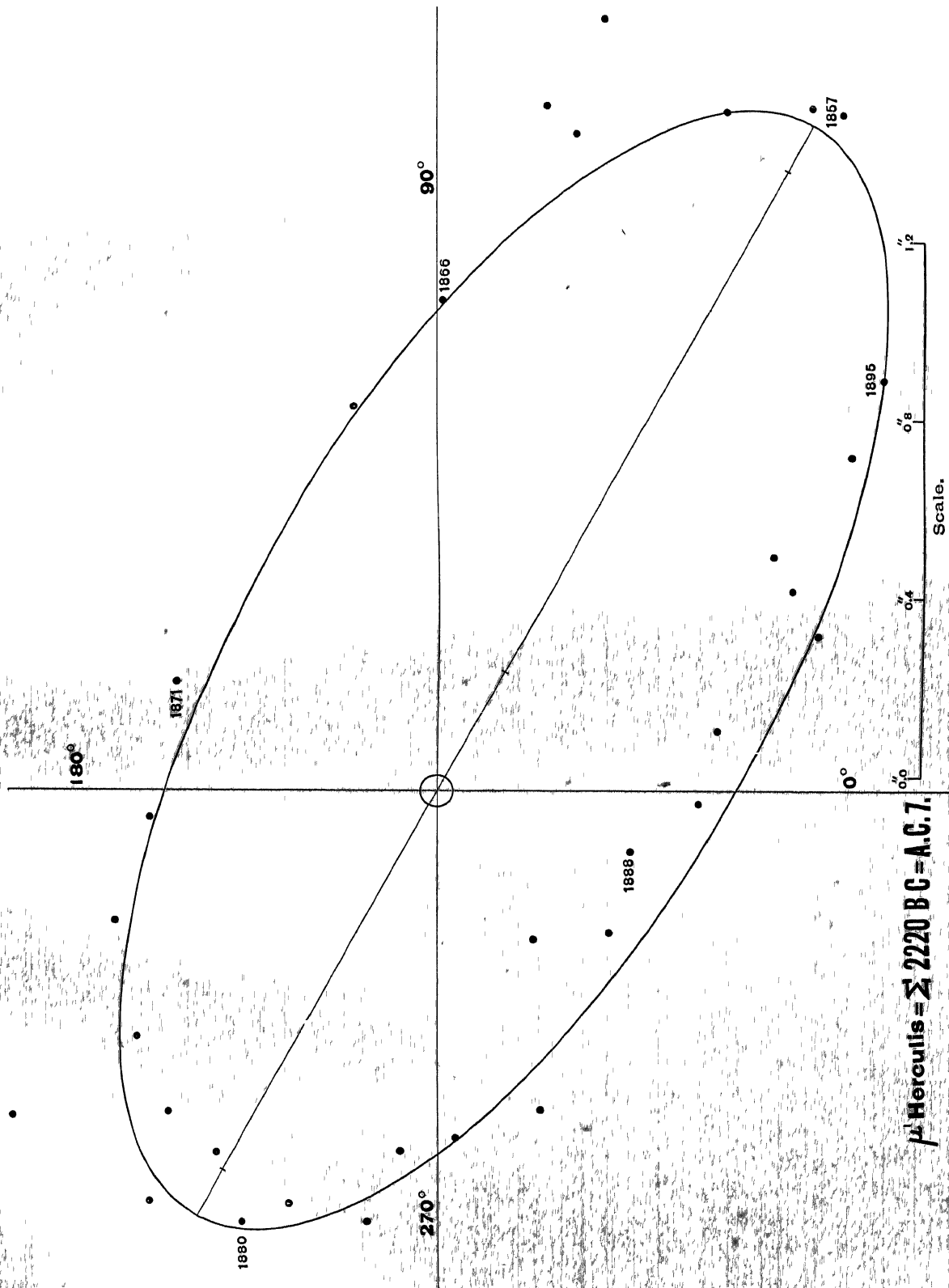


In July, 1856, ALVAN CLARK discovered that the bluish companion of  $\mu$  *Herculis* =  $\Sigma$  2220 is a close double star; he estimated the magnitudes of the component to be 10 and 11. The object was first measured by DAWES who predicted the binary character of the system; by repeating his observations in 1859 and 1864, he was able to announce a decided orbital motion. The object has since received considerable attention from the best observers, and the material now available for an orbit is sufficient to define the elements in a very satisfactory manner. Owing to the faintness and difficulty of the pair, the measures must be carefully combined in order to get a satisfactory set of mean places, the distances of some observers are notably too small, and hence they are omitted in forming the yearly means. Most of the early observations of DAWES seem to be affected by sensible errors, and hence we give his work in full.

$t$	$\theta_0$	$\rho_0$	
1857 472	58 97	1 853	
1857 562	60 08	1 75 $\pm$	
1859 650	58 91	2 304	distance indifferent
1859 691	59 51	1 422	observation very poor
1859 757	62 02	2 040	difficult in distance
1864 431	77 59	1 806	undoubtedly binary

While measuring the wide pair in 1857, he observed that "the stars  $B$  and  $C$  certainly point rather to the north of  $\mu$ ." He gives the angle of  $\mu$  *Herculis* relative to  $BC$  as  $242^\circ 2$ , and hence we gather that the angle of the pair  $BC$  must have been at least  $63^\circ 0$ . Since the alignment of the two faint stars with  $\mu$  *Herculis* would probably be more exact than even micrometer settings, it seems certain that most of DAWES' measured angles are too small; we have therefore chosen certain nights only in making up the means, and have selected the distances with some regard to the subsequent motion of the star. This selection of DAWES' material is necessary in order to represent satisfactorily the whole series of observations by an orbit based on both angles and distances. The following list gives the elements published by previous computers:

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
$+54^\circ 25'$	1877 13	0 3023	1 46	$57^\circ 95'$	$60^\circ 72'$	$156^\circ 35'$	Doberck, 1879	A N, 2287
$+45^\circ 39'$	1880 142	0 2139	1 369	$62^\circ 11'$	$67^\circ 01'$	$181^\circ 97'$	Leuschner, 1889	Pub A S P, p 46
$+48^\circ 65'$	1839 585	0 14853	1 2807	$63^\circ 38'$	$65^\circ 18'$	$182^\circ 05'$	Celoria, 1890	A N, 2949
$42^\circ 09'$	1880 43	0 16922	1 356	$62^\circ 65'$	$63^\circ 82'$	$183^\circ 87'$	Hall, 1894	A J, No 324





We find the following elements of  $\mu^1$  *Herculis* BC.

$$\begin{array}{ll} P = 45.0 \text{ years} & \Omega = 61^\circ 4' \\ T = 1879.80 & i = 64^\circ 28' \\ e = 0.219 & \lambda = 180^\circ 0' \\ a = 1''.390 & n = +8^\circ 0' \end{array}$$

Apparent orbit.

$$\begin{array}{ll} \text{Length of major axis} & = 2''.78 \\ \text{Length of minor axis} & = 1''.148 \\ \text{Angle of major axis} & = 61^\circ 4' \\ \text{Angle of periastron} & = 241^\circ 4' \\ \text{Distance of star from centre} & = 0''.304 \end{array}$$

The period here given can hardly be in error by more than one year, while the uncertainty of the eccentricity probably does not surpass  $\pm 0.02$ . The elements are therefore well defined, and may indeed be regarded as extraordinarily good for an object of such difficulty.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1857.47	63.0	61.8	—	1.69	+1.2	—	1	Dawes
1857.56	60.1	62.0	1.75 ±	1.69	-1.9	+0.06	1	Dawes
1859.70	62.0	66.9	1.73	1.65	-4.9	+0.08	1-2	Dawes
1860.30	67.7	68.4	1.64	1.63	-0.7	+0.01	1	O. Struve
1862.83	78.5	75.3	1.50	1.46	+3.2	+0.04	1	O. Struve
1864.56	78.2	81.1	1.76	1.30	-2.9	+0.46	2-3	Dawes 1, Englemann 0-1, Winn. 1
1865.44	81.3	84.9	1.55	1.20	-3.6	+0.35	7-6	Knott 2-1, Dembowski 5
1866.68	89.5	91.2	1.10	1.05	-1.7	+0.05	2	O. Struve
1867.58	95.4	97.2	—	0.94	-1.8	—	4	Searle 3, Winlock 1
1868.56	102.0	105.1	0.88	0.82	-3.1	+0.06	2-1	O. Struve 1, Winlock 1-0
1869.73	121.3	118.0	—	0.69	+3.3	—	3	Winlock 1, Peirce 2
1871.52	156.8	148.5	0.62	0.57	+8.3	+0.05	1	O. Struve
1873.50	185.5	182.3	0.63	0.60	+3.2	+0.03	1	O. Struve
1874.48	202.4	200.5	0.76	0.70	+1.9	+0.06	4-2	Newcomb
1875.66	217.8	213.6	1.18	0.82	+4.2	+0.36	12-3	Sch. 6-0, Hall 5-3, Holden 1
1876.62	219.7	221.6	0.85	0.92	-1.9	-0.07	8-12	Hall 4, OΣ 0-4, Dembowski 4
1877.59	231.0	228.4	0.96	1.00	+2.6	-0.04	13-8	OΣ 2, Sch 5-0, Hall 2, Dem 4
1878.50	235.6	234.0	1.11	1.06	+1.6	+0.05	8-7	β 6, Hall 2-0, OΣ 0-1
1879.50	239.0	239.7	0.94	1.08	-0.7	-0.14	19-8	β 5, Hall 3, Seabroke 11-0
1880.63	246.2	245.9	1.05	1.07	+0.3	-0.02	14	β 7, Hall 4, Frisby 3
1881.49	250.6	251.0	0.97	1.03	-0.4	-0.06	10	β 5, Hall 5
1882.55	261.2	258.0	0.97	0.96	+3.2	+0.01	18-6	H1 4-0, HΣ 1-0, Ho 3, OΣ 3, Sch 7-0
1883.64	264.5	266.5	0.80	0.85	-2.0	-0.05	16-8	β 3, Ho 3, H1 3-0, Frisby 2, Sch 5-0
1884.65	273.0	276.6	0.77	0.74	-3.6	+0.03	4-1	Hall 3-0, OΣ 1
1885.56	288.0	288.5	0.88	0.65	-0.5	+0.23	5-4	Holetschek 2-1, Hall 3
1886.60	302.1	305.5	0.39	0.58	-3.4	-0.19	5	Hall
1887.56	319.9	324.4	0.49	0.55	-4.5	-0.06	9-5	Schiaparelli 6-5, Hall 3-0
1888.57	342.3	343.8	0.44	0.58	-1.5	-0.14	15-11	Tarrant 0-2, Sch 11-9, Hall 4-0
1889.58	359.3	0.7	0.57	0.66	-1.4	-0.09	8-7	β 4, Schiaparelli 0-3, Hall 4-0
1890.57	12.5	12.0	0.62	0.75	+0.5	-0.13	11-7	β 4, Hall 4-0, Schiaparelli 3
1891.57	23.3	22.5	0.90	0.87	+0.8	+0.03	9-3	Schiaparelli 2-0, Hall 4-0, Big 3
1892.62	30.4	29.9	0.89	1.00	+0.5	-0.11	14-5	Com 4-0, Sch 5-4, Big 1, Hall 4-0
1893.62	36.0	35.4	0.90	1.12	+0.6	-0.22	1	Bigourdan
1894.55	39.9	39.7	1.17	1.23	+0.2	-0.06	17-13	Bar 7, Ho 4-0; Stone 3, Com 3 [See 1
1895.55	43.3	43.5	1.34	1.33	-0.2	+0.01	9-1	See 1-0, Sch 2-0, Com 3-0, See 2-0,

We remark the star is now wider than most observers have indicated by their recent measures. The distance for 1895 is based upon two nights' work, one of the observations being taken by SCHIAPARELLI, the other by the writer at Madison and accidentally omitted in *Astronomical Journal*, No 359. This observation is.

1895 732     $43^{\circ} 2$      $1'' 34$      $1n$     See

The images are noted as "good but faint" There is no doubt that the distance is now at least  $1''.3$ , and it will increase for some years. Observers should follow this system carefully. The following is an ephemeris

$t$	$\theta_o$	$\rho_o$	$t$	$\theta_o$	$\rho_o$
1896 60	$47^{\circ} 0$	$1.43''$	1899 60	$55^{\circ} 1$	$1.63''$
1897 60	$49^{\circ} 9$	$1.51''$	1900 60	$57^{\circ} 5$	$1.67''$
1898 60	$52^{\circ} 6$	$1.58''$			

### $\tau$ OPHIUCHI = $\Sigma 2262$ .

$\alpha = 17^h 57^m 6$  ,  $\delta = -8^{\circ} 11'$   
5, yellowish , 6, yellowish

*Discovered by Sir William Herschel, April 28, 1783*

#### OBSERVATIONS

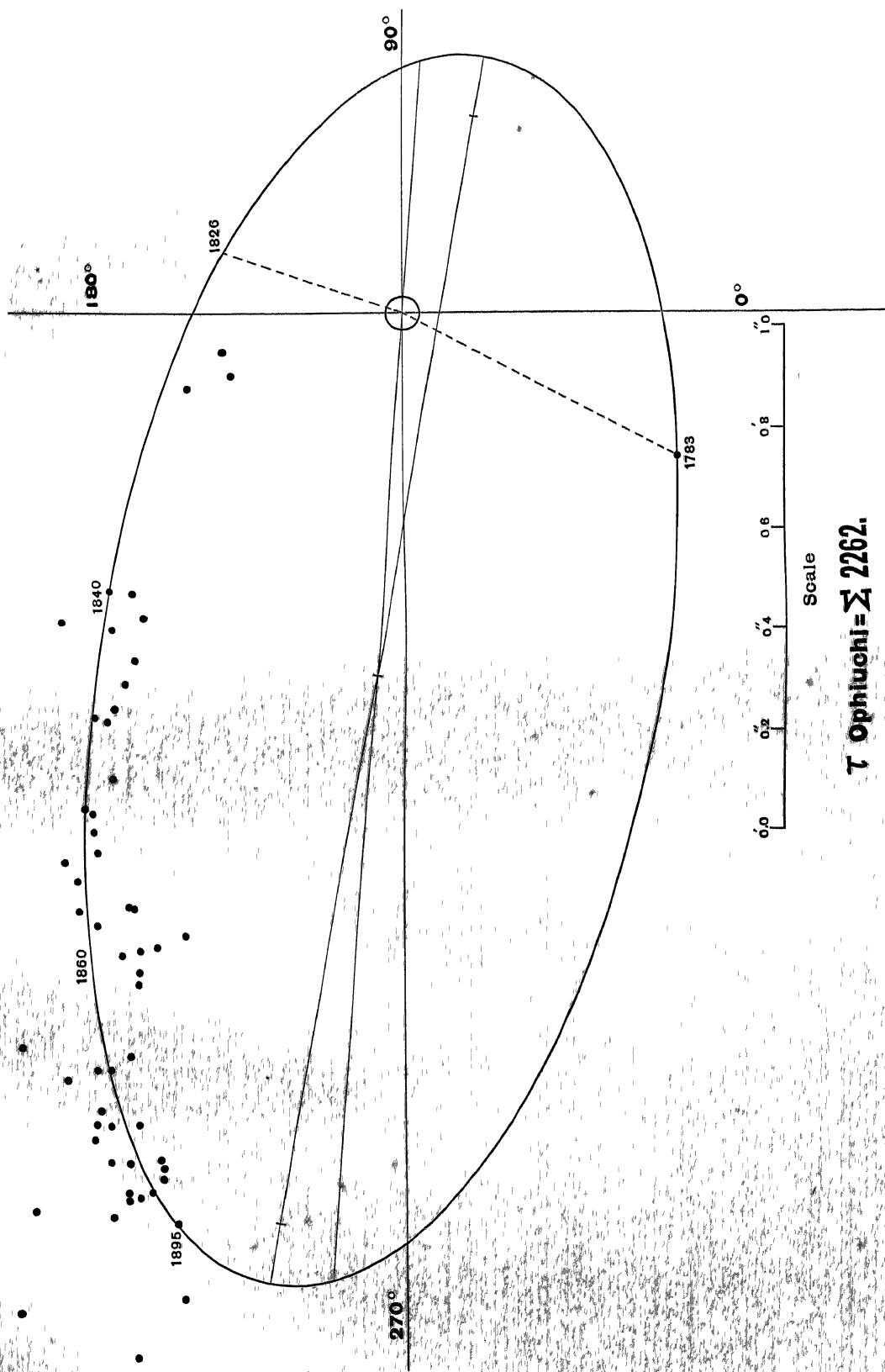
$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Observers
1783 34	$331^{\circ} 6$	elong	1	Herschel	1843 11	$224^{\circ} 6$	$0.80''$	—	Kaiser
1802 74	$360 \pm$	elong	1	Herschel	1843 54	$228^{\circ} 8$	$0.80''$	11	Mädler
1804 44	$360 \pm$	elong	1	Herschel	1843 61	$229^{\circ} 0$	$0.95 \pm$	2	Dawes
1825 71	$176^{\circ} 0$	cuneata	1	Struve	1844 34	$229^{\circ} 8$	$0.79''$	2	Mädler
1827 28	$146^{\circ} 0$	oblonga	1	Struve	1844 74	$218^{\circ} 7$	$0.79''$	1	Challis
1835 68	$192^{\circ} 9$	$0.35''$	6-2	Struve	1845 65	$232^{\circ} 4$	$0.87''$	1	O Struve
1836 62	$199^{\circ} 9$	$0.44''$	5	Struve	1846 22	$239^{\circ} 5$	$1.00''$	—	Jacob
1837 70	$200^{\circ} 8$	$0.35''$	1	Struve	1846 51	$229^{\circ} 4$	$0.78''$	8	Mitchell
1840 51	$223^{\circ} 1$	$0.94''$	1	O Struve	1846 69	$230^{\circ} 7$	$0.97''$	2	O Struve
1840 68	$221^{\circ} 5$	$0.88''$	4-1	Dawes	1847 82	$233^{\circ} 9$	$0.97''$	1	O Struve
1841 53	$217^{\circ} 3$	$0.75''$	8	Mädler	1848 10	$229^{\circ} 7$	$1.18''$	2	Mitchell
1841 60	$228^{\circ} 1$	$0.87''$	3-2	O Struve	1848 66	$232^{\circ} 7$	$1.01''$	1	Dawes
1841 66	$225^{\circ} 7$	$0.79''$	5-1	Dawes	1850 77	$234^{\circ} 0$	$1.0''$	21	Jacob
1842 57	$225^{\circ} 6$	$0.77''$	5	Mädler	1851 66	$239^{\circ} 4$	$1.0''$	—	Fletcher
1842 64	$226^{\circ} 9$	—	1	Dawes	1851 67	$238^{\circ} 2$	$1.19''$	1	O Struve

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1852 65	239 5	1 10	2	Jacob	1870 04	247 3	1 43	8	Dembowski
1852 65	239 7	1 23	2	O Struve	1870 71	250 7	1 26	1	Dunér
1852 66	238 6	1 27	4-3	Madler	1871 66	251 0	1 31	2	Dunér
1853 79	238 3	1 17	4	Madler	1872 01	247 8	1 55	8	Dembowski
1854 67	238 0	1 22	1	Dawes	1872 58	248 1	1 69	1	O. Struve
1854 70	236 1	1 20	1	O Struve	1873 54	248 9	2 12	1	Leyton Obs
1854 74	238 2	1 09	3	Madler	1874 08	248 5	1 60	8	Dembowski
1855 49	238 1	1 30	3	Dembowski	1874 57	250 7	1 48	1	Leyton Obs
1855 55	236 9	1 27	2	Secchi	1874 67	251 1	1 63	1	O Struve
1855 67	240 4	1 31	2	O Struve	1875 61	248 9	1 61	8	Schiaparelli
1856 24	240 7	1 20	4	Secchi	1876 02	249 3	1 67	10	Dembowski
1856 58	240 5	1 20	6	Dembowski	1876 60	247 6	1 73	3	Schiaparelli
1856 62	242 6	1 29	1	Winnecke	1876 62	250 4	2 05	1	Stone
1857 55	239 6	1 26	3	Secchi	1876 64	251 1	1 72	3	Hall
1857 63	241 4	1 20	4	Dembowski	1876 67	248 2	1 78	1	Waldo
1857 67	240 2	1 44	2	O Struve	1876 70	246 5	1 58	1	O Struve
1858 20	243 6	1 41	—	Jacob	1877 55	249 0	1 53	4	Hall
1858 52	241 8	1 20	6	Dembowski	1877 61	250 5	1 90	8	Cincinnati
1858 64	240 7	1 33	3	Madler	1877 66	248 6	1 64	7	Schiaparelli
1858 71	240 9	1 47	1	O Struve	1878 02	250 4	1 72	8	Dembowski
1859 63	242 7	1 64	1	O Struve	1878 52	254.1	1 69	2	Doberck
1860 77	245 8	1.30	1	Secchi	1879 35	247 9	1 63	2	Burnham
1861 60	244.4	1 29	3	Madler	1879 41	250 1	1 78	26-25	Cincinnati
1861 63	242.9	1 43	1	O Struve	1879 72	250 3	1 74	5	Schiaparelli
1863 05	244 6	1 40	13	Dembowski	1880 07	249.7	1 78	3	Cincinnati
1863 57	246 5	1 20	4	Knott	1880 65	251 6	1 80	6	Schiaparelli
1864 47	247 8	1 92	2	Englemann	1880 66	251 1	1 64	2	Hall
1865 52	249 4	1 40	—	Kaiser	1880 67	252 2	1 89	3	Jedrzejewicz
1865 60	243 1	1 23	1-2	Leyton Obs	1881 55	251 3	1 71	3	Hall
1865 72	244 1	1 51	1	O Struve	1881 79	252 7	1 67	2	Smith
1865 89	245 9	1 42	13	Dembowski	1882 49	252 0	2.05	3	H C Wilson
1866 43	246 3	1 66	3-2	Leyton Obs	1882 54	253 3	1 73	3	Hall
1866 58	247 5	2 48	3-2	Winlock	1882 60	252 1	1 86	7	Schiaparelli
1866 59	247 7	1 65	2-3	Searle	1882 62	250 8	2 13	1	O Struve
1866 62	243 3	1 75	1	O Struve	1883 38	254.5	1 84	9	Englemann
1866 72	247 6	1 60	2	Secchi	1883 51	252 1	1 66	3	Perrotin
1867 56	251 5	2 49	2-1	Winlock	1883 53	253 0	2 37	2-1	H C Wilson
1867 98	246 0	1 43	9	Dembowski	1883 55	253 8	1 60	1	Seabroke
1868 57	247 6	1 29	3	C S Peirce	1883 58	253 4	1 78	5	Hall
1868 58	246 4	—	1	Leyton Obs	1883 61	252.0	1 83	6	Schiaparelli
1868 61	249 5	1 44	1	Winlock	1883.66	254 8	1 79	3	Jedrzejewicz
1869 56	248 4	—	1	Leyton Obs	1884 41	253 5	1 94	1	H C Wilson
1869.64	248 2	1 41	6	Dunér	1884 60	253 0	1 82	3	Hall
1869 73	245 0	1 41	1	C S Peirce	1884 78	251.6	1.74	6	Schiaparelli

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1885 48	258 1	1 79	3	Tarrant	1890 57	254 6	1 78	1	Hayn
1885 56	253 5	1 66	4	Hall	1891 48	257 6	2 0 $\pm$	1	See
1885 57	251 2	1 76	4	de Ball	1892 65	255 2	1 75	4	Schiaparelli
1885 58	256 0	2 01	5	Jedrzejewicz	1892 58	254 6	1 78	4	Comstock
1886 22	254 8	1 98	7	Englemann	1893 50	254 1	1 81	3	Maw
1886 54	254 0	1 67	4	Hall	1893 70	254 7	1 83	1	Bigoudan
1886 62	256 2	1 85	6	Jedrzejewicz	1894 59	254 4	1 88	2	Glasenapp
1887 09	252 0	1 72	4	Schiaparelli	1894 77	254 7	1 64	3	Comstock
1887 57	252 5	1 81	4	Hall	1894 78	253 2	1 91	1	Bigoudan
1888 56	253 1	1 70	5	Hall	1895 56	256 1	1 78	3	Schiaparelli
1888 61	254 4	1 71	4	Schiaparelli	1895 58	255 4	1 98	2	Collins
1888 71	255 2	1 80	3	Maw	1895 59	253 4	1 94	5	Schwarzschild
1889 57	255 6	2 23	2	Glasenapp	1895 72	254 7	1 86	4	See
1889 68	253 5	1 69	1	Schiaparelli	1895 72	257 8	1 90 $\pm$	2	Moulton

Since the discovery of this double star in 1783, the radius vector of the companion has swept over an arc of  $285^\circ$ . But while the length of the arc would ordinarily be sufficient to fix the character of the orbit, it happens unfortunately in this case that the observations are neither very consistent nor very well distributed over the arc; and since by far the greater number of observed positions lie in the sixty degrees described since 1836, a satisfactory determination of the elements is embarrassed by difficulties of a somewhat formidable character. But when we examine HERSCHEL's angle of 1783 in the light of his remarks, there seems to be every reason to regard it as fairly correct. In his notes on the observation of  $\tau$  *Ophiuchi*, he says. "The closest of all my double stars can only be suspected with 460, but 932 confirms it to be a double star. It is wedge-formed with 460; with 932 one-half of the small star, if not three-quarters, seems to be behind the large star. The morning is so fine that I can hardly doubt the reality; but according to custom, I shall put it down as a phenomenon that may be a deception." If we depend on the approximate accuracy of HERSCHEL's earliest measure, and deduce the areal velocity from the most recent observations, where both angles and distances can be relied upon, we are led to an orbit which will not differ greatly from the truth. The following orbits have been published by previous investigators:

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
87 036	1840 07	0 03746	0 8178	55 5	51 47	145 40	Madler, 1847	Fixt Syst, I, 255
120 0	1824 8	0 575	—	130 0	48 30	146 6	Hind, 1849	M N, IX, p 145
185 2	1820 63	0 5818	1 111	69 31	53 5	28 35	Doberck, 1877	A N, 2037
217 87	1818 50	0 6055	1 193	67 1	46 8	36 26	Doberck, 1877	A N, 2041



$\tau$  Ophiuchi =  $\Sigma$  2262.



We find the following elements of  $\tau$  *Ophiuchi*.

$$\begin{array}{ll} P = 230.0 \text{ years} & \Omega = 76^\circ 4 \\ T = 1815.0 & i = 57^\circ 6 \\ e = 0.592 & \lambda = 18^\circ 05 \\ a = 1'' 2495 & n = +1^\circ 5652 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 2'' 46 \\ \text{Length of minor axis} & = 1'' 09 \\ \text{Angle of major axis} & = 80^\circ 0 \\ \text{Angle of periastron} & = 85^\circ 8 \\ \text{Distance of star from centre} & = 0'' 712 \end{array}$$

The accompanying table shows that this orbit gives a very satisfactory representation of both angles and distances.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1783 34	331.6	313.7	elongated	0.75	+17.9	—	1	Heischel
1802 74	360 ±	31.8	elongated	0.49	—31.8	—	1	Heischel
1804 44	360 ±	45.5	elongated	0.51	—45.5	—	1	Herschel
1826 50	161.0	164.6	oblonga	0.37	— 3.6	—	2	Struve
1835 68	192.9	211.2	0.35	0.61	—18.3	—0.26	6-2	Struve
1836 62	199.9	213.8	0.44	0.64	—13.9	—0.20	5	Struve
1837 70	200.8	216.6	0.35	0.68	—15.8	—0.33	1	Struve
1840 60	222.3	222.3	0.91	0.78	± 0.0	+0.13	5-2	O Struve, Dawes 4-1
1841 60	223.7	224.2	0.80	0.82	— 0.5	—0.02	16-11	Madler 8, OΣ 3-2, Dawes 5-1
1842 60	226.2	225.7	0.77	0.84	+ 0.5	—0.07	6-5	Madler 5, Dawes 1-0
1843 41	227.5	227.0	0.85	0.88	+ 0.5	—0.03	12+	Kaiser —, Madler 11, Dawes 2
1844 54	229.8	228.3	0.79	0.91	+ 1.5	—0.12	2-3	Madler 2, Challis 0-1
1845 65	232.4	229.9	0.87	0.95	+ 2.5	—0.08	1	O Struve
1846 47	233.2	230.8	0.92	0.98	+ 2.4	—0.06	10+	Jacob —, Mitchell 8, OΣ 2
1847 82	233.9	232.4	0.97	1.02	+ 1.5	—0.05	1	O Struve
1848 66	232.7	233.2	1.01	1.04	— 0.5	—0.03	1	Dawes
1850 77	234.0	235.2	1.00	1.10	— 1.2	—0.10	21obs	Jacob
1851 66	238.2	236.0	1.09	1.13	+ 2.2	—0.04	1+	Fletcher —, OΣ 1
1852 65	239.3	236.8	1.20	1.16	+ 2.5	+0.04	8-7	Jacob 2, OΣ 2, Madler 4-3
1853 79	238.3	237.6	1.17	1.19	+ 0.7	—0.02	4	Madler
1854 70	237.4	238.5	1.17	1.22	— 1.1	—0.05	5	Dawes 1, OΣ 1, Madler 3
1855 57	238.5	239.1	1.28	1.24	— 0.6	+0.04	7	Dembowski 3, Secchi 2, OΣ 2
1856 48	240.6	239.7	1.23	1.26	+ 0.9	—0.03	11-10	Secchi 4, Dembowski 6, Winn 1
1857 62	240.4	240.5	1.30	1.30	— 0.1	±0.00	9	Secchi 3, Dembowski 4, OΣ 2
1858 52	241.7	241.1	1.35	1.32	+ 0.6	+0.03	10+	Jacob —, Dem 6, Madler 3, OΣ 1
1859 63	242.7	241.8	1.64	1.34	+ 0.9	+0.30	1	O Struve
1860 77	245.8	242.6	1.30	1.37	+ 3.2	—0.07	1	Secchi
1861 62	243.7	243.3	1.36	1.39	+ 0.4	—0.03	4	Madler 3, OΣ 1
1863 31	245.5	243.9	1.30	1.42	+ 1.6	—0.12	17	Dembowski 13, Knott 4
1864 47	247.8	244.6	1.92	1.45	+ 3.2	+0.47	2	Englemann
1865 68	246.5	245.2	1.39	1.47	+ 1.3	—0.08	14-16	Kaiser —, Ley 1-2, OΣ 1, Dem 13
1866 59	246.5	245.6	1.66	1.49	+ 0.9	+0.17	8	Ley 3-2, Wk 3-0, Si 2-3, OΣ 1,
1867 77	248.7	246.2	1.43	1.51	+ 2.5	—0.08	11-9	Winlock 2-0, Dembowski 9 [Sec 2
1868 59	247.8	246.6	1.37	1.53	+ 1.2	—0.16	4-3	Peirce 3, Leyton 1-0, Winlock 1
1869 64	248.3	247.0	1.41	1.55	+ 1.3	—0.14	7	Leyton 1-0, Dunér 6, Peirce 1
1870 37	249.0	247.3	1.35	1.56	+ 1.7	—0.21	9	Dembowski 8, Dunér 1

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1871 66	251 0	248 0	1 31	1 59	+3 0	-0 28	2	Dunér
1872 30	248 0	248 3	1 62	1 60	-0 3	+0 02	9	Dembowski 8, $O\Sigma$ 1
1873 54	248 9	248 8	2 12	1 62	+0 1	+0 50	1	Leyton Observers
1874 44	250 1	249 1	1 57	1 63	+1 0	-0 06	10	Dembowski 8, Leyton 1, $O\Sigma$ 1
1875 61	248 9	249 6	1 61	1 65	-0 7	-0 04	8	Schiaparelli [ $O\Sigma$ 1
1876 54	249 6	250 0	1 75	1 67	-0 4	+0 08	17-19	Dem 10; Sch 3; St 1; Hl 3; Wdo 1,
1877 61	249 4	250 4	1 69	1 68	-1 0	+0 01	19	Hall 4, Cincinnati 8, Schiaparelli 7
1878 27	250 4	250 6	1 71	1 69	-0 2	+0 02	8-10	Dembowski 8, Doberck 2
1879 49	249 4	251 1	1 72	1 71	-1 7	+0 01	33-32	$\beta$ 2, Cincinnati 26-25, Sch 5
1880 51	251 1	251 5	1 78	1 72	-0 4	+0 06	14	Cin 3, Sch 6, Hall 2, Jed 3
1881 67	252 0	251 9	1 69	1 74	+0 1	-0 05	5	Hall 3, Smith 2
1882 56	252 1	252 2	1 88	1 75	-0 1	+0 13	14-13	H C W 3, Hl 3, Sch 7, $O\Sigma$ 1-0
1883 53	252 8	252 6	1 84	1 76	+0 2	+0 08	17-28	En 9, Per 3, H C W 2-1, Sea 1, Hl 5,
1884 60	252 7	252 9	1 83	1 77	-0 2	+0 06	10	H C W 1, Hl 3; Sch 6 [Sch 6; Jed 3
1885 55	253 5	253 2	1 81	1 78	+0 3	+0 03	13-16	Tar 3, Hall 4; deBall 4, Jed 5
1886 46	254 4	253 6	1 83	1 80	+0 8	+0 03	11-17	Englemann 7, Hall 4, Jed 6
1887 33	252 3	253 9	1 77	1 81	-1 6	-0 04	8	Schiaparelli 4, Hall 4
1888 64	254 2	254 3	1 75	1 82	-0 1	-0 07	8	Hall 5, Maw 3
1889 57	255 6	254 6	2 13	1 83	+1 0	+0 30	2-1	Glasenapp
1890 57	254 6	254 9	1 78	1 84	-0 3	-0 06	1	Hayn
1891 48	257 6	255 2	2 $\pm$	1 85	2 4	+0 15 $\pm$	1	See
1892 58	254 6	255 5	1 78	1 85	-0 9	-0 07	4	Comstock
1893 50	254 1	255 8	1 81	1 86	-1 7	-0 05	3	Maw
1894 68	254 5	256 2	1 76	1 87	-1 7	-0 11	5	Glasenapp 2; Comstock 3
1895 72	256 2	256 5	1 86	1 88	-0 3	-0 02	6-4	See 4, Moulton 2-0

The following is an ephemeris for the next five years:

$t$	$\theta_o$	$\rho_o$	$t$	$\theta_o$	$\rho_o$
1896 50	256 7	1 88	1899 50	257 6	1 90
1897 50	257 0	1 89	1900 50	257 9	1 91
1898 50	257 3	1 90			

It will be evident from what has been said that this orbit is still open to some uncertainty, but a material improvement in the elements will not be possible for many years. Since the companion is at present nearing the apastron of the apparent ellipse, the motion will continue to be very slow; yet the pair will still be worthy of occasional attention from observers. While the period found above is perhaps uncertain to the extent of 15 years, it does not seem probable that the eccentricity can be in error by more than  $\pm 0.05$ . Accordingly there is no probability that even the lapse of ages will radically change these elements of  $\tau$  Ophiuchi.

70 OPHIUCHI =  $\Sigma 2272$ . $\alpha = 18^{\text{h}} 0^{\text{m}} 4$  ,  $\delta = +2^{\circ} 33'$ 

4 5, yellow , 6, purplish

*Discovered by Sir William Herschel, August 7, 1779.*

## OBSERVATIONS

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1779 77	90	—	1	Herschel	1836 42	128 9	6 38	8	Madler
1781 74	80 8	4 45	1-2	Herschel	1836 51	127 7	6 48	4	Encke
1802 34	336 1	—	1	Herschel	1836 52	129 5	6 34	5	Bessel
1804 42	318 8	—	2	Herschel	1836 66	129 5	6 15	8	Struve
1819 64	168 5	—	5	Struve	1837 13	127 7	6 47	3	Dawes
1820 77	160 2	—	2	Struve	1837 46	128 3	6 74	4	Encke
1821 74	157 6	—	5	Struve	1837 60	127 5	6 46	16	Bessel
1822 42	154 8	4 27	2	H and So	1837 72	128 0	6 15	4	Struve
1822 64	153 9	—	3	Struve	1838 57	126 6	6 64	7	Galle
1825 56	148 1	4 76	14	South	1839 52	125 2	6 78	2	Galle
1825 57	148 2	3 98	14	Struve	1839 65	125 9	6 55	2	Dawes
1827 02	145 1	4 37	2	Struve	1840 35	128 0	6 00	—	Kaiser
1828 58	140 6	5 37	1	Herschel	1840 48	126 6	6 52	10	O Struve
1828 71	140 2	4 78	4	Struve	1840 59	124 9	6 63	4	Dawes
1829 59	138 1	5 08	6	Struve	1841 50	125 4	6 40	8	Madler
1829 60	140 6	—	1	Herschel	1841 65	123 4	6 54	5	Kaiser
1830 39	138 2	6 01	9	Herschel	1841 68	123 4	6 63	4	Dawes
1830 50	135 8	5 47	10	Bessel	1841 74	123 8	6 85	7	Be and Sel
1830 57	137 3	5 53	6	Dawes	1842 31	125 1	6 63	8	O Struve
1830 84	135 7	5 31	2	Struve	1842 53	124 6	6 25	3	Madler
1831 53	136 5	5 94	8-6	Herschel	1842 53	123 3	6 72	2	Dawes
1831 53	134 0	5 68	7	Bessel	1842 59	122 6	6 48	22	Kaiser
1831 68	134 7	5 41	5	Struve	1842 60	123 5	6 79	18	Schlüter
1832 55	133 8	5 71	3	Dawes	1843 47	122 0	—	1	Dawes
1832 57	135 4	5 35	4-3	Herschel	1843 52	121 1	6 70	3	Encke
1832 69	133 0	5 79	5	Bessel	1843 58	123 1	6 44	16	Madler
1833 42	132 8	6 14	1	Dawes	1844 36	120 7	6 84	5	Encke
1834 47	131 1	5 85	4	Struve	1844 52	122 0	6 48	5	Madler
1834 57	130 6	6 13	7	Dawes	1845 43	120 8	6 77	9	Hind
1834 61	130 8	6 13	7	Bessel	1845 48	121 0	6 56	5	O Struve
1835 60	130 7	6 11	5	Struve	1845 54	120 8	6 58	16	Madler
					1846 25	120 2	6 83	1	Jacob
					1846 46	120 1	6 14	7	Hind
					1846 56	117 1	7 43	—	Durham obs
					1846 58	119 8	6 65	10	Madler

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1847 25	120 5	6 56	4	O Struve	1857 13	110 6	6 45	3	Jacob
1847 45	117 2	7 19	—	Durham obs	1857 41	112 5	6 19	1	Winnecke
1847 59	120 3	—	1	Mitchell	1857 51	110 4	6 20	4	Secchi
1847 60	118 5	6 79	8	Madler	1857 58	110 3	6 52	2	Dawes
1848 12	118 8	6 80	3	Dawes	1857 64	109 5	6 25	4	Dembowski
1848 49	118 4	6 84	4	Madler	1857 67	110 2	6 15	2	Morton
1848 52	118 0	6 8	2	Bond	1857 69	110 1	6 40	4	O Struve
1849 39	118 1	6 64	5	O Struve	1858 12	109 7	6 10	3	Jacob
1850 42	116 8	6 88	8	Radcliffe	1858 39	108 6	6 08	2	Morton
1850 49	115 2	6 86	2	Worster & Ja	1858 44	109 3	6 04	4	Dembowski
1850 64	116 7	6 94	4	Madler	1858 63	108 9	5 83	9	Madler
1850 66	117 0	6 46	4	Fletcher	1859 30	109 0	6 20	5	O Struve
1851 20	115 2	6 65	4	Madler	1859 72	109 3	6 24	4	Dawes
1851 58	115 8	6 38	8	Fletcher	1859 75	109 0	6 44	5	Auwers
1851 67	115 4	6 34	5	O Struve	1859 76	107 8	6 10	5	Powell
1851 73	115 5	6 67	7	Madler	1859 80	107 0	6 25	1	Madler
1852 63	116 0	6 36	6	Fletcher	1860 61	106 3	6 07	3	Secchi
1852 67	115 0	6 55	5	O Struve	1860 74	109 0	6 41	—	Luther
1852 71	114 7	6 56	11	Madler	1860 76	106 7	6 52	5	Auwers
1852 74	114 0	6 73	15	Jacob	1861 46	107 0	5 89	1	Radcliffe
1853 55	113 6	—	9	Powell	1861 69	106 6	5 92	7	Madler
1853 55	116 5	6 36	6	Dembowski	1861 74	106 0	6 21	6	Auwers
1853 62	114 6	6 49	6	Dawes	1861 81	105 4	5 8	3	Powell
1854 08	113 6	6 36	21	Jacob	1862 40	105 6	5 86	3	O Struve
1854 24	113 0	6 51	2	Jacob	1862 55	106 0	6 05	1	Winnecke
1854 24	113 3	6 51	6	O Struve	1862 62	105 5	5 72	9	Dembowski
1854 64	113 4	6 23	12	Dembowski	1862 72	105 2	5 69	6	Madler
1854 67	113 0	6 27	10	Madler	1863 47	104 0	6 07	11	Adolph
1854 73	113 7	6 34	3	Dawes	1863 51	104 1	5 28	2	Secchi
1854 78	112 9	—	3	Powell	1863 51	104 2	5 60	9	Dembowski
1855 03	115 3	6 86	2	Luther	1863 55	104 5	5 76	1	Talmage
1855 45	111 6	6 25	3	Searle	1863 58	106 2	5 19	1	Ferguson
1855 56	114 2	6 34	1	Winnecke	1863 64	105 8	5 82	5	Hall
1855 63	112 7	6 33	5	Madler	1864 48	104 8	5 42	2	Englemann
1855 69	113 3	6 47	2	Dawes	1864 60	103 5	5 45	11	Dembowski
1855 75	112 4	—	7	Powell	1865 30	102 6	5 27	8	Englemann
1855 82	—	7 23	1	Schmidt	1865 51	102 7	5 43	4	Secchi
1856 09	111 8	6 44	5	O Struve	1865 51	102 3	5 35	9	Dembowski
1856 33	111 5	6 40	7	Jacob	1865 56	103 9	5 24	2	Talmage
1856 50	111 5	6 32	3	Madler	1865 62	100 6	5 31	20	Kaiser
1856 50	112 6	6 40	8	Winnecke	1866 13	101 6	5 26	8	Dembowski
1856 55	111 2	6 12	3	Secchi	1866 29	101 0	5 29	5	O Struve
1856 63	111 8	6 38	6	Dembowski	1866 49	101 8	5 26	5	Talmage
					1866 54	100 8	5 50	4	Harvard
					1866 69	101 1	5 27	3	Secchi

<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1867 41	98 1	5 33	1	Radcliffe	1875 52	83 7	3 48	9	Dembowski
1867 44	99 8	5 22	2	Knott	1875 62	84 1	3 44	8	Schiaparelli
1867 52	100 5	—	1	Talmage	1875 68	84 8	3 84	4	Radcliffe
1867 57	100 4	5 06	7	Dembowski	1876 48	82 1	3 55	5	Schur
1867 57	99 2	5 10	3	Harvard	1876 52	78 9	3 46	2	Doberck
1868 47	98 4	4 85	7	Dembowski	1876 52	80 9	3 32	7	Dembowski
1868 56	98 5	4 98	2	Knott	1876 54	80 2	3 55	3	Plummer
1868 57	99 9	5 11	2	Radcliffe	1876 59	81 3	3 39	6	Schiaparelli
1868 64	101 1	5 41	1	Talmage	1876 64	80 9	3 56	3	Hall
1868 72	97 6	4 84	4	Dunér	1876 64	81 5	3 27	4	Jedrzejewicz
1868 72	99 1	4 69	2	O Struve	1876 66	79 7	3 72	1	Waldo
1868 90	98 0	4 92	5	Brünnow	1877 51	77 6	3 08	8	Dembowski
1869 68	100 2	5 31	1	Talmage	1877 52	77 6	3 47	2	Doberck
1869 69	96 9	4 59	3	Dunér	1877 55	75 8	3 36	4	Hall
1869 73	98 1	5 12	1	Peirce	1877 58	79 4	3 18	10	Jedrzejewicz
1869 91	96 5	4 70	8	Dembowski	1877 65	78 5	3 39	8	Plummer
1870 51	94 0	4 4	2	Gledhill	1877 66	77 3	3 12	10	Schiaparelli
1870 51	94 1	4 55	8	Dembowski	1877 68	78 5	3 12	4	Cincinnati
1870 52	94 4	4 62	2	Talmage	1877 68	79 5	3 15	4	Schur
1871 48	92 6	4 30	2	W & S	1878 51	74 5	2 96	7	Dembowski
1871 49	94 9	4 42	2	Radcliffe	1878 54	75 3	3 04	3	Seabroke
1871 51	90 8	4 61	2	Peirce	1878 54	75 5	3 03	4	Doberck
1871 53	92 6	4 27	8	Dembowski	1878 72	71 9	3 13	4	Goldney
1871 55	96 7	4 36	1	Talmage	1879 41	69 2	2 84	18	Cincinnati
1871 59	94 9	4 30	3	Knott	1879 50	69 8	2 84	10	Schiaparelli
1871 64	92 7	4 29	3	Gledhill	1879 59	71 3	2 93	5	Hall
1871 72	92 6	4 20	1	Dunér	1879 64	67 9	2 94	5	Cincinnati
1872 47	91 8	4 19	2	Brünnow	1879 65	70 3	3 04	4	Seabroke
1872 49	91 5	4 30	3	Ferrari	1879 66	68 6	3 01	5	Jedrzejewicz
1872 49	90 8	4 28	2	Radcliffe	1880 47	65 8	2 44	3	Doberck
1872 49	90 7	4 04	9	Dembowski	1880 49	62 1	2 69	6	Franz
1872 51	91 5	4 29	3	W & S	1880 57	65 5	2 75	6	Hall
1872 60	93 6	4 08	4	O Struve	1880 66	64 9	2 69	10	Schiaparelli
1873 51	89 5	3 90	1	Gledhill	1880 66	62 8	2 75	6	Jedrzejewicz
1873 51	88 8	3 89	8	Dembowski	1880 74	62 7	2 55	2	Seabroke
1873 51	88 8	4 10	1	W & S	1881 23	61 7	2 80	2	Doberck
1873 55	84 7	3 95	1	Talmage	1881 53	60 6	2 49	5	Hall
1873 71	88 8	4 22	3	Radcliffe	1881 72	56 3	2 45	2	Bigourdan
1874 48	88 8	4 01	4	Radcliffe	1881 77	62 7	2 45	2	Seabroke
1874 57	86 1	3 66	8	Dembowski	1882 49	52 3	2 92	1	Wilson
1874 58	88 6	3 67	1	Talmage	1882 52	55 7	2 29	2	Doberck
1874 69	87 5	3 79	3	O Struve	1882 57	56 1	2 31	7	Hall
1874 73	87 5	3 92	1	Gledhill	1882 61	51 8	2 33	9	Schiaparelli
					1882 62	48 8	2 25	4	Jedrzejewicz
					1882 69	51 2	2 96	3	Seabroke
					1882 72	51 6	2 31	4	Englemann

<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1883 49	45 6	2 28	4	Perrotin	1890 42	338 5	2 40	2	Glasenapp
1883 58	40 0	2 36	8	Seagrave	1890 49	338 3	2 42	8	Giacomelli
1883 62	43 7	2 21	15	Schiaparelli	1890 56	335 8	2 13	7	Hall
1883 64	42 2	2 22	6	Jedrzejewicz	1890 61	336 5	2 01	3	Maw
1883 68	45 2	2 51	3	Küstner	1890 61	336 6	2 16	1	Wellmann
1883 68	44 0	2 30	3	Seabrooke	1890 70	334 8	2 02	6	Schur
1883 72	43 6	2 25	6	Englemann	1890 70	334 9	2 22	16	Bigourdan
1884 41	37 6	2 30	1	Wilson	1890 73	336 1	2 15	9	Schiaparelli
1884 53	35 9	2 18	1	Pritchett	1891 54	328 3	2 11	4	Maw
1884 56	34 5	2 09	6	Perrotin	1891 56	327 5	2 23	6	Hall
1884 59	37 6	2 16	7	Hall	1891 58	329 1	2 16	6	Schur
1884 62	35 3	2 07	8	Schiaparelli	1891 59	326 0	2 33	6	Knorre
1884 69	35 2	2 20	5	Englemann	1891 60	328 5	2 15	6	Schiaparelli
1884 70	34 8	2 45	3-1	Seabroke	1891 63	327 2	2 37	2	See
1885 50	26 0	2 08	4	Perrotin	1891 65	326 7	2 21	9	Bigourdan
1885 55	25 1	1 97	4-2	Sea & Sm	1892 37	321 9	2 28	4	Burnham
1885 57	29 5	1 88	7	Hall	1892 41	320 5	2 36	1	Collins
1885 64	24 3	2 07	8	Englemann	1892 49	321 7	2 26	3	Maw
1885 65	26 5	2 07	2	Schiaparelli	1892 57	321 3	2 19	4	Comstock
1885 71	23 4	2 19	5	Jedrzejewicz	1892 62	319 3	2 25	5	Bigourdan
1886 53	13 8	1 98	7	Hall	1892 64	321 0	2 24	6	Schur
1886 56	15 3	1 97	7	Perrotin	1892 65	320 3	2 22	17	Schiaparelli
1886 66	13 7	2 01	7	Jedrzejewicz	1893 47	313 8	2 22	3	Maw
1886 66	14 1	1 81	14	Schiaparelli	1893 58	313 4	2 41	3	Tucker
1886 67	14 8	1 88	7	Englemann	1893 62	313 6	2 27	4	Schur
1886 67	15 6	2 01	4-2	Smith	1893 62	312 5	2 34	5	Comstock
1887 55	359.6	—	1	Smith	1893 69	309 2	2 22	1	H C Wilson
1887 61	3.6	1 92	6	Hall	1893 70	312 3	2 21	11	Schiaparelli
1887 63	4 3	1.87	18	Schiaparelli	1894 50	309 8	2 47	8	Ebell
1887 81	3 5	1 91	4	Tarrant	1894 54	307 4	2 29	3	Maw
1888 41	352 7	2 07	3	Comstock	1894 59	304 6	2 38	12-11	Knorre
1888 55	354 5	2 17	4	Maw	1894 60	306 3	2 26	4	Schur
1888 57	353.4	2.02	6	Hall	1894 75	302 5	2 30	4	Comstock
1888 62	355 4	2 00	3	Giacomelli	1894 77	301 3	2 45	5-6	Callandreaux
1888 64	355 1	1.88	10-9	Schiaparelli	1894 77	303 2	2 21	6	Schiaparelli
1888.65	352.4	2 14	1	Leavenworth	1894 79	302 5	2 33	5	Bigourdan
1888.66	354 7	2 66	3	Copeland	1895 32	298 6	2 22	3	See
1888.85	353 1	1 92	6	Tarrant	1895 50	298 2	2 53	2	Glasenapp
1889 30	348 7	2 16	2	Burnham	1895 51	301 6	2 31	5	Schur
1889 48	344.9	1.60	2	Hodges	1895 55	298 7	2 14	9	Schiaparelli
1889.50	345 7	2 18	5	Comstock	1895 58	296 9	2 26	4	Maw
1889.57	344 5	2 10	6	Hall	1895 60	297 0	2 35	4	Schwarzschild
1889.64	346 4	1 96	5	Maw	1895 62	295 0	2 24	5	Hough
1889 70	344 9	1 99	17-16	Schiaparelli	1895 70	296 0	2 01	5	See
1889 77	343.6	1 84	4	Schur	1895 72	296 3	2 01	3-1	Moulton

*Researches on the Orbit of 70 Ophiuchi, and on a Periodic Perturbation in the Motion of the System Arising from the Action of an Unseen Body\**

While engaged recently in the observation of double stars at the Leander McCormick Observatory of the University of Virginia, I took occasion to measure 70 *Ophiuchi* on three good nights (*A J.* 349). On comparing the results with SCHUR's ephemeris, four months later, I noticed with surprise that the observed angle was over four degrees in advance of the theoretical place. As the Virginia measures had been made under favorable conditions and with extreme care, it became evident that even the orbit to which PROFESSOR SCHUR had devoted so much attention would need revision. Accordingly, after all the observations had been collected from original sources and tabulated in chronological order, I proceeded to investigate the orbit in the usual manner, and obtained a set of elements very similar to those which BURNHAM has given in *Astronomy and Astrophysics* for June, 1893. On comparing the computed with the observed places there appeared to be a sensible irregularity in the angular motion, and as the observed places were admittedly exact to a very high degree, it was impossible to attribute such large and continued deviations to errors of observation. It was also observed that the sign of  $\theta_o - \theta_c$  showed a peculiar periodicity; the residuals being for many years steadily of one sign, and then as uniformly of the other. After making some unsuccessful efforts to correct the apparent orbit, from which the elements had been derived, by the method of KLINKERFUES, I decided to project the orbit found by SCHUR, so as to compare his apparent ellipse directly with the places given by the mean observations for each year. Though I was aware that SCHUR's orbit had been based wholly on angles of position, I was not a little surprised to find that the distances had been vitiated in the remarkable periodic manner indicated by the pointed ellipse in the accompanying diagram. And since I had uniformly adhered to the use of both angles and distances in deriving the orbits of double stars, it was not allowable to violate the distances as PROFESSOR SCHUR had done, nor could we pass over such remarkable periodic errors in the residuals of the angles. We were thus confronted with a case in which it was apparently impossible to satisfy both angles and distances. A closer examination of the diagram suggested the idea of a periodic perturbation, alternately in angle and then in distance; and the drawing, in conjunction with the computations, enabled me to see that the case is one worthy of special attention. After some delay (*A J.* 358) the additional observations

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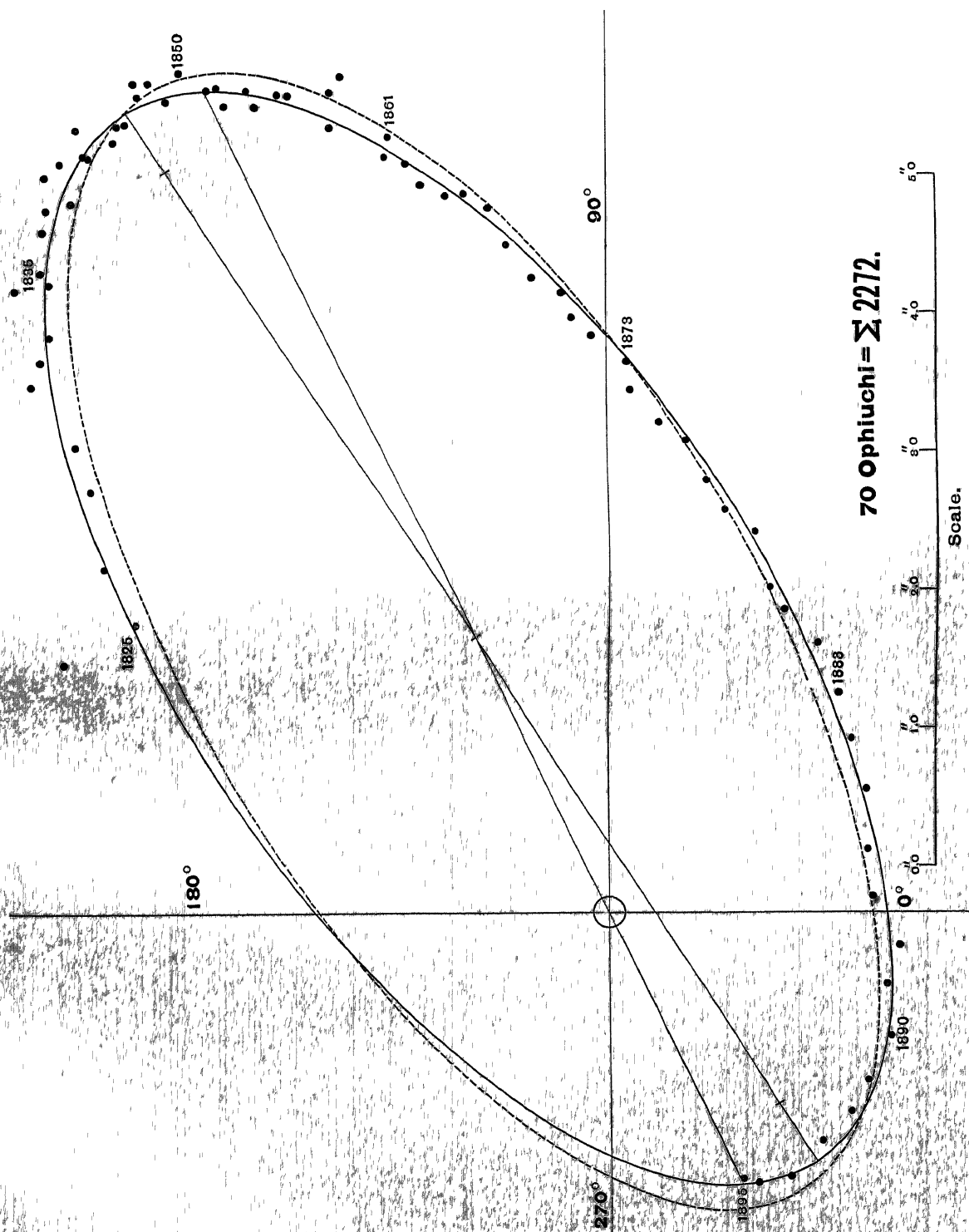
\* *Astronomical Journal*, 363

placed at my disposal by PROFESSORS HOUGH and COMSTOCK, in conjunction with the independent measures made at Madison by MR. MOULTON and myself (A. J. 359) confirmed the correctness of the Virginia measures, and left no doubt of the rapid deviation of the companion from SCHUR's orbit. Before considering the physical cause of this unexpected phenomenon, I desire to remark that, in the preparation of this paper, my friend MR. ERIC DOOLITTLE, C. E., has rendered valuable assistance. He has carried out the calculations entrusted to him not only with care and accuracy, but also with zeal and enthusiasm, and has, therefore, contributed in no small degree to the early completion of this investigation.

Since SIR WILLIAM HERSCHEL's discovery of this beautiful system the companion has described considerably more than one revolution. More orbits have been computed for this binary than for any other in the northern sky, but, in spite of the immense labor which astronomers have bestowed upon this star, the motion has proved to be so refractory and so anomalous that the companion has departed from every orbit heretofore obtained. It follows from the phenomena disclosed in this paper that the system contains a dark body, and that no satisfactory orbit can be obtained until this disturbing cause is taken into account. The following list of the orbits found by previous investigators will be of interest to astronomers; in most cases the data have been taken from original sources, but in a few instances we have relied upon the table of elements given by GORE in his useful "Catalogue of Binary Stars for which Orbits have been Computed."

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
73 862	1806.88	0.430	4.3284	147.2	46.42	283.1	Encke, 1829	B J, 1832
79 091	1814.155	0.34737	5.554	128.15	64.2	259.4	Encke, 1830	B. J., 1832, p 295
80 34	1807.06	0.4667	4.392	137.03	48.1	145.77	Herschel, 1833	Mem R A S, vol V, p 217
80 61	1806.746	0.47715	4.3159	133.8	42.87	287.23	Madler, 1835	A N, 289
92.869	1812.73	0.4438	5.316	126.9	64.86	279.8	Madler, 1842	A N, 444, Dorp Obs., IX, 185
87 52	1807.60	0.482	4.675	128.55	51.5	293.3	Jacob	
88.48	1807.48	0.4973	—	122.23	47.33	294.1	Hind, 1849	M N, IX, p. 145
92.338	1810.671	0.4445	4.966	127.35	61.05	212.97	Villarcceau, 1851	C R, XXXII, p 51
98.146	1806.92	0.546	4.48	111.7	49.93	187.5	Powell, 1855	M N, XV, p 42
98.10	1808.12	0.4894	—	124.53	55.27	159.53	Jacob, 1857	A N, 1082
98.966	1808.27	0.4935	4.731	128.13	57.35	160.53	Klinkerf, 1858	A N, 1135
94.37	1808.79	0.49149	4.704	125.4	57.9	155.7	Schur, 1868	A N, 1682
92.77	1807.9	0.3859	4.88	122.0	62.0	163.0	Flammario 1874	C R, LXXXIX, p 1248
94.35	1809.64	0.47286	4.770	127.37	60.0	149.72	Tisserand, 1876	Flam Cat Et Doub., p 166
94.35	1808.90	0.4672	4.790	127.38	58.08	151.92	Pritchard, 1878	Oxf. Obs., I, p. 63
94.34	1807.65	0.4912	4.50	120.08	58.47	171.75	Gore, 1888	M N, XLVIII, No. 5
94.35	1895.23	0.4994	4.45	120.8	57.0	174.92	Mann, 1890	Sid Mes, Nov, 1890
94.35	1808.0707	0.4751	4.60	121.31	60.08	168.3	Schur, 1893	A N, 3220-21 [1893]
94.35	1895.6	0.50	4.56	123.5	58.3	190.8	Burnham 1893	Astron. and Astroph., June,
94.35	1895.58	0.500	4.548	125.7	58.42	198.25	See 1895	A J, 363





70 Ophiuchi =  $\Sigma 2272$ .



An inspection of this table discloses the fact that the early investigations, so far as they are reliable, led to periods sensibly less than 90 years, while the determinations made between 1845 and 1880, or, when the companion was describing the apastron of the real ellipse, favored a period of at least 94 years. Thus TISSERAND and PRITCHARD, so lately as 1876 and 1878, find periods of 94.93 and 94.44 years, respectively. In 1868 SCHUR obtained a period of 94.37 years, and similar periods before and since have been deduced by other trustworthy computers.

There is thus unmistakable evidence of a retardation in the motion of the companion near apastron, more recently this inequality has become an acceleration. It was observed by GORE in 1888 that the old orbits did not represent recent measures satisfactorily, and, accordingly, he derived a new set of elements with a period of 87.84 years, which was substantially confirmed by subsequent work of MANN and BURNHAM. Finally PROFESSOR SCHUR made an exhaustive investigation of all the observations up to 1893, and adjusted his orbit by the method of least squares to about 400 mean observations of position-angle. He says that in this work he could not advantageously employ the measures of distance, owing to the differences of the individual observers. The angles, however, were admitted to be admirably adapted to a fine determination of the elements, and, accordingly, PROFESSOR SCHUR's able discussion of 400 observations inspired the belief that his orbit would give good places of the companion for a great many years, if not for an almost indefinite period. But this just expectation has not been realized, owing to the action of an unseen body which disturbs the elliptical motion of the companion. To establish the existence and general character of the perturbations thus disclosed we submit the following considerations:

(1) A reference to PROFESSOR SCHUR's able and exhaustive paper in the *Astronomische Nachrichten*, No. 3220, 21, will enable the reader to judge of the improbability of an orbit based on such a multitude of good measures proving to be defective within two years of its completion, unless disturbing causes were at work to produce the sudden acceleration in angular motion. It is inconceivable that this rapid deviation could take place without a true physical cause. The error in the angle now amounts to about five degrees.

(2) In regard to the older observations we may remark, as PROFESSOR SCHUR and others before him have done, that SIR WILLIAM HERSCHEL's angles are open to some uncertainty, owing to a possible error in the reading or in the records, so that his observations do not give an exact or trustworthy criterion for the period. HERSCHEL says, however, explicitly, that on "Oct. 7,

1779, the stars were exactly in the parallel, the following star being the largest," and, as it does not seem that any sensible error could affect the angle which he has thus recorded, we see from the measures in 1872-3 that the resulting period would be approximately 92 years. This is an additional indication that the period of this star is not constant. A careful examination of the other early measures shows that the first really good position is that of STRUVE in 1825. These measures are so uniform and consistent, and appear in every way so worthy of entire confidence, that I quote the record from the *Mensurae Micrometricae* in full.

$t$	$\theta_0$	$\rho_0$		$t$	$\theta_0$	$\rho_0$	
	$^{\circ}$	$''$			$^{\circ}$	$''$	
1825 42	150 1	3 89	4, 6	1825 61	149 3	4 05	
1825 43	147 0	4 05		1825 62	146 8	3 92	
1825 44	149 1	3 94		1825 63	147 3	3 85	
1825 48	148 8	4 05		1825 63	148 4	3 99	
1825 50	146 4	4 21		1825 64	147 0	4 01	
1825 60	148 1	3 90		1825 66	148 5	4 01	4, 6
1825 60	149 5	3 85		1825 71	148 8	4 02	
Mean				1825 56	148 2	3 98	14 <i>n</i> Struve

An examination of these separate measures clearly indicates that the error in the mean result does not surpass  $0^{\circ}.5$  in angle, and  $0'' 1$  in distance. By SCHUR's orbit the angle is corrected two degrees, and when the radius vector is thus thrown forward to  $146^{\circ}.2$  the computed and observed distances are nearly identical. As STRUVE took special pains to secure good measures on a large number of nights, and obtained the foregoing beautiful and consistent results, we may regard his mean position as one of the highest precision. The probable error of such measures would evidently be very small.

(3) We see from the diagram illustrating the apparent ellipse that SCHUR's orbit falls within the positions given by the measures prior to 1845; so that nearly all the observations of STRUVE, BESSEL, DAWES, MADLER, etc., require a sensible negative correction in distance. In figure *B* the differences  $\rho_0 - \rho_0$  of the individual measures used by SCHUR are plotted to scale, and a glance at the figure will show the improbability of such classic observers as STRUVE, BESSEL and DAWES making the constant errors here indicated. It would be still more remarkable if the observers between 1845 and 1870 have as uniformly erred in the opposite direction. How has it happened that from 1825 to 1845 the distances were steadily over-measured by the best observers, while during the next period the distances were constantly under-measured? Individual observers have what may be called a personal equation (though this is far from constant and is difficult to determine with any certainty) but it

could not happen that all the best observers would err alike, although in opposite directions, during the two periods. PROFESSOR SCHUR'S corrections are evidently inadmissible.

(4) The peculiar periodic manner in which SCHUR'S apparent ellipse crosses and re-crosses the general path which best represents the mean positions, first suggested to my mind the hypothesis of a disturbing body. Figure *C* is based upon these mean positions, and a comparison with the curve in *B* shows that the mean positions are typical of all the observations for any given year. Since I was desirous of avoiding any possible prejudice of the material used, I have retained, without alteration, the mean positions which had been formed in August before suspecting the existence of a disturbing influence.

(5) We suggest that the companion of 70 *Ophiuchi* is attended by a dark satellite, and that the visible companion, therefore, moves in a sinuous curve about the common centre of gravity of the new system, with a period somewhat less than 40 years, and in a retrograde direction. As SCHUR'S orbit is based on a least-square adjustment of all the observations extending over two entire revolutions of the invisible body, it may reasonably be inferred that his apparent ellipse will represent very nearly the true motion of the centre of gravity, while the apparent ellipse which best represents the observed distances will give a general outline of the path of the visible star in its sinuous motion. Let us recur to the diagram of the apparent ellipse and imagine that the visible companion and the centre of gravity are in the tangent to the ellipse at the epoch of intersection in 1818. Then, the motion of the visible star being retrograde, we perceive that it will gain steadily on the centre of gravity, and, in 1836, the two will be in line with the original position, after half a sidereal revolution, from 1836 to 1845 the satellite will make another quarter revolution, and again the bright companion will be in the tangent to the apparent ellipse and in advance of the common centre of gravity. As the visible star will now steadily fall behind in its retrograde motion about the centre of gravity, it is clear that from 1845 to 1872, which is three-fourths of a revolution, the motion of the bright body *will appear to be abnormally slow*. This is the apparent retardation previously mentioned as giving rise to the long periods found by computers who used observations extending over the apastron portion of the real orbit. Assuming that the motion is undisturbed, and hence that the areas are constant, PROFESSOR SCHUR was compelled to run his ellipse further out in this part of the orbit in order to represent the observed angles. From the diagram we see that the retrograde motion of the visible star continues after 1872, and, as this apparently accelerates the visible

motion of the companion relative to the central star, SCHUR's ellipse is drawn inside of most of the observations of this period. The falling of the measured distances beyond SCHUR's orbit shows plainly the periodic motion of the visible star in accordance with the above theory. From this sketch of the effects of the disturbing body it is evident that, at the time SCHUR completed his orbit, the visible star and the unseen body were nearly in line with the central star. And since the visible companion in 1825, according to STRUVE, had an angle of  $148^{\circ} 2$ , whereas SCHUR makes it  $146^{\circ} 2$ , or, substantially the same as the centre of gravity at that epoch, it follows that our hypothesis, making SCHUR's orbit represent the motion of the centre of gravity, is indeed very nearly correct. Any slight correction that may be required for the periastron of SCHUR's ellipse in order to make it represent the true path of the centre of gravity, had better be deferred until additional observations disclose more clearly the nature and extent of the perturbations.

(6) We may fix the approximate elements of the visible companion about the centre of gravity as follows: From 1818 to 1890, or 72 years, is the time required for two revolutions, as explained in the preceding paragraph, and hence we see that *the period is approximately thirty-six years*. The motion is retrograde, and from the diagram of the apparent orbit, we may conclude that the distance of the visible star from the common centre of gravity is about  $0''.3$ . It is natural to suppose that the plane of the orbit is not greatly inclined to that found by SCHUR, but existing data will not fix all the elements with the desired precision. Perhaps until the path of the centre of gravity is known with great accuracy, the simple hypothesis of a circular orbit, with node and inclination identical with the similar elements of the visible pair, will be sufficient to explain phenomena, and it follows that *both angles and distances are comparatively well represented by this hypothesis*.

It is found, however, on more detailed examination that the representation can be somewhat improved by the adoption of the following elements:

$P' = 36$ years	$\Omega' = 151^{\circ} 0$
$T' = 1822.0$	$i' = 60^{\circ} 1$
$e' = 0.475$	$\lambda' = 191^{\circ} 7$
$a' = 0''.30$	$n' = 10^{\circ} 0$

While this orbit gives a good representation of the motion of the bright body about the common centre of gravity, the data are so rough that the determination of such delicate elements must be regarded as provisional only.

In the following table we have compared SCHUR's elements with the mean

positions for each year; the residuals are given in the columns headed  $\theta_0 - \theta_1$  and  $\rho_0 - \rho_1$ . It is at once evident that the angles are beautifully represented down to 1893, after which the error in angle rapidly accumulates until it now amounts to nearly *five degrees*! The errors in distance are illustrated in diagram *C*, which shows the same general features as diagram *B*, where the points represent the individual measures employed by SCHUR.

The elements of the orbit which best represents the observed distances are as follows:

$$\begin{array}{lcl} P = 88\,3954 \text{ years} & \left. \vphantom{\begin{array}{l} P \\ T \\ e \\ a \end{array}} \right\} \text{SCHUR's values} & \Omega = 125^\circ 7' \\ T = 1808\,0707 & & i = 58^\circ 42' \\ e = 0.500 & & \lambda = 198^\circ 25' \\ a = 4'' 548 & & n = -4^\circ 0728 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 9'' 00 \\ \text{Length of minor axis} & = 4'' 17 \\ \text{Angle of major axis} & = 122^\circ 9' \\ \text{Angle of periastron} & = 295^\circ 8' \\ \text{Distance of star from centre} & = 2'' 198 \end{array}$$

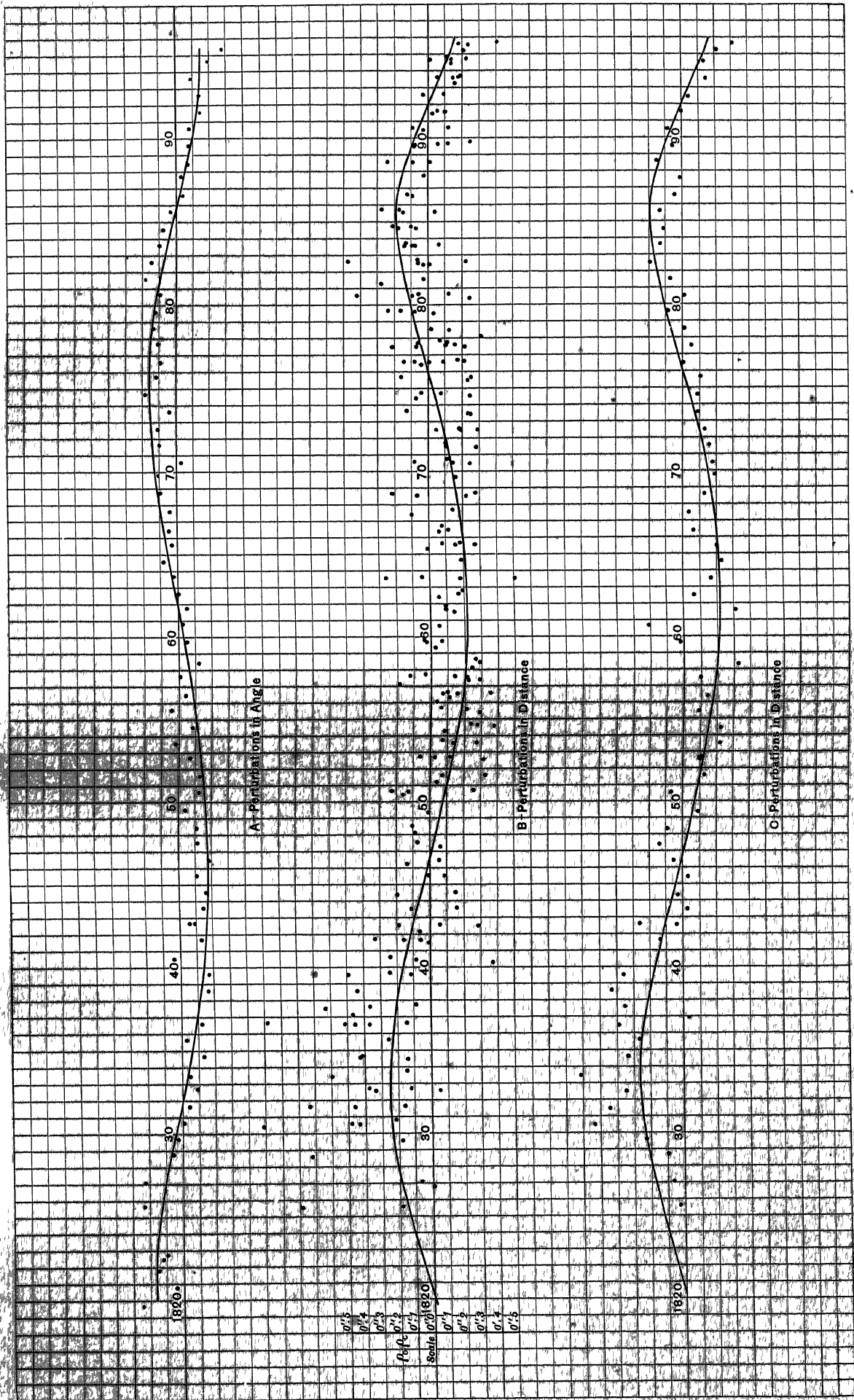
COMPARISON OF COMPUTED WITH OBSERVED PLACES ACCORDING TO THE TWO SETS OF ELEMENTS

$t$	$\theta_0$	$\rho_0$	$\theta_0 - \theta_1$	$\rho_0 - \rho_1$	$\theta_0 - \theta_2$	$\rho_0 - \rho_2$	$d\theta''$	$n$	Observers
1779 77	90 0	—	—	—	—8 11	—	—0 708	1	Herschel
1781 74	99 2	4 49	+1 6	—0 27	+4 40	—0 11	+0 359	1-2	Herschel
1802 34	336 1	—	+1 4	—	+0 71	—	+0 027	1	Herschel
1804 42	318 8	—	—0 3	—	—3 15	—	—0 128	2	Herschel
1819 64	168 5	—	—0 9	—	+5 08	—	—0 244	5	Struve
1820 77	160 2	—	—2 7	—	+0 78	—	+0 042	2	Struve
1821 74	157 6	—	—0 8	—	+2 65	—	+0 154	5	Struve
1822 42	154 8	4 27	—0 8	+1 12	+2 04	+0 72	+0 126	2	Herschel and South
1822 64	153 9	—	—1 0	—	+1 75	—	+0 109	3	Struve
1825 56	148 2	3 98	+2 0	+0 03	+3 17	—0 37	+0 238	28-14	South 14-0, Struve 14
1827 02	145 1	4 37	+2 1	+0 07	+2 82	—0 28	+0 227	2	Struve
1828 71	140 2	4 78	+0 3	+0 10	+0 72	—0 22	+0 062	4	Struve
1829 59	138 1	5 08	—0 3	+0 23	+0 39	—0 15	+0 035	6	Struve
1830 57	136 8	5 58	0 0	+0 54	—0 02	+0 24	—0 002	27	II <sub>2</sub> 9, Bessel 10, Dawes 6, W Struve 2
1831 58	135 1	5 68	—0 3	+0 45	—0 38	+0 36	+0 036	20-18	II 8-6, Bessel 7, W Struve 5
1832 62	133 4	5 75	—0 6	+0 35	—0 84	+0 08	—0 083	8	Dawes 3, Bessel 5
1833 42	132 8	6 14	—0 2	+0 61	—0 34	+0 34	—0 034	1	Dawes
1834 55	130 8	6 04	—0 8	+0 34	—1 25	+0 12	—0 128	18	W Struve 4, Dawes 7, Bessel 7
1835 60	130 7	6 11	+0 2	+0 26	—0 22	+0 07	—0 023	5	Struve
1836 52	128 9	6 34	—0 5	+0 38	—1 08	+0 20	—0 115	25	Madler 8, Encke 4, Bessel 5, W Struve 8
1837 64	127 9	6 45	—0 4	+0 35	—0 96	+0 21	—0 104	27	Dawes 3, Encke 4, Bessel 16, W Struve 4
1838 59	126 6	6 64	—0 7	+0 43	—1 38	+0 32	—0 152	7	Galle
1839 58	125 5	6 66	—0 9	+0 36	—1 51	+0 25	—0 169	4	Galle 2, Dawes 2
1840 47	126 5	6 38	+1 0	0 00	+0 33	—0 09	+0 037	14+	Kaiser —, O Struve 10, Dawes 4
1841 64	124 0	6 60	—0 5	+0 14	—1 13	+0 10	—0 129	26	Madler 8, Kaiser 5, Dawes 4, Be and Schl 7
1842 57	123 8	6 57	+0 1	+0 04	—0 50	—0 01	—0 057	35	O Struve 8, Madler 3, Dawes 2, Kaiser 22
1842 60	123 5	6 79	—0 1	+0 26	—0 78	+0 20	—0 090	20	Schluter
1843 55	122 1	6 57	—0 7	—0 02	—1 26	—0 06	—0 146	20-19	Dawes 1-0, Encke 3, Madler 16
1844 44	121 3	6 66	—0 7	+0 03	—1 36	+0 01	—0 158	10	Encke 5, Madler 5

$t$	$\theta_0$	$\rho_0$	$\theta_0 - \theta_1$	$\rho_0 - \rho_1$	$\theta_0 - \theta_2$	$\rho_0 - \rho_2$	$d\theta''$	$n$	Observers
1845 48	120 9 6 64	-0 3	-0 03	-0 86	-0 03	-0 101	30	Hind 9, O Struve 5, Madler 16	
1846 46	119 3 6 76	-0 7	+0 06	-1 63	+0 07	-0 190	18+	Jacob 1, Hind 7, Dur Obs —, Madler 10	
1847 47	119 1 6 85	-0 3	+0 14	-0 96	+0 16	-0 112	13+	O Struve 4, Dur Obs —, Mitchell 1, Ma 8	
1848 38	118 4 6 81	-0 3	+0 09	-0 96	+0 13	-0 112	9	Dawes 3, Madler 4, Bond 2	
1849 39	118 1 6 64	+0 3	-0 09	-0 31	-0 03	-0 036	5	O Struve	
1850 55	116 4 6 78	-0 5	+0 07	-1 01	+0 13	-0 118	18	Rad 8, W & J 2, Madler 4, Fletcher 4	
1851 54	115 5 6 57	-0 5	-0 13	-1 04	-0 04	-0 121	24	Madler 4, Fletcher 8, O Struve 5, Madler 7	
1852 69	114 9 6 56	-0 2	-0 11	-0 63	0 00	-0 073	37	Fletcher 6, O Struve 5, Madler 11, Jacob 15	
1853 57	114 9 6 42	+0 6	-0 22	+0 12	-0 11	+0 014	21-12	Powell 9-0, Dem 6, Dawes 6 [Po 3 0	
1854 48	113 2 6 37	-0 3	-0 22	-0 71	-0 11	-0 081	57-54	Ja 21, Ja 2, OZ 6, Dem 12, Ma 10, Da 3,	
1855 52	113 3 6 45	+0 6	-0 09	+0 35	+0 04	+0 039	20-13	Lu 2, Si 3, Winn 1, Ma 5, Da 2, Po 7 0	
1856 43	111 7 6 34	+0 1	-0 14	-0 41	0 00	-0 046	32	OZ 5, Ja 7, Ma 3, Winn 8, Sec 3, Dem. 6	
1857 52	111 0 6 31	+0 2	-0 10	-0 04	+0 05	-0 005	20	Ja 3, Winn 1, Sec 4, Da 2, Dem 1, Ma 2,	
1858 39	109 1 6 01	-0 9	-0 32	-1 07	-0 16	-0 116	18	Ja 3, Mo 2, Dem 4, Ma 9 [OZ 4	
1859 66	108 4 6 24	-0 4	+0 02	-0 45	+0 18	-0 048	20	OZ 5, Dawes 4, Auwers 5, Powell 5, Ma 1	
1860 70	107 3 6 33	-0 4	+0 21	-0 29	+0 40	-0 030	8+	Secchi 3, Luther —, Auwers 5	
1861 67	106 2 5 70	-0 5	-0 31	-0 50	-0 14	-0 052	17	Rad 1, Madler 7, Auwers 6, Powell 3	
1862 59	105 6 5 83	-0 2	-0 07	-0 02	+0 09	-0 002	19	O Struve 3, Winnecke 1, Dem 9, Madler 6	
1863 54	104 8 5 62	0 0	-0 17	+0 19	+0 01	+0 019	29	Adh 11, Sec 2, Dem 9, Ta 1, Fer. 1, Ill 5	
1864 54	104 1 5 43	+0 5	-0 23	+0 84	-0 06	+0 082	13	Englemann 2, Dembowski 11	
1865 50	102 4 5 32	0 0	-0 20	+0 22	-0 03	+0 021	43	En 8, Secchi 4, Dem 9, Ta 2, Kaiser 20	
1866 43	101 2 5 31	0 0	-0 07	+0 48	+0 11	+0 044	25	Dem 8, OZ 5, Ta 5, Ilv. 4, Secchi 3	
1867 50	99 6 5 18	-0 2	-0 03	+0 43	+0 14	+0 038	14-13	Rad 1, Kn 2, Ta 1-0, Dem 7, Ilv. 3	
1868 65	98 6 4 90	+0 5	-0 13	+1 26	+0 05	+0 101	22	Dem 7, Kn 2, Rad 2, Du 4, OZ. 2, Brw. 5	
1869 80	96 7 4 64	+0 4	-0 19	+1 32	-0 03	+0 109	11	Dunér 3, Dembowski 8	
1870 51	94 2 4 52	-1 0	-0 18	-0 40	-0 08	-0 032	12	Gledhill 2, Dem 8, Ta 2, [Gl. 3; Du. 1	
1871 56	93 4 4 34	+0 1	-0 16	+1 27	-0 03	+0 099	22	W & S 2, Rad 2, Pel 2, Dem 8, Ta. 1; Kn. 3;	
1872 51	91 6 4 20	+0 2	-0 13	+1 41	-0 01	+0 105	23	Brw 2, Fer 3, Rad 2, Dem 9, W & S. 3; OZ. 1	
1873 56	88 1 4 01	-0 1	-0 09	+0 43	0 00	+0 031	14	Gl 1, Dem 8, W & S 1, Ta 1, Rad. 3	
1874 61	87 7 3 81	+1 1	-0 09	+2 71	+0 01	+0 183	17	Rad 4, Dem 8, Ta 1, OZ 3; Gledhill 1	
1875 61	84 2 3 59	+0 1	-0 10	+1 74	-0 04	+0 113	21	Dem 9, Sch 8, Rad 4 [Jed. 4; Wdo 1	
1876 57	80 7 3 48	-0 6	0 00	+1 53	+0 07	+0 093	31	Sh 5, Dk 2, Dem 7, Pl 3, Sch. 6; Hall 3;	
1877 60	77 4 3 23	-0 5	-0 05	+1 83	+0 02	+0 104	50	Dem 8, Dk 2, Ill 4, Jed 10; Pl. 8; Sch. 10,	
1878 58	74 3 3 05	+0 1	-0 01	+2 49	+0 03	+0 134	18	Dem 7, Sea 3, Dk 4, Gold. 4 [Cin 4; Sh. 4	
1879 57	69 5 2 95	-0 4	+0 09	+2 28	+0 12	+0 115	47	Cin 18, Sch 10, Ill. 5, Cin. 5, Sea. 4; Jed 5	
1880 59	64 0 2 64	-0 9	-0 01	+1 98	-0 01	+0 093	33	Dk 3, Fr 6, Ill 6, Sch 10, Jed. 6; Sea. 2	
1881 56	60 3 2 55	+1 0	+0 08	+3 91	+0 06	+0 172	11	Doberck 2, Hall 5, Big 2; Sea. 2 [En 4	
1882 60	52 5 2 48	+0 2	+0 20	+3 35	+0 16	+0 137	30	II C W 1, Dk 2, Ill 7; Sch 9; Jed. 4; Sea 3,	
1883 62	44 0 2 31	-0 3	+0 18	+2 42	+0 11	+0 094	45	Per 4, Seag 8; Sch 15, Jed. 6, Kü 3, Sea. 3; En 6	
1884 56	36 0 2 17	+0 3	+0 16	+2 01	+0 07	+0 077	31-29	II C W 1, Pr 1, Per 6, Hl. 7, Sch. 8, En. 5; Sea 3-1	
1885 61	25 9 2 06	+0 1	+0 13	+0 72	+0 02	+0 026	30-28	Per 4, Sea 4-2, Ill 7, En 8, Sch 2; Jed. 5	
1886 61	14 3 1 93	-0 9	+0 04	-0 89	-0 07	-0 031	46-44	Hl 7, Per 7, Jed 7, Sch 14, En 7, Sm. 4 2	
1887 68	3 8 1 91	-0 1	+0 01	-1 03	-0 09	-0 036	29-28	Sm 1-0, Ill 6, Sch 18, Tar. 4 [Cop. 3, Tar. 6	
1888 62	353 9 2 11	-0 4	+0 15	-2 10	+0 07	-0 075	36-35	Com 3, Maw 4, Ill 6, Giac 3; Sch. 10-9; Iv. 1,	
1889 53	345 9 2 08	+0 6	+0 06	-2 26	-0 01	-0 082	37-34	$\beta$ 2, Hod 2-0, Com 5, Ill 6, Maw 5; Sch. 17-16	
1890 57	336 7 2 21	+0 6	+0 08	-2 38	+0 04	-0 090	46	Glas 2, Giac 8, Hl 7, Maw 3; Well. 1, Big. 16;	
1891 59	327 4 2 23	-0 6	0 00	-3 58	-0 02	-0 141	33	Maw 4, Hl 6, Knr 6, Sch. 6; See 2, Big. 9 [Sch 9	
1892 52	320 8 2 26	-0 4	-0 04	-3 52	-0 05	-0 142	34	$\beta$ 4, Col 1, Maw 3, Com. 4; Big. 5; Sch. 17	
1893 62	312 9 2 25	-0 8	-0 15	-2 30	-0 10	-0 094	19-20	Maw 3, Com 5, II C W 0-1; Sch. 11	
1894 69	304 2 2 30	-2 7	-0 14	-4 80	-0 03	-0 195	30-29	Maw 3, Knr 12-11, Com. 4; Sch. 6; Big. 5	
1895 32	298 6 2 22	-4 3	-0 21	-6 98	-0 09	-0 280	3	See	
1895 64	296 1 2 14	-4 8	-0 28	-5 62	-0 12	-0 221	20-18	Maw 4, Com 3, Ho 5, See 5; Moulton 3-1	

The values of  $P$  and  $T$  are taken from SCHUR's orbit, because the values of these elements derived from so many observations may be regarded as very nearly the mean of all the periods and epochs which result from the observa-







tions prior to 1893. The residuals which follow from the use of these elements are given in the columns marked  $\theta_0 - \theta_2$  and  $\rho_0 - \rho_2$ . In the case of the second elements the periodic errors in angle are very noticeable, but, as the simple differences  $\theta_0 - \theta_2$  would not be strictly comparable at different distances, we have reduced all these angular displacements to seconds of the arc of a great circle by the formula

$$\frac{r''(\theta_0 - \theta_2)^\circ}{57^\circ 3} = d\theta''$$

where  $r''$  denotes the apparent length of the radius vector in seconds of arc, and  $(\theta_0 - \theta_2)^\circ$  the residuals of position-angle expressed in degrees. The displacement  $d\theta''$  is tabulated and also illustrated graphically in diagram *A*. It will be seen that the maximum or minimum displacement in angle is practically identical in time with the zero of the curves of distance in *B* and *C*, and that the zero of the curve of angles corresponds to the maximum or minimum of the curve of distances. This displacement of phase would be a necessary consequence of the orbital motion of the visible companion about the common centre of gravity, and may be said to establish completely the reality of that phenomenon. The present theory does not require the several phases of the curves to be of equal length, since the tangent to the ellipse itself revolves very unequally in different parts of the orbit, and the zero of the curve of distance, for example, depends on the coincidence of this tangent with the line connecting the bright with the dark body.

(8) The problem here presented of finding the elements of the orbit of the visible companion from irregularities in the elliptical motion is very much more difficult than those arising from the irregular proper motions of perturbed stars, such as *Sirius* and *Procyon*. In the case of the phenomena first investigated by BESSEL, the centre of gravity of the system moves uniformly on the arc of a great circle, but in this case the centre of gravity moves on the arc of a very small ellipse and with a velocity which follows a very complex law. Indeed the velocity at any point of the orbit is inversely as the perpendicular from the central star to the tangent to the ellipse at the point in question; and, as the central star may in general occupy any point whatever of the apparent ellipse, we see that the velocity varies in an extremely complicated manner. In view of these facts it seems best, especially from the point of view of practical double-star work, to determine first of all the path of the centre of gravity and the elements of its orbit. Suppose we designate the rectangular coordinates of this centre, relative to the principal star, by  $x'$ ,  $y'$ ; and the coordinates of the visible companion referred to the same origin by

$x, y$ ; then if  $\alpha$  and  $\beta$  denote the differences of these coordinates, the observations will furnish a series of equations of the form.

$$\begin{array}{ll}
 \alpha_1 = x_1' - x_1 & \beta_1 = y_1' - y_1 \\
 \alpha_2 = x_2' - x_2 & \beta_2 = y_2' - y_2 \\
 \alpha_3 = x_3' - x_3 & \beta_3 = y_3' - y_3 \\
 \alpha_4 = x_4' - x_4 & \beta_4 = y_4' - y_4 \\
 \alpha_5 = x_5' - x_5 & \beta_5 = y_5' - y_5 \\
 \hline
 \alpha_n = x_n' - x_n & \beta_n = y_n' - y_n
 \end{array}$$

Five points, each determined by two such equations, are theoretically sufficient to fix the elements of the orbit of the visible star about the common centre of gravity, a larger number of equations, when combined in an advantageous manner, so as to render the errors of observation a minimum, will make the determination more exact, and define the elements with the desired precision. In the case of 70 *Ophiuchi*, SCHUR'S orbit is to all appearances a good first approximation to the path of the centre of gravity, but it does not seem worth while to enter upon the more refined analysis here indicated until additional measures of the visible companion have confirmed the accuracy of this hypothesis. Apart from these theoretical difficulties, the sensible perturbations of the central star upon the motion of its attendant system will give rise to obstacles which are scarcely less formidable.

(9) While we have spoken of the dark body as attending the companion, it is clear that similar phenomena would result from the action of a body revolving round the central star. In this case, however, the considerable distance which would result from a period of 36 years might render the stability of the system somewhat precarious, especially if the orbit be eccentric like that of the visible companion. And as there is every reason to suppose that the system is the outgrowth of nebular condensation, and is, therefore, adjusted to conditions of stability and permanence, it is more natural to regard the companion as the binary. In this case the small mass might give rise to a period of 36 years even if the pair be very close. The separation of the new system is not likely to be less than  $0''.4$ , and it may be more than twice that distance. If we adopt the parallax of  $0''.162$  found by KRUEGER it will follow that the major semi-axis of the orbit of the visible companion is  $28.07$  astronomical units, and the combined mass is  $2.83$  that of the sun; and hence we conclude that the orbit of the visible companion about the common centre of gravity has a major semi-axis of  $1.84$  astronomical units. Therefore, while the bright companion describes an eccentric orbit with a major axis which is slightly less than that of *Neptune*, the action of the dark body causes it to

describe another ellipse, which in size considerably surpasses that of the planet *Mars*.

(10) With regard to the position of the dark body we remark that an exact prediction is difficult, but the general indications are that at the epoch 1896 50 it lies approximately in the direction of  $260^\circ$ \*. As the companion is now near periastron, the present is a favorable opportunity for searching for the dark body, since in this position the orbit will be expanded owing to the perturbations of the central star. In case it should be imagined that the unseen body attends the central star, it would be natural to locate it in the direction of  $160^\circ$ .

(11) Many years ago a disturbing body in the system of 70 *Ophiuchi* was suspected by MADLER, JACOB and SIR JOHN HERSCHEL, and on two occasions, more recently, BURNHAM has searched for it without success. After examining both stars with the Dearborn 18-inch refractor in 1878 he adds: "Both stars round," while a still more critical search with the Lick 36-inch refractor led him to remark "I could not see any third component and both stars appeared to be round, with all powers." In spite of this negative evidence, observers with great telescopes will find this system worthy of special examination. Whatever be the result of optical search for the unseen body, it will now become a matter of great interest to measure the visible companion with the most scrupulous care until the nature and extent of its perturbations are fully established.

### 99 HERCULIS = A.C. 15.

$\alpha = 18^h 3^m 2$  ,  $\delta = +30^\circ 33'$   
60, yellow , 117, purple

*Discovered by Alvan Clark, July 10, 1859*

OBSERVATIONS									
<i>t</i>	$\theta_o$	$\rho_o$	<i>n</i>	Observers	<i>t</i>	$\theta_o$	$\rho_o$	<i>n</i>	Observers
1859 61	347 4	1 61	1	Dawes	1872 56	6 0	1 46	1	O Struve
1859 65	347 0	1 80	1	Dawes	1877 56	22 0	1 19	1	O Struve
1860 30	342 3	2 28	1	O Struve	1878 46	24 4	1 09	3-1	Burnham
1866 68	360 8	1 73	1	O Struve	1879 47	26 5	1 13	1	Burnham
1868 50	358 6	1 69	1	O Struve					

\* The estimated position given in *A J* 303 for 1895 was  $330^\circ$ , the retrograde motion would diminish the angle considerably, but the principal change in the theoretical position results from the elements above referred to

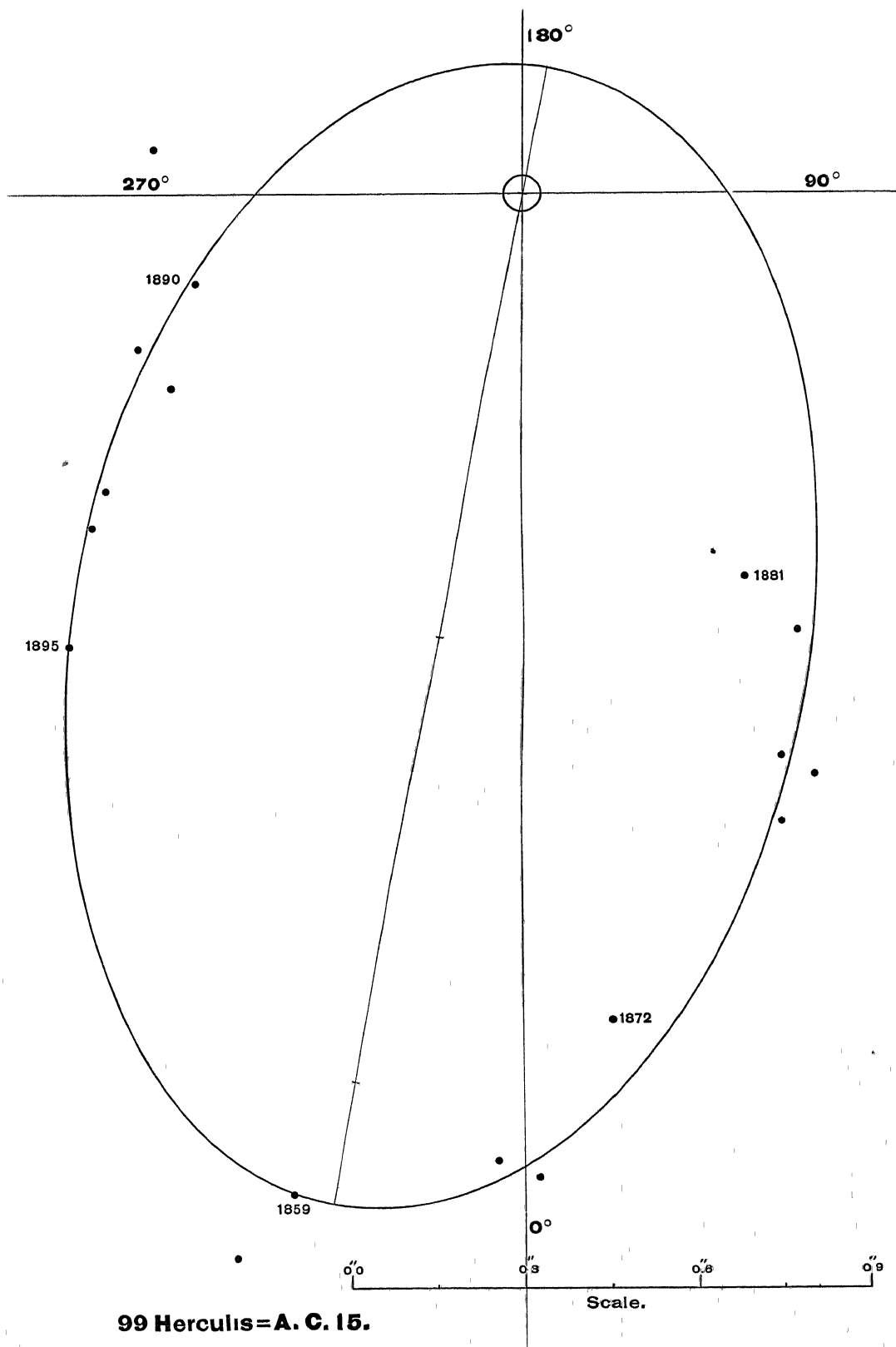
$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1880 53	31° 6'	0.90"	2-1	Burnham	1891 56	292° 0'	0.72"	2-3	Burnham
1881 43	29 4	0.51	1	Burnham	1892 40	299 2	0.70	3	Burnham
1883 60	72 9	1.30	1	O. Struve	1894 74	305 7	0.88	1	Comstock
1883 70	82 4	1.04	1	O. Struve	1895 47	309 5	1.04	6	Barnard
1888 54	77 4	1.05	1	O. Struve	1895 50	308 0	0.95	2	See
1889 50	281 2	0.65	1	Burnham	1895 73	315 2	1.12	3	See
1890 45	285 1	0.59	3-2	Burnham	1895 73	313 4	1.00	2-1	Moulton

This difficult double star was discovered by CLARK while testing the telescope he had just made for DAWES, at the latter's private observatory.\* The physical connection of the pair was suspected, and during the same year two sets of good measures were obtained by DAWES. OTTO STRUVE began to give his attention to the pair the following year, and continued his measures from time to time until 1888. His first observations are very satisfactory, and of the highest value in fixing the elements of the orbit, but the later measures are less trustworthy, owing to the great inequality and closeness of the components. The series of measures begun by BURNHAM in 1878, and continued until the close of his work in California, is of great importance, and in conjunction with STRUVE's observations and those recently made by the writer at Madison, enables us to fix the elements with a relatively high degree of precision.

In order to obtain a good orbit from such measures, the means must be formed in a judicious manner, regard being had to the known motion of the companion. After careful study of all the observations, we have formed a suitable set of mean places, and deduced the corresponding elements. The orbits previously found for this system are:

GORE, 1890	SEE, 1895
$M N$ , Nov 1893	unpublished
$P = 53.55$ years	57.5 years
$T = 1885.58$	1887.30
$e = 0.7928$	0.806
$a = 1''.12$	1''.163
$\Omega = 50^\circ 1'$	77° 0'
$i = 38^\circ 6'$	35° 5'
$\lambda = 110^\circ 73'$	90° 0'

\* *Astronomical Journal*, 366



99 Herculis = A. C. 15.

The adopted elements of 99 *Herculis* are as follows.

$$\begin{array}{ll}
 P = 54.5 \text{ years} & \Omega = \text{indeterminate} \\
 T = 1887.70 & i = 0^\circ 0' \\
 e = 0.781 & \text{Angle of periastron} = 169^\circ 5' \\
 a = 1''.014 & n = +6^\circ 60'55''
 \end{array}$$

The apparent is the same as the real orbit.

$$\begin{array}{ll}
 \text{Length of major axis} & = 2''.028 \\
 \text{Length of minor axis} & = 1''.278 \\
 \text{Angle of major axis and periastron} & = 169^\circ 5'
 \end{array}$$

TABLE OF COMPUTED AND OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1859 65	347 0	348 4	1 80	1 81	- 1 4	-0 01	1	Dawes
1859 96	344 8	348 9	1 94	1 81	- 4 1	+0 13	2	Dawes 1, O Struve 1
1866 68	360 8	357 8	1 73	1 74	+ 3 0	-0 01	1	O Struve
1868 50	358 6	360 5	1 69	1 70	- 1 9	-0 01	1	O Struve
1872 56	6 0	7 0	1 46	1 56	- 1 0	-0 10	1	O Struve
1877 56	22 0	17 4	1 19	1 29	+ 4 6	-0 10	1	O Struve
1878 46	24 4	20 1	1 04	1 21	+ 4 3	-0 17	3-1	Burnham
1879 47	26 5	23 2	1 13	1 14	+ 3 3	-0 01	1	Burnham
1880 53	31 6	27 3	0 90	1 04	+ 4 3	-0 14	2-1	Burnham
1881 43	29 4	31 0	0 77	0 96	- 1 6	-0 19	1-2	Burnham 1, O Struve 0-1
1889 50	257 4	262 7	0 65	0 42	- 5 3	+0 23	1-1	Burnham 0-1, O Struve 1-0
1890 45	285 1	280 5	0 59	0 56	+ 4 6	+0 03	3-2	Burnham
1891 56	292 0	292 9	0 72	0 71	- 0 9	+0 01	2-3	Burnham
1892 40	299 2	299 2	0 70	0 80	0 0	-0 10	3	Burnham
1894 74	305 7	311 3	0 88	1 04	- 5 6	-0 16	1	Comstock
1895 50	308 0	314 2	0 95	1 11	- 6 2	-0 16	2	See
1895 73	315 2	315 1	1 12	1 13	+ 0 1	-0 01	3	See

EPIHEMERIS

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 50	317 5	1 18	1899 50	325 3	1 39
1897 50	320 4	1 26	1900 50	327 6	1 45
1898 50	323 0	1 33			

While this orbit may need slight modification in the course of time, it does not seem probable that a sensible improvement can be effected for a good many years, as the motion is now very slow, and chiefly in the direction of the radius vector. The orbit is remarkable for its high eccentricity, and for having no sensible inclination. This circumstance enables us to contemplate directly the real orbit, and renders 99 *Herculis* an object of the highest interest. The pair is always rather difficult, owing to the inequality of the components, and exact measurement is seldom possible. But at present the star is relatively easy, and ought to be given some attention by observers.



## ζ SAGITTARII.

$$\alpha = 18^{\text{h}} 56^{\text{m}} 3, \quad \delta = -30^{\circ} 1$$

$$3.9, \text{ yellow}, \quad 4.4, \text{ yellow}$$

*Discovered by Winlock in July, 1867*

OBSERVATIONS									
$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1867 59	257.7	0.86	1	Winlock	1888 66	259.3	0.67	7	$\beta$ & Lv
1867 80	260.8	0.48	1	Newcomb	1889 41	255.1	0.81	5	Burnham
1878 70	84.2	0.42	1	Burnham	1890 49	251.1	0.76	3	Burnham
1879 71	54.8	0.3±	1	Burnham	1891 53	246.5	0.61	3	Burnham
1880 62	62.1	0.55	2	Burnham	1892 39	245.1	0.60	3	Burnham
1881 61	36.1	0.31	2	Burnham	1895 32	194.7	0.35	3	See
1886 62	271.3	0.65	4	Hall	1895 62	193.6	0.13	2	Bainard
1886 74	271.1	—	1-0	Pollock	1895 74	193.1	0.20±	1	See
1887 64	265.3	—	5-0	Pollock					

Owing to the great southern declination of  $\zeta$  *Sagittarii*, which renders it inaccessible to European observers, and makes observations difficult even in the United States, the object was comparatively neglected for a number of years. The first observations were made by WINLOCK and NEWCOMB in the year of its discovery. The pair was not again observed until 1878, when BURNHAM began to give it regular attention\*. His series of measures now show that  $\zeta$  *Sagittarii* belongs to the class of bright, close binaries with short periods. This object has therefore become one of particular interest to American observers.

The first investigation of the orbit was made by MR J E GORE, who published the following elements (*Monthly Notices*, R A S, 1886, p 444)

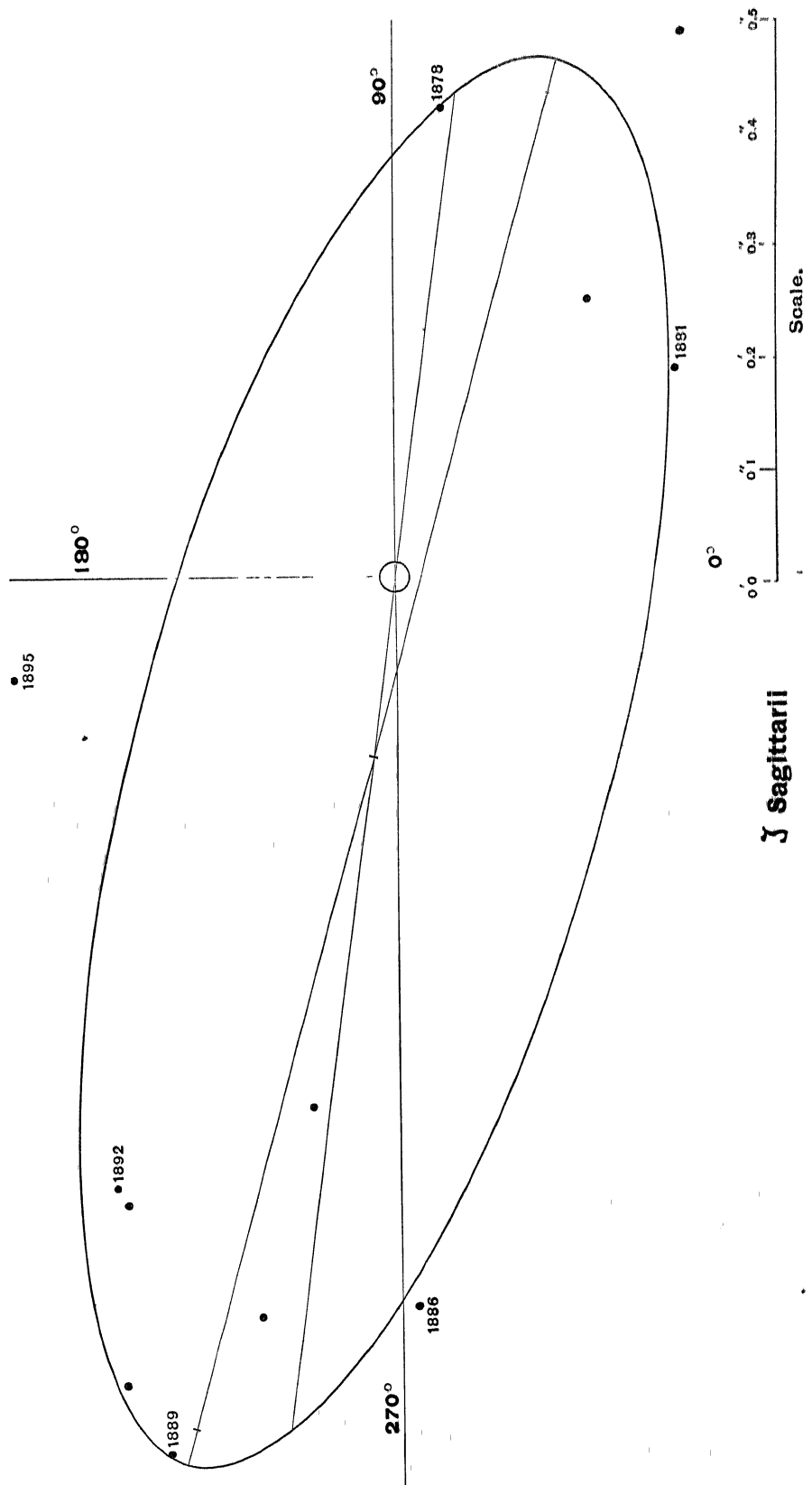
$$\begin{aligned} P &= 18.69 \text{ years} & i &= 58^{\circ} 8 \\ T &= 1882.86 & \Omega &= 83^{\circ} 37 \\ e &= 0.1698 & \lambda &= 263^{\circ} 35 \\ a &= 0''.53 \end{aligned}$$

MR J W FROLEY has more recently examined this orbit (*Astronomy and Astrophysics*, June, 1893), and obtained a set of elements which do not require any large corrections:

$$\begin{aligned} P &= 17.715 \text{ years} & \Omega &= 75^{\circ} 35 \\ T &= 1878.62 & i &= 73^{\circ} 95 \\ e &= 0.30 & \lambda &= 327^{\circ} 35 \\ a &= 0''.68 \end{aligned}$$

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\* *Astronomical Journal*, 355



♐ Sagittarii

While in Virginia recently, I took occasion to measure this star, and, although the object was seen with difficulty, owing to its low altitude, I could discover a distinct elongation in the direction  $194^{\circ} 7$ , the distance could not be fixed with much confidence, but my settings of the micrometer gave  $0'' 35$ . The estimates of distance were substantially the same, but I am now convinced, from my distinct recollection of the appearance of the object, that both the measure and the estimate were too large. The star could not be separated, although it was sharply elongated with a power of 1300, the distance was probably less than  $0'' 25$ .

From an examination of all the measures of this pair, we have derived the following elements.

$$\begin{array}{ll} P = 18\ 85 \text{ years} & \Omega = 69^{\circ} 3 \\ T = 1878\ 80 & \iota = 67^{\circ} 32 \\ e = 0\ 279 & \lambda = 328^{\circ} 1 \\ a = 0''\ 686 & n = -19^{\circ} 098 \end{array}$$

Apparent orbit.

$$\begin{array}{ll} \text{Length of major axis} & = 1''\ 300 \\ \text{Length of minor axis} & = 0''\ 423 \\ \text{Angle of major axis} & = 74^{\circ} 8 \\ \text{Angle of periastron} & = 82^{\circ} 8 \\ \text{Distance of star from centre} & = 0''\ 168 \end{array}$$

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1867 80	260 8	254 8	0 48	0 81	+ 6 0	-0 33	1	Newcomb
1878 70	84 2	85 6	0 42	0 41	- 1 4	+0 01	1	Burnham
1879 71	54 8	69 4	0 3 ±	0 48	-14 6	-0 18	1	Burnham
1880 62	62 1	57 7	0 55	0 42	+ 4 4	+0 13	2	Burnham
1881 61	36 1	42 2	0 31	0 33	- 6 1	-0 02	2	Burnham
1886 62	271 3	273 7	0 65	0 59	- 2 4	+0 06	4	Hall
1888 66	259 3	260 9	0 67	0 79	- 1 6	-0 12	7	Burnham 6, Leavenworth 1
1889 41	255 1	256 7	0 81	0 81	- 1 6	±0 00	5	Burnham
1890 49	251 1	251 7	0 76	0 78	- 0 6	-0 02	3	Burnham
1891 53	246 5	246 5	0 61	0 70	± 0 0	-0 09	3	Burnham
1892 39	245 1	241 8	0 60	0 60	+ 3 3	±0 00	3	Burnham
1895 32	194 7	194 3	0 35	0 22	+ 0 4	+0 13	3	See

EPIHEMERIS

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 50	118 1	0 24	1899 50	49 8	0 37
1897 50	76 5	0 47	1900 50	29 9	0 28
1898 50	63 5	0 46			

When we consider the small number of observations, and the discordant character of some of them, we must regard these elements as highly

satisfactory. It is not likely that they will be materially changed by future observations, but for some time this rapid binary will deserve careful attention. The eccentricity of the orbit appears to be fairly well defined, and is rather smaller than usual; good observations during the next five years will enable us to fix this element with the desired precision. The star is now very difficult, and will remain so for several years, but it is constantly within reach of our large refractors

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$\gamma$  CORONAE AUSTRALIS = H<sub>2</sub> 5084.

$\alpha = 18^h 59^m 6$  ,  $\delta = -37^\circ 12'$   
 5 5, yellowish , 5 5, yellowish

*Discovered by Sir John Herschel, June 20, 1834*

## OBSERVATIONS

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1834 47	37 1	3 $\pm$	1	Herschel	1859 72	338 1	1 5 $\pm$	4-2	Powell
1835 43	37 0	—	1	Herschel	1861 69	328 8	1 5 $\pm$	4-1	Powell
1835 56	36 7	—	1	Herschel	1862 27	325 3	1 5 $\pm$	5-1	Powell
1836 43	34 5	3 67	1	Herschel	1863 84	318 1	—	4	Powell
1837 35	32 0	2 63	1	Herschel	1870 19	286 9	—	2	Powell
1837 44	33 9	2 76	1	Herschel	1871 22	281 9	—	1	Powell
1837 45	32 2	2 04	1	Herschel	1875 65	257 4	1 45	4	Schiaparelli
1837 46	32 7	2 40	1	Herschel	1876 64	253 1	1 67	—	Stone
1847 32	14 1	2 30	1	Jacob	1877 43	248 4	1 49	5	Schiaparelli
1850 51	5 9	2 29	4	Jacob	1877 63	246 6	1 44	4-3	Stone
1851 48	4 4	2 26	6	Jacob	1878 49	242 6	1 36	2	Stone
1852 27	3 4	1 89	3	Jacob	1880 46	233 1	1 15	1	Russell
1853 52	359 1	1 83	—	Jacob	1880 67	232 4	1 32	1	Hargrave
1853 71	358 6	2 $\pm$	4-1	Powell	1881 72	225 5	1 42	3-2	H C Wilson
1854 26	356 2	1 71	3	Jacob	1883 62	217 7	1 66	4-1	H C Wilson
1854 78	355 6	—	3	Powell	1886 58	200 3	1 37	6	Pollock
1855 77	352 9	—	5	Powell	1886 70	203 5	1 52	1	Russell
1856 22	350 8	1 68	8-7	Jacob	1887 69	196 6	1 16	4	Pollock
1856 67	348 1	1 66	3	Jacob	1887 73	196 2	1 68	4-1	Tebbutt
1857 21	348 4	1 67	5	Jacob	1888 61	189 3	1 71	6-3	Tebbutt
1857 66	346 3	1 55	3	Jacob	1888 71	188 0	1 2	1	Leavenworth
1858 20	343 4	1 53	3	Jacob					

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1889 41	185 4	1 70	4-3	Burnham	1891 75	176 7	1 54	9-4	Tebbutt
1889 84	185 4	2 30	4-1	Tebbutt	1892 64	172 9	1 65	5-2	Tebbutt
1890 59	182 9	1 61	4	Tebbutt	1894 80	165 5	1 62	5-6	Tebbutt
1890 65	180 3	0 99	6-4	Sellers	1895 73	161 9	1 59	1-2	See
1891 53	176 9	1 68	3	Burnham	1895 73	164 1	1 55	2-1	Moulton
1891 70	177 6	1 33	3	Sellois					

During his sojourn at Feldhausen HERSCHEL made careful measures of this object with the seven-foot equatorial, and on two occasions swept over it with the twenty-foot reflector\*. In sweep 461 he saw the pair under specially favorable conditions, and estimated the distance of the components at 3". This value is therefore adopted in the table of observations instead of the distance (1" 23<sup>p</sup>) indicated by the micrometer, which was vitiated by troublesome hitching of the threads, and had to be rejected as worthless. HERSCHEL showed from his observations that the system had a considerable retrograde motion, and hence it was subsequently followed by JACOB, POWELL, RUSSELL, TEBBUTT and other southern observers. At the present time the arc described amounts to 238°, and even if the observations are not very numerous, they are sufficient, both in point of quantity and quality, to give an orbit which will undoubtedly prove to be substantially correct.

The components are nearly equal in magnitude, and, as they are never closer than 1" 42, the pair is always comparatively easy, and even if difficulties arise in the measurement of distance, there will be practically no difficulty, as HERSCHEL remarks, in determining the angle with the necessary accuracy. In dealing with the orbit of a bright pair with equal components, it is clear that unusual weight should be given to the position angles, and especially when the stars are fairly wide, but the measured distances are affected by relatively large errors. The orbit of this star is therefore based mainly on the angles, but the distances have been of no small service in the final definition of the elements. Some of the orbits which have been published by previous investigators are as follows:

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
100 8	1863 08	0 602	2 549	352 2	53 6	266 4	Jacob, 1858	M N, XV, p 208
55 58	1882 77	0 6989	2 400	229 15	111 45	—	Schiaparelli, 1876	A N, 2073
54 98	1883 20	0 6974	2 44	227 4	69 3	283 95	Downing, 1883	M N, XLIII, p 368
81 78	1886 53	0 322	1 885	45 4	47 43	141 0	Gore, 1885	M N, XLVI, p 103
78 80	1887 40	0 324	1 85	41 0	50 5	—	Wilson, 1886	Gore's Catalogue, p H
93 34	1885 19	0 303	2 034	49 3	48 8	153 4	Powell, 1890	Gore's Catalogue, p H
121 24	1879 33	0 331	2 191	57 95	35 62	181 1	Sellois, 1892	M N, LIII, p 45
154 41	1876 84	0 4244	2 55	77 23	35 6	175 3	Gore, 1892	M N, LII, p 503

\* *Astronomische Nachrichten*, 3323

An investigation of all the observations has led to the following elements of  *$\gamma$  Coronae Australis*

$P = 152\ 7\ \text{years}$   
 $T = 1876\ 80$   
 $e = 0\ 420$   
 $a = 2''\ 453$

$\Omega = 72^\circ\ 3$   
 $i = 34^\circ\ 0$   
 $\lambda = 180^\circ\ 2$   
 $n = -2^\circ\ 3575$

Apparent orbit.

Length of major axis = 4'' 906

Length of minor axis = 3'' 661

Angle of major axis = 72° 2

Angle of periastron = 252° 1

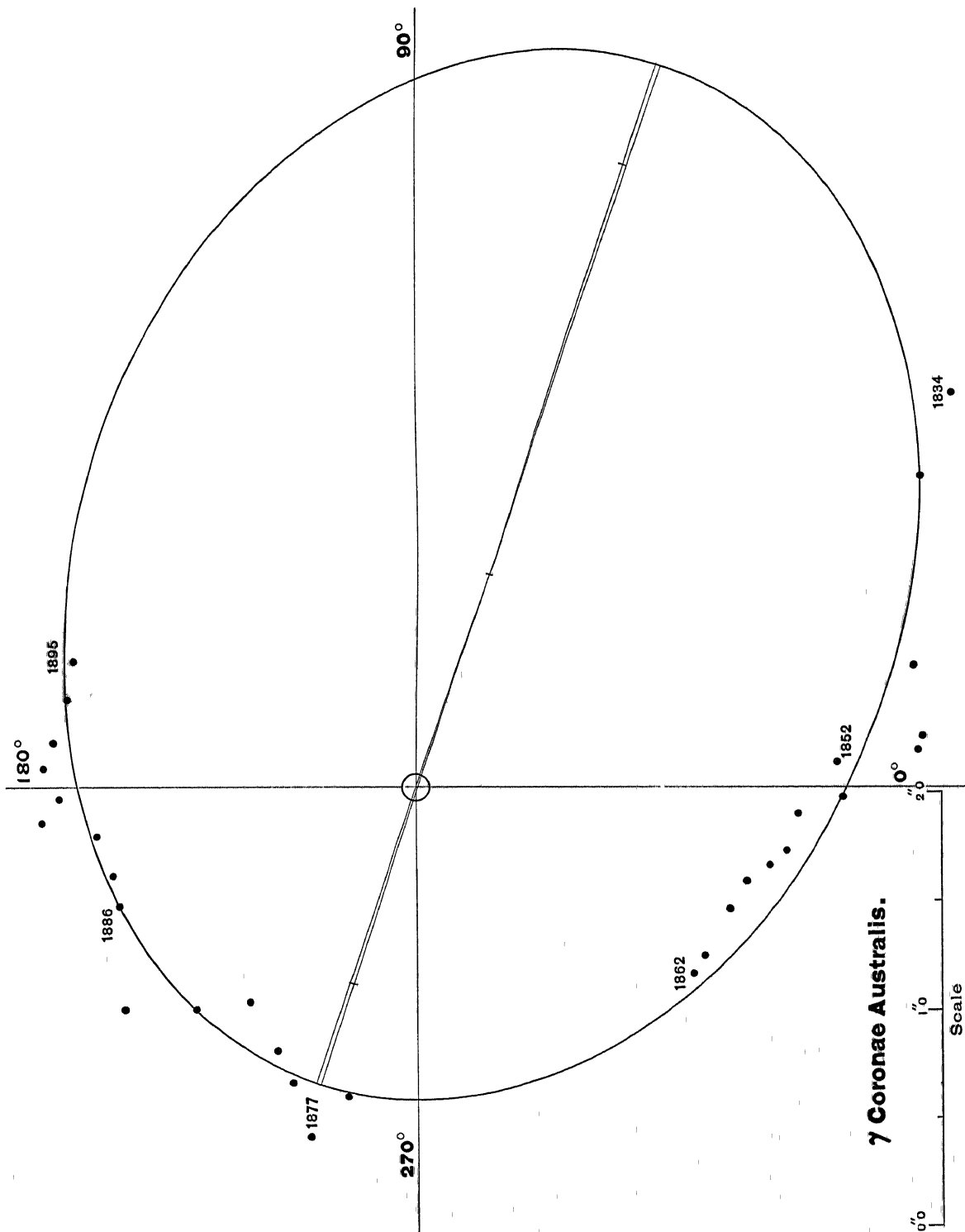
Distance of star from centre = 1'' 033

COMPARISON OF COMPUTED WITH OBSERVED PLACES

<i>t</i>	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	<i>n</i>	Observers
1834 47	37 1	37 1	3 ±	2 80	± 0 0	+ 0 20	1	Herschel
1837 42	32 7	32 7	2 66	2 66	± 0 0	± 0 00	4	Herschel
1847 32	14 1	14 6	2 30	2 20	- 0 5	+ 0 10	1	Jacob
1850 51	5 9	5 9	2 29	2 03	± 0 0	+ 0 26	4	Jacob
1851 48	4 4	4 4	2 26	2 00	± 0 0	+ 0 26	6	Jacob
1852 27	3 4	2 3	1 89	1 96	+ 1 1	- 0 07	3	Jacob
1853 61	358 8	358 2	1 91	1 90	+ 0 6	+ 0 01	6 ±	Jacob, Powell 4-1
1854 52	355 9	355 5	1 71	1 86	+ 0 4	- 0 15	6-3	Jacob 3, Powell 3-0
1856 44	349 5	349 3	1 67	1 78	+ 0 2	- 0 11	11-10	Jacob 8-7, Jacob 3
1857 43	347 3	345 5	1 61	1 73	+ 1 8	- 0 12	8	Jacob 5, Jacob 3
1858 20	343 4	342 6	1 53	1 70	+ 0 8	- 0 17	3	Jacob
1859 72	338 1	336 8	1 5 ±	1 64	+ 1 3	- 0 14	4-2	Powell
1861 69	328 8	328 5	1 5 ±	1 58	+ 0 3	- 0 08	4-1	Powell
1862 27	325 3	325 8	1 5 ±	1 56	- 0 5	- 0 06	5-1	Powell
1863 84	318 1	319 0	—	1 52	- 0 9	—	4	Powell
1870 19	286 1	287 0	—	1 44	- 0 9	—	2	Powell
1871 22	281 9	281 3	—	1 43	+ 0 6	—	1	Powell
1875 65	257 4	258 1	1 45	1 43	- 0 7	+ 0 02	4	Schiaparelli
1876 64	253 1	253 0	1 67	1 43	+ 0 1	+ 0 24	—	Stone
1877 53	247 5	247 9	1 47	1 42	- 0 4	+ 0 05	9-7	Schiaparelli 5, Stone 4-3
1878 49	242 6	243 0	1 36	1 43	- 0 4	- 0 07	2	Stone
1880 57	232 7	232 0	1 24	1 43	+ 0 7	- 0 19	2	Russell 1, Hargrave 1
1881 72	225 5	226 1	1 42	1 43	- 0 6	- 0 01	3-2	H C Wilson
1883 62	217 7	216 3	1 66	1 43	+ 1 4	+ 0 23	4-1	H C Wilson
1886 64	201 9	200 6	1 44	1 44	+ 1 3	± 0 00	7	Pollock 6, Russell 1
1887 71	196 4	195 2	1 42	1 46	+ 1 2	- 0 04	8-5	Pollock 4, Tebbutt 4-1
1888 66	188 6	190 6	1 46	1 47	- 2 0	- 0 01	7-3	Tebbutt 6-3, Leavenworth 1
1889 62	185 4	186 1	1 70	1 49	- 0 7	+ 0 21	8-3	Burnham 4-3, Tebbutt 4-0
1890 62	181 6	181 6	1 61	1 51	± 0 0	+ 0 10	10-4	Tebbutt 4, Sellors 6-0
1891 53	176 9	177 0	1 68	1 54	- 0 1	+ 0 14	3	Burnham
1892 64	172 9	172 3	1 65	1 57	+ 0 6	+ 0 08	5-2	Tebbutt
1894 80	165 5	163 5	1 62	1 65	+ 2 0	- 0 03	5-6	Tebbutt
1895 73	159 2	159 9	1 59	1 69	- 0 7	- 0 10	2	See

EPHEMERIS

<i>t</i>	$\theta_c$	$\rho_c$	<i>t</i>	$\theta_c$	$\rho_c$
1896 50	157 4	1 71	1899 50	147 2	1 85
1897 50	154 0	1 76	1900 50	143 8	1 90
1898 50	150 6	1 80			



$\gamma$  Coronae Australis.





It will be seen that my orbit is quite similar to that found by GORE. Though the period is not defined with the greatest accuracy, it does not seem probable that the value given above can be uncertain by more than five years. The eccentricity will certainly be in the immediate neighborhood of the value here assigned, and an error exceeding  $\pm 0.02$  is very improbable. The orbit of  $\gamma$  *Coronae Australis* is therefore comparatively well determined, and yet as great accuracy in the orbits of double stars is ultimately desirable, southern observers will find this system worthy of constant attention.

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 $\beta$  DELPHINI =  $\beta$  151.

$\alpha = 20^{\text{h}} 32^{\text{m}} 9$  ,  $\delta = +14^{\circ} 15'$   
4, yellow , 6, yellowish

*Discovered by Burnham with his celebrated six-inch Clark Refractor in August, 1873*

OBSERVATIONS									
$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1873 60	$355^{\circ} \pm$	$0.7''$	1	Burnham	1885 61	$222.9^{\circ}$	$<0.4''$	1	H. Struve
1874 66	15.5	0.65	5	Dembowski	1885 95	216.6	0.38	8	Englemann
1874 70	13.6	0.49	3-1	Newcomb	1886 78	257.8	obl	1	H. Struve
1874 73	8.0	0.69	1	O. Struve	1886 88	238.1	$0.22 \pm$	7	Schiaparelli
1875 61	14.7	0.42	4	Schiaparelli	1886 91	219.5	0.39	4	Englemann
1875 65	20.1	0.54	4	Dembowski	1887 55	278.5	0.36	5	Tarrant
1876 65	25.8	0.48	4	Dembowski	1887 66	272.0	0.39	5	H. Struve
1877 27	17.7	0.35	2	Schiaparelli	1887 75	308.1	$0.3 \pm$	1	Hough
1877 71	29.7	0.51	5	Dembowski	1887 85	287.8	$0.2 \pm$	8	Schiaparelli
1877 79	40.8	0.32	2	Burnham	1888 65	304.0	0.30	5	Burnham
1878 65	53.7	0.24	4	Burnham	1888 76	300.9	0.35	3	H. Struve
1878 75	59.2	—	1	Dembowski	1888 84	311.5	0.25	17	Schiaparelli
1879 56	$90 \pm$	along doubtful	2	Burnham	1889 50	314.2	0.31	5	Burnham
1880 68	133.6	0.26	3	Burnham	1889 78	318.5	0.43	6	H. Struve
1880 75	214.5	$0.2 \pm$	2	Hall	1889 86	319.2	$0.37 \pm$	11	Schiaparelli
1881 54	149.2	0.26	5	Burnham	1890 49	324.2	0.45	4	Burnham
1881 88	154.7	—	1	Bigourdan	1890 89	326.5	0.43	12	Schiaparelli
1882 60	167.5	0.26	3	Burnham	1891 45	331.6	0.38	4	Burnham
1883 25	183.9	0.19	7	Englemann	1891 64	330.1	0.39	3	Hall
1883 55	182.5	0.23	3	Burnham	1891 76	334.0	0.48	5	H. Struve
1884 69	195.9	0.32	3	Hall	1891 85	158.2	—	1	Bigourdan
1884 71	197.7	0.32	4	Englemann	1891 87	333.7	0.43	9	Schiaparelli
1884 77	199.2	0.29	5	Burnham	1892 39	338.7	0.50	4	Burnham
					1892 88	337.6	0.49	2	Barnard
					1892 93	340.7	0.52	5	Schiaparelli

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1893 52	339 2	0 58	2	Leavenworth	1894 79	348 6	—	1	H C Wilson
1893 53	338 8	0 73	2	H C Wilson	1894 83	347 2	0 48	13	Schiaparelli
1893 62	335 3	0 57	3	Hough					
1893 70	342 2	0 56	5	Barnard	1895 31	351 8	0 50	1	See
1893 79	346 8	0 51	3	Comstock	1895 42	349 8	0 73	6	Barnard
1893 87	344 2	0 49	13	Schiaparelli	1895 61	352 1	0 80	1	See
1893 95	345 8	—	1	Bigourdan	1895 61	352 1	0 64	1	See
					1895 66	350 8	0 58	3	Comstock
1894 51	346 3	0 56	8	Barnard					

When discovered in 1873 the companion was near its maximum elongation, and was easily measured by DEMBOWSKI in 1874. The measures of the next few years showed that the pair had a rapid direct motion \*. In 1879–80 the distance of the components became so small (about 0' 20) that the object could be elongated only by the most powerful telescopes. The measures at this time are therefore few in number, and necessarily of doubtful accuracy.

Since the epoch of DEMBOWSKI's measures in 1874, the radius-vector of the companion has swept over 335 degrees of position-angle, and the intervening observations enable us to determine the orbit with a comparatively high degree of precision. The following table gives the orbits hitherto published for this star

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
<sup>yrs</sup> 26 07	1882 19	0 357	0 55	163 6	54 9	354 6	Dubiago, 1884	A N, 2602
30 91	1882 25	0 337	0 517	2 67	59 33	327 8	Goie, 1885	Proc R I A, IV, no 5
16 95	1885 80	0 096	0 460	10 9	61 6	220 9	Celoria, 1888	A N, 2824
22 97	1882 37	0 260	0 501	174 2	64 1	343 9	Glasenapp, 1893	A N, 3177
24 16	1882 38	0 284	0 51	174 4	64 64	344 2	Glasenapp, 1893	A N, 3177

From an investigation of all the observations we find the following elements for  $\beta$  Delphin.

$P = 27.66 \text{ years}$   
 $T = 1883.05$   
 $e = 0.373$   
 $a = 0''.6724$

$\Omega = 3^\circ 9'$   
 $i = 61^\circ 35'$   
 $\lambda = 164^\circ 93'$   
 $n = +13^\circ 015'$

Apparent orbit.

Length of major axis = 1'' 060

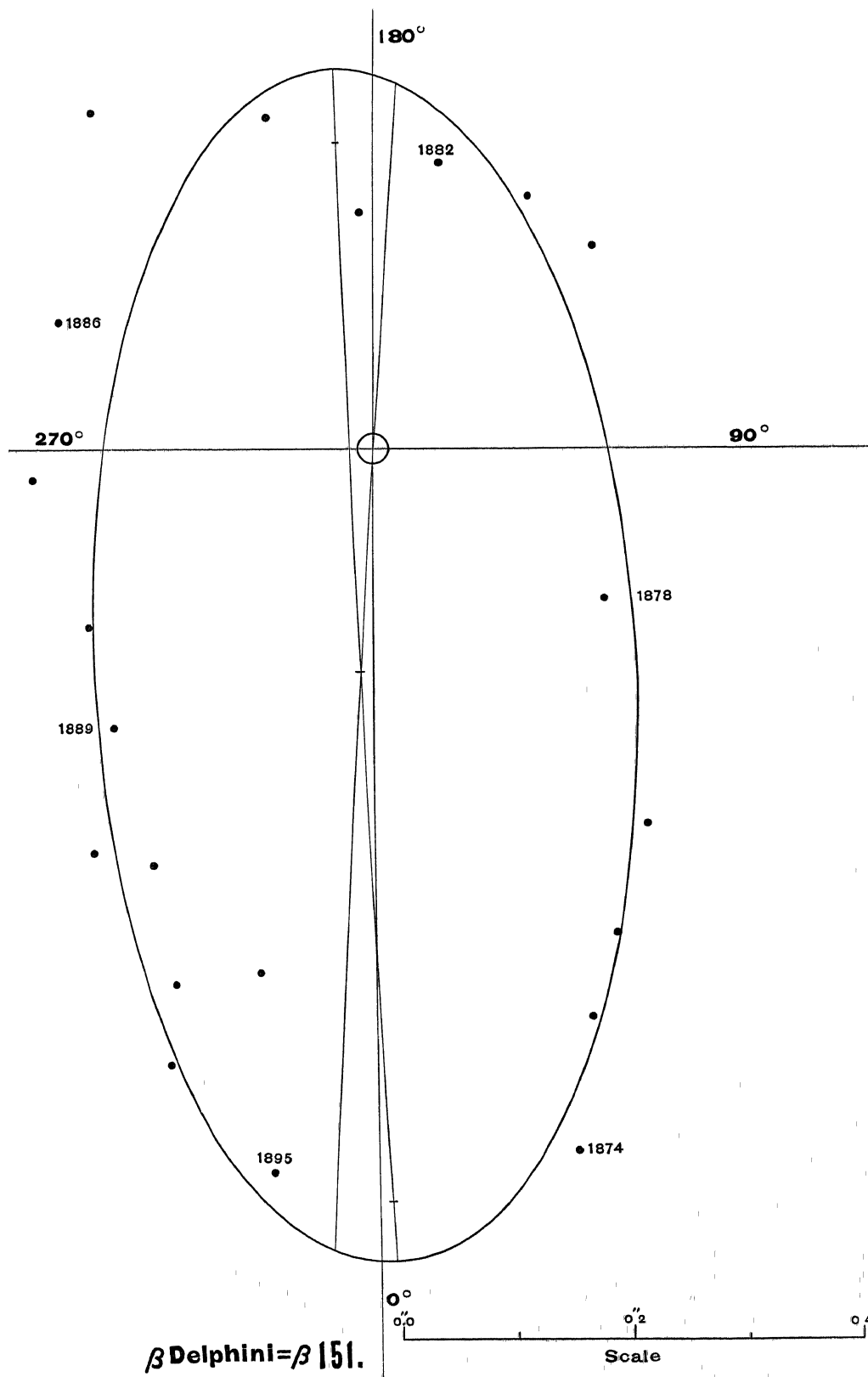
Length of minor axis = 0'' 477

Angle of major axis = 2° 5'

Angle of periastron = 176° 6'

Distance of star from centre = 0'' 194

\* *Astronomical Journal*, 357.





The accompanying table of computed and observed places shows that these elements are extremely satisfactory. The only large residual is that of 1880, which is probably due to an error of observation incident to the excessive closeness of the components

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1874 66	15 5	15 2	0 65	0 62	+ 0 3	+0 03	5	Dembowski
1875 65	20 1	20 0	0 54	0 55	+ 0 1	-0 01	4	Dembowski
1876 65	25 8	26 2	0 48	0 48	- 0 4	$\pm$ 0 00	4	Dembowski
1877 75	35 3	36 2	0 41	0 38	- 0 9	+0 03	7	Dembowski 5, Burnham 2
1878 70	56 4	50 1	0 24	0 29	+ 6 3	-0 05	5-4	Burnham 4, Dembowski 1-0
1879 56	90 $\pm$	71 3	0 23	0 23	—	—	2	Burnham
1880 68	133 6	114 7	0 26	0 20	+18 9	+0 06	3	Burnham
1881 54	149 2	145 6	0 26	0 24	+ 3 6	+0 02	5	Burnham
1882 60	167 5	169 1	0 26	0 31	- 1 6	-0 05	3	Burnham
1883 40	183 2	181 7	0 21	0 33	+ 1 5	-0 12	10	Englemann 7, Burnham 3
1884 72	197 6	201 2	0 31	0 33	- 3 6	-0 02	12	Hall 3, Englemann 4, Burnham 5
1885 95	219 8	220 7	0 39	0 28	- 0 9	+0 11	9	Englemann 8, II Struve 1
1886 86	248 0	247 4	0 30	0 24	+ 0 6	+0 06	8-11	Sch 7, Englemann 0-4, II Struve 1-0
1887 70	275 2	271 8	0 31	0 24	+ 3 4	+0 07	18-19	Tar 5, Ho 1, Schiaparelli 8, II Struve 5
1888 65	302 4	296 3	0 30	0 27	+ 6 1	+0 03	8-5	Burnham 5, II Struve 3-0
1889 68	317 3	313 8	0 34	0 34	+ 3 5	$\pm$ 0 00	16	$\beta$ 5, Schiaparelli 11, II Struve 0-0
1890 69	325 3	325 2	0 44	0 41	+ 0 1	+0 03	16	Burnham 4, Schiaparelli 12
1891 68	332 4	333 5	0 42	0 48	- 1 1	-0 06	21	$\beta$ 4, III 3, Schiaparelli 9, H Struve 5
1892 66	339 7	339 9	0 51	0 54	- 0 2	-0 03	9	Schiaparelli 5, Burnham 4 [Big 1-0
1893 71	341 7	344 8	0 58	0 61	- 3 1	-0 03	24-23	Lv 2, II C W 2, Ho 3, Com 3, Sch 13,
1894 81	347 9	349 3	0 48	0 65	- 1 4	-0 17	14-13	H C Wilson 1-0, Schiaparelli 13
1895 51	352 0	351 9	0 65	0 68	+ 0 1	-0 03	3	See

The present orbit is somewhat more eccentric than those heretofore published, and in this respect it conforms better to the general rule among binaries. That the orbit has an eccentricity of about this magnitude is evident from the rapid motion of the radius-vector in the periastral region, and its slow motion at the present time. The slow, angular motion of the radius-vector during recent years indicates, of course, that the distance of the companion is much increased, and this leads us to remind observers that the present distance is sensibly larger than some have indicated by their measures. At present the distance is probably over 0" 65, and for some years will slightly augment.

It does not seem at all probable that the true elements of this remarkable binary can differ materially from those here obtained. Nevertheless, additional exact measures will be valuable in fixing the orbit with great accuracy, and as the star will be relatively easy for several years, observers should give it regular attention. The following is a short ephemeris:

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 51	355 3	0 71	1899 51	4 9	0 72
1897 51	358 6	0 72	1900 51	8 2	0 69
1898 51	1 7	0 72			

4 AQUARI =  $\Sigma 2729$ .

$\alpha = 20^h 46^m 1$  ,  $\delta = -8^\circ 1'$   
6, yellow , 7, yellow

*Discovered by Sir William Herschel, September 3, 1782*

OBSERVATIONS									
$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1783 55	351 5	—	1	Herschel	1875 62	157 0	0 4 $\pm$	4	Schiaparelli
1802 65	28 9	—	2	Herschel	1877 15	148 7	0 56	3	Dembowski
1825 60	27 5	0 80	2	Struve	1877 70	158 5	0 5 $\pm$	1	Cincinnati
1830 92	13 4	0 69	1	Struve	1879 44	156 4	0 57	5-1	Cincinnati
1832 73	46 0	0 67	2-1	Herschel	1879 76	155.9	0.40	4	Hall
1832 90	23 0	oblonga	1	Struve	1880 78	165 5	0 51	2	Pritchett
1833 77	31 2	0 67	1	Struve	1881 54	159 6	0 52	3	Burnham
1836 05	46 3	0 41	4	Struve	1883 84	182 1	—	1	Seabroke
1839 68	62 2	—	2	Dawes	1884 77	166 8	—	7	Seabroke
1840 72	65 5	0 6 $\pm$	2	Dawes	1885 64	156 1	—	1	Seabroke
1841 51	24 6	0 6 $\pm$	1	Mädler	1885 74	167 9	0 46	3	Hall
1841 80	72 7	—	1	Dawes	1886 69	162 5	—	1	Seabroke
1842 82	27 2	0 45	2-1	Mädler	1886 74	168 3	0 54	3-2	Leavenworth
1843 70	31 9	0 5 $\pm$	3	Mädler	1886 84	174 8	0 47	2	Hall
1843 76	81 7	—	1	Dawes	1887 28	173 4	0 41	7	Schiaparelli
1844 90	23 1	0 5 $\pm$	1	Mädler	1887 79	175 9	0 53	3	Hall
1853 70	95 9	0 5 $\pm$	1	Dawes	1887 82	170 5	0 52	2	Tarrant
1854 75	101 7	0 3 $\pm$	1	Dawes	1888 81	172 4	0 48 $\pm$	5	Schiaparelli
1855	—	—	1	Secchi	1889 51	155 5	—	1	Seabroke
1856 81	107 8	0 3 $\pm$	1	Secchi	1889 88	176 7	0 49 $\pm$	2	Schiaparelli
1862 68	137 5	oblonga	3	Dembowski	1890 78	178 2	0 49	2	Tarrant
1865 71	125 $\pm$	cuneo	1	Secchi	1891 77	178 1	0 50 $\pm$	1	Schiaparelli
1865 74	143.6	—	1	Talmage	1892 70	184 5	0 55	3	Tarrant
1866 08	139 6	oblonga	3	Dembowski	1892 80	181 7	0 33	2-1	Comstock
1866.65	125 5	—	3	Searle	1892 91	187 0	0 4 $\pm$	1	Schiaparelli
1866.66	110 0	—	5	Winlock	1893 81	182 4	0 35 $\pm$	2-1	Comstock
1867.86	141 1	0 30	1	Newcomb	1894 86	186 5	0 38 $\pm$	3	Schiaparelli
1872 88	147.5	oblonga	5	Dembowski	1895 61	193 9	0 30 $\pm$	1	Comstock
					1895 73	184.2	0 33	3	See

THIS double star is always an exceedingly close and difficult object. SIR WILLIAM HERSCHEL measured the position-angle in 1783, and on repeating his observation in 1802, concluded that in nineteen years the motion had amounted to  $37^{\circ} 4$  (*Phil Trans*, 1804, p 371). In 1825 the star was measured by STRUVE on two nights, his observations gave  $\theta = 25^{\circ} 0$ ,  $\rho = 0'' 81$ ,  $\theta = 30^{\circ} 0$ ,  $\rho = 0'' 80$ . These results do not accord well with those of 1802, but we may infer with DAWES (*Mem R A S.*, vol xxxv p 427) that HERSCHEL'S second observation is erroneous. For it is clear that the angle could not have been the same in 1802 as in 1825, and the subsequent motion of the star shows that STRUVE'S first position is essentially correct. All the early and some of the more recent measures of 4 *Aquarii* are extremely discordant, and great difficulty is experienced in determining what measures ought to be relied upon. Careful sifting of the observations and judicious combinations of individual results will alone insure suitable mean places for the derivation of a satisfactory set of elements. We have relied principally upon the work of SIR WILLIAM HERSCHEL, STRUVE, SIR JOHN HERSCHEL, DAWES, MADLER, SECCHI, DEMBOWSKI, HALL, BURNHAM, SCHIAPARELLI and COMSTOCK.

The following elements of 4 *Aquarii* have been published by previous computers

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
$\frac{129.8}{126.65}$	1752 0 1899 88	0 46 0 543	$\frac{0.72}{0.7036}$	$\frac{340.2}{177.4}$	$\frac{56.6}{68.51}$	$\frac{235.0}{74.25}$	Doberck, 1877 See, 1895	A N, 2287 A J, 341

A revision of my former orbit of this star gives the following elements

$$\begin{aligned}
 P &= 129.0 \text{ years} & \Omega &= 177^{\circ} 7 \\
 T &= 1899.40 & i &= 72^{\circ} 53 \\
 e &= 0.514 & \lambda &= 68^{\circ} 63 \\
 a &= 0'' 732 & n &= +2^{\circ} 7907
 \end{aligned}$$

Apparent orbit

$$\begin{aligned}
 \text{Length of major axis} &= 1'' 288 \\
 \text{Length of minor axis} &= 0'' 43 \\
 \text{Angle of major axis} &= 0^{\circ} 3 \\
 \text{Angle of periastron} &= 215^{\circ} 2 \\
 \text{Distance of star from centre} &= 0'' 173
 \end{aligned}$$

The accompanying table of computed and observed places shows a very satisfactory agreement. The present orbit is narrower than the one recently published in the *Astronomical Journal*, 341, but the great discordance of results of individual observers shows that the object has always been extremely close;

and hence we think the chances favor the present orbit, which differs from the previous one chiefly in the higher inclination. It is noticeable that the representation of the more recent observations is sensibly improved.

COMPARISON OF COMPUTED WITH OBSERVED PLACES

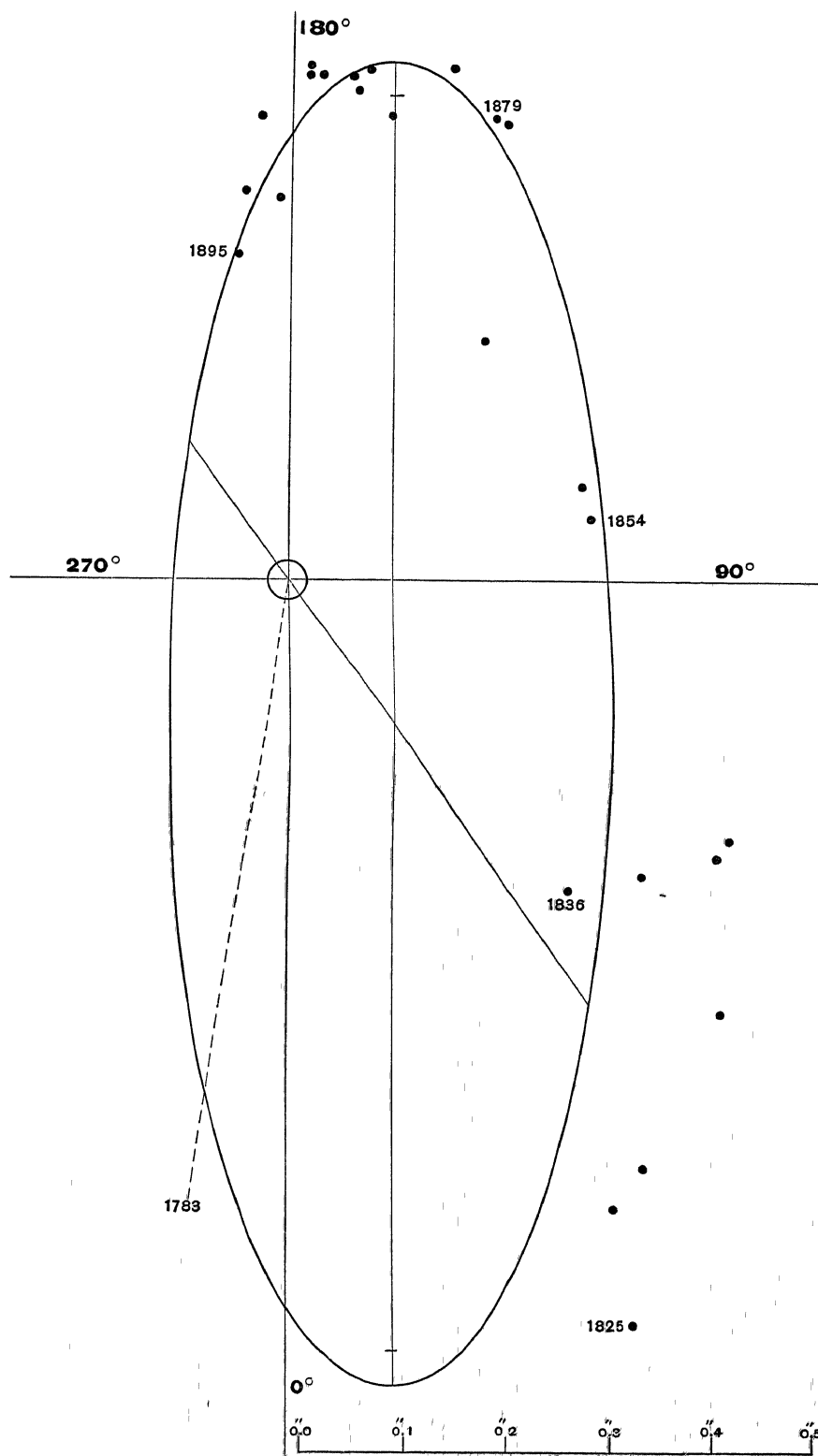
$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1783 55	351.5	352.2	—	0.53	— 0.7	—	1	Herschel
1825 60	25 0	24 0	0.80	0.64	+ 1.0	+0.16	1-2	Struve
1832 18	27 5	31 4	0.69	0.55	— 3.9	+0.14	4-1	Struve 1, Herschel 2-1, Struve 1
1833 77	31 2	33 5	0.67	0.53	— 2.3	+0.14	1	Struve
1836 05	40 9	37 0	0.41	0.50	+ 3.9	—0.09	4	Struve
1841 12	45 0	46 4	0.6±	0.43	— 1.4	+0.17	3	Dawes 2, Madler 1
1842 31	49 9	50 3	0.45	0.40	— 0.4	+0.05	3-1	Dawes 1, Madler 2-1
1843 73	56 8	53 0	0.5±	0.39	+ 3.8	+0.11	4-3	Madler 3, Dawes 1
1849 30	59 5	68 8	0.5±	0.34	— 9.3	+0.16	2	Madler 1, Dawes 1
1854 75	101 7	85 9	0.3±	0.31	+15.8	—0.01	1	Dawes
1856 81	107 8	97 8	0.3±	0.31	+10.0	—0.01	1	Secchi
1864 20	131 2	125.4	cuneo	0.34	+ 5.8	—	4	Dembowski 3, Secchi 1
1866 08	139 6	131 2	oblonga	0.36	+ 8.4	—	3	Dembowski 3
1867 86	141 1	136 2	0.30	0.38	+ 4.9	—0.08	1	Newcomb
1872 88	147 5	142.6	oblonga	0.41	+ 4.9	—	5	Dembowski 3
1876 82	154 7	155 2	0.49	0.47	— 0.5	+0.02	8	Schiaparelli 4, Dembowski 3, Cinn. 1
1879 60	156 2	159 7	0.49	0.49	— 3.5	±0.00	9-5	Cincinnati 5-1, Hall 4
1881 16	162 5	162 1	0.52	0.50	+ 0.4	+0.02	5	Pritchett 2, Burnham 3
1885 74	167 9	168 9	0.46	0.51	— 1.0	—0.05	3	Hall
1886 79	171 5	170 5	0.50	0.51	+ 1.0	—0.01	5-4	Leavenworth 3-2, Hall 2
1887 63	173 3	172 2	0.49	0.50	+ 1.1	—0.01	12	Schiaparelli 7, Hall 3, Tarrant 2
1888 81	172 4	173 5	0.48	0.49	— 1.1	—0.01	5	Schiaparelli
1889 88	176 7	175 3	0.49±	0.48	+ 1.4	+0.01	2	Schiaparelli
1890 78	178 2	176 9	0.49	0.47	+ 1.3	+0.02	2	Tarrant
1891 77	178 1	178 5	0.50±	0.45	— 0.4	+0.05	1	Schiaparelli
1892 85	181 7	181 0	0.37	0.42	+ 0.7	—0.05	2	Comstock 2-1, Schiaparelli 0-1
1893 25	183 4	181 8	0.45	0.41	+ 1.6	+0.04	5-4	Tarrant 3, Comstock 2-1
1894 86	186 5	185 3	0.38±	0.37	+ 1.2	+0.01	3	Schiaparelli
1895 67	189 0	188 8	0.32	0.33	+ 0.2	—0.01	4	Comstock 1, See 3

The period here indicated is not likely to be in error by more than five years, while a variation of  $\pm 0.03$  in the eccentricity does not seem probable. It is therefore unlikely that future observations will greatly alter the present elements, but as some improvement is still desirable, astronomers should continue to give this star careful attention. During the next few years the motion will be very rapid, and the object excessively difficult, but for this very reason observations will be the more valuable.

The following is an ephemeris for five years:

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 80	193.5	0.28	1899 80	224.0	0.14
1897 80	199.4	0.24	1900 80	244.1	0.12
1898 80	208.1	0.19			





4 Aquarii =  $\Sigma$  2729.

Scale.

$\delta$  EQUULEI =  $\sigma$  535.

$\alpha = 21^h 9^m 6$  ,  $\delta = +9^\circ 37'$   
 4 5, yellow , 5 0, yellow

*Discovered by Otto Struve, August 19, 1852*

## OBSERVATIONS

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1852 64	22 5	0 45	1	O Struve	1881 46	22 1	0 38	4	Burnham
1852 67	18 8	0 43	1	O Struve	1882 63	9 8	0 29	3	Burnham
1853 91	11 9	0 27	1	O Struve	1883 55	307 6	0 21	3	Burnham
1854 69	simple		1	O Struve	1886 84	203 5	0 47	2	Hall
1856 57	simple		1	O Struve	1886 87	204 6	0 35	6-2	Schiaparelli
1857 67	207 6	0 21	1	O Struve	1886 91	203 2	0 47	4	Englemann
1857 67	211 8	0 23	1	O Struve	1887 78	195 2	0 49	2-1	Hough
1858 59	16 8	0 40	1	O Struve	1887 79	195 8	0 44	5	Tarrant
1859 65	13 5	0 39	1	O Struve	1887 80	198 7	0 41	4	Hall
1861 57	236 ?	oblong	1	O Struve	1887 86	195 0	0 33	11-8	Schiaparelli
1865 91	203 3	<0 5	1	O Struve	1888 69	189 9	0 25	4	Burnham
1869 74	15 6	—	6-0	Harvard	1888 90	187 0	0 15	14-10	Schiaparelli
1870 73	8 0	—	1-0	Dunér	1889 51	163 2	0 10 $\pm$	1	Burnham
1874 67	24 0	oblong	1-0	O Struve	1889 82	193 1	0 2 $\pm$	1	Hough
1874 73	1 8	cuneiforme	1-0	O Struve	1889 84	175 0	0 15	3	Schiaparelli
1874 75	221 2	0 33	1	O Struve	1890 88	single	—	3	Schiaparelli
1877 76	156 4	0 2 $\pm$	1	Burnham	1891 63	31 6	0 20	5	Burnham
1878 65	elong. doubtful		2	Burnham	1891 85	23 4	0 21	5	Schiaparelli
1879 76	150 0	doubtful	2	Hall	1892 39	26 6	0 35	4	Burnham
1880 60	29 1	0 35	5	Burnham	1892 91	22 8	0 30	2	Schiaparelli
					1893 93	16 8	0 25	6	Schiaparelli
					1893 97	200 2	—	1	Bigoudan
					1894 85	simple	—	4	Schiaparelli

The pair was first measured in 1852, and when the observations were repeated the following year it was found that there was a slight diminution in the angle of position as well as in the distance. In 1854 and in 1856 the star was noted as single, but in 1857 the companion appeared in the opposite quadrant, and hence it became evident that the star is a binary in rapid retrograde motion. Continued observation disclosed the fact that the orbit is highly

inclined upon the visual ray, and STRUVE's measures seemed to indicate a period of 6.5 or 13 years. Since 1877 the star has been carefully followed by BURNHAM, and by means of his fine series of observations we are enabled to derive a very satisfactory orbit.

The two orbits heretofore published for this star are as follows

$P$	$T$	$e$	$a$	$\Omega$	$i$	$\lambda$	Authority	Source
$\frac{\text{yrs}}{11.48}$	1892.0	0.20	0".41	$24^{\circ}0'$	$81^{\circ}8'$	$26^{\circ}6'$	Wrublewsky, 1887	A. N., 2771
11.45	1892.80	0.14	0".452	$22^{\circ}2'$	$79^{\circ}05'$	$0^{\circ}00'$	See, 1895	A. N., 3290

An investigation of all the observations leads to the following elements of  $\delta$  Equulei

$$\begin{aligned}
 P &= 11.45 \text{ years} & \Omega &= 22^{\circ}2' \\
 T &= 1892.80 & i &= 79^{\circ}0' \\
 e &= 0.165 & \lambda &= 0^{\circ}0' \\
 a &= 0''.452 & n &= -31^{\circ}.441
 \end{aligned}$$

#### Apparent orbit

$$\begin{aligned}
 \text{Length of major axis} &= 0''.904 \\
 \text{Length of minor axis} &= 0''.171 \\
 \text{Angle of major axis} &= 22^{\circ}2' \\
 \text{Angle of periastron} &= 22^{\circ}2' \\
 \text{Distance of star from centre} &= 0''.075
 \end{aligned}$$

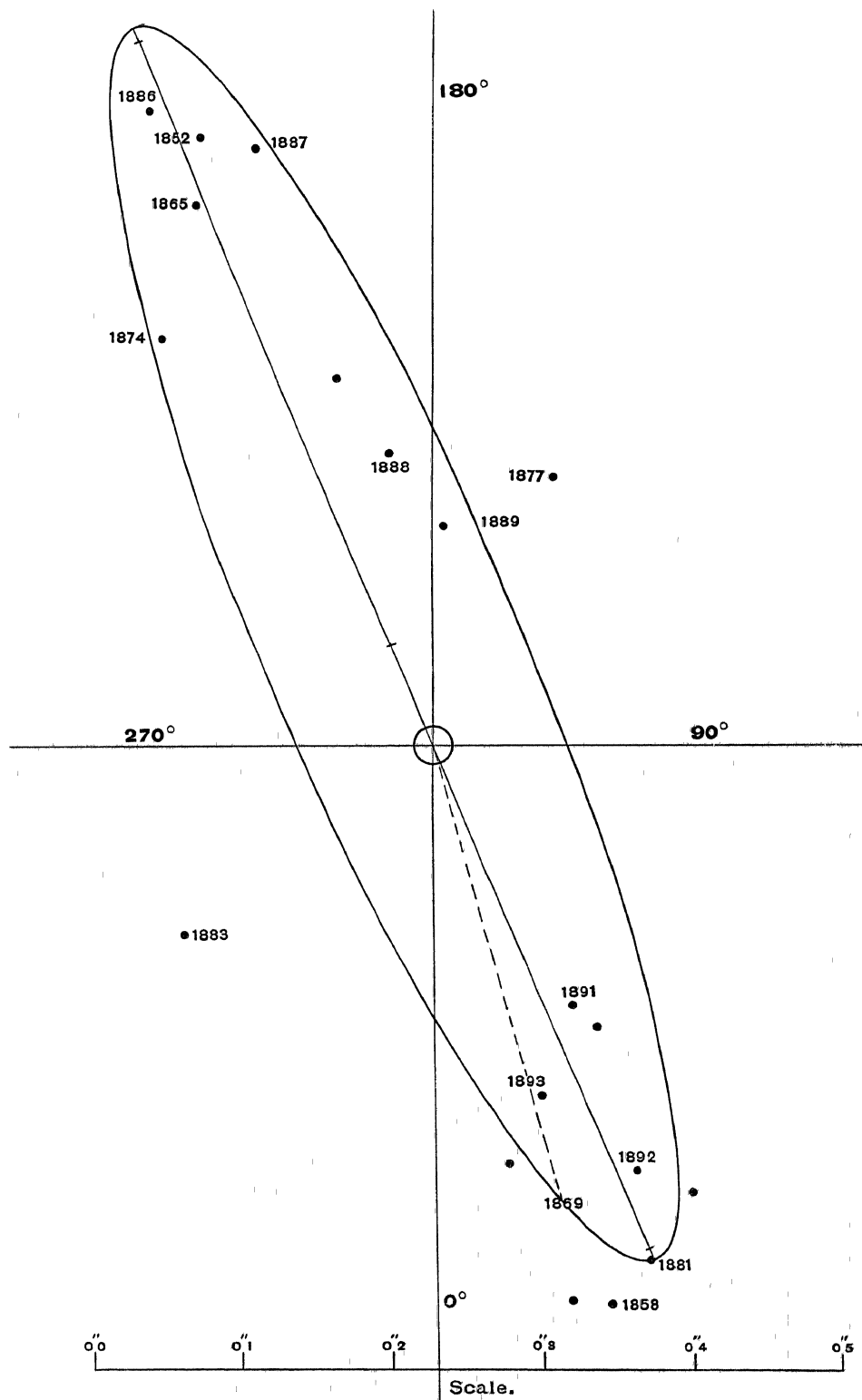
The following table gives a comparison of the computed with the observed places, and shows that the present elements will never require any considerable correction. Only a few large deviations occur, and these are probably to be explained by the extreme difficulty of the object.\*

BURNHAM's measure of 1877 is marked "doubtful," and is practically only an estimate, as the object was very difficult to separate.

It will be seen that the eccentricity of this orbit is considerably smaller than that generally found among double stars. It is also remarkable that the real major axis coincides with the line of nodes, so that  $\lambda$  is zero.

$\delta$  Equulei and  $\kappa$  Pegasi are the most rapid binaries in the heavens, and on this account are worthy of special attention from observers who have large telescopes. The elements given here need to be tested by further observation. It is especially important to determine the maximum distances of the companion when the angles are about  $22^{\circ}$  and  $202^{\circ}$  respectively, as this would furnish a more exact determination of the eccentricity and the major axis.

\* *Astronomische Nachrichten*, 3290



$$\sum \text{Equulei} = 0 \sum 535 = \sum 2777 \text{ AB}$$



$\kappa$  PEGASI =  $\beta$  989

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
	$^{\circ}$	$^{\circ}$	$''$	$''$	$^{\circ}$	$''$		
1852 65	200 6	202 5	0 44	0 52	- 1 9	-0 08	2	O Struve
1853 91	191 9	196 6	0 27	0 46	- 4 7	-0 19	1	O Struve
1857 67	29 7	29 6	0 22	0 33	+ 0 1	-0 11	2	O Struve
1858 59	16 8	21 0	0 40	0 38	- 4 2	+0 02	1	O Struve
1859 65	13 5	8 7	0 39	0 25	+ 4 8	+0 14	1	O Struve
1865 91	203 3	192 8	0 4 $\pm$	0 38	+10 5	+0 02 $\pm$	1	O Struve
1869 74	15 6	23 5	—	0 38	- 7 9	—	6	Harvard
1870.73	8 0	14 2	—	0 31	- 6 2	—	1	Dunér
1874 74	212 6	206 2	0 33	0 48	+ 6 4	-0 15	2-1	O Struve
1877 76	156 4	187 9	0 2 $\pm$	0 30	-31 5	-0 10 $\pm$	1	Burnham
1880 60	29 1	29 3	0 35	0 33	- 0 2	+0 02	5	Burnham
1881 46	22 1	21 2	0 38	0 37	+ 0 9	+0 01	4	Burnham
1882 63	9 8	7 1	0 29	0 24	+ 2 7	+0 05	3	Burnham
1883 55	307 6	302 2	0 21	0 09	+ 5 4	+0 12	3	Burnham
1886 87	203 8	203 1	0 47	0 52	+ 0 7	-0 05	12-6	Hall 2, Schiaparelli 6-2, Engleman
1887 81	196 2	198 9	0 42	0 50	- 2 7	-0 08	22-18	Ho 2-1, Tar 5, Hall 4, Schiaparelli
1888 80	188 5	192 9	0 20	0 49	- 4 4	-0 29	18-14	Burnham 4, Schiaparelli 14-10
1889 72	177 1	180 0	0 15	0 22	- 2 9	-0 07	5	Burnham 1, Hough 1, Schiaparelli
1890 88	single	65 4	—	0 12	—	—	3	Schiaparelli
1891 74	27 5	35 0	0 20	0 26	- 7 5	-0 06	10	Burnham 5, Schiaparelli 5
1892 65	24 7	23 5	0 32	0 38	+ 1 2	-0 06	6	Burnham 4, Schiaparelli 2
1893 93	16 8	10 0	0 25	0 26	+ 6 8	-0 01	6	Schiaparelli
1894 85	simple	324 8	—	0 10	—	—	—	Schiaparelli

The following is a short ephemeris:

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 85	211 1	0 39	1899 85	195 8	0 44
1897 85	205 2	0 50	1900 85	186 4	0 28
1898 85	200 8	0 52			

$\kappa$  PEGASI =  $\beta$  989.

$\alpha = 21^{\text{h}} 40^{\text{m}} 1$  ,  $\delta = +25^{\circ} 11'$   
43, yellowish , 5 0, yellowish

Discovered by Burnham, August 12, 1880

OBSERVATIONS

$t$	$\theta_o$	$\rho_o$	$n$	Observers	$t$	$\theta_o$	$\rho_o$	$n$	Obs
	$^{\circ}$	$''$				$^{\circ}$	$''$		
1880 68	137 9	0 27	4	Burnham	1891 61	150 0	0 10	3	Burn
1883 02	116 0	0 16	1	Englemann	1891 81	144 6	0 13	4	Burn
1888 78	274 7	0 23	3	Burnham	1891 92	159 0	0 18	3	Schi
1889 51	262 3	0 14	4	Burnham	1892 39	132 8	0 18	4	Burn
1890 57	187.1	0 10	4	Burnham	1892 88	131 0	0 20	1	Bar
					1892 96	135 1	0 20	4	Sch

$t$	$\theta_0$	$\rho_0$	$n$	Observers	$t$	$\theta_0$	$\rho_0$	$n$	Observers
1893 51	121 0	0 29	3	Leavenworth	1894 51	117 6	0 19	7 6	Barnard
1893 77	127 5	0 20	2	Barnard	1894 83	114 8	0 11	4	Lewis
1893 82	130 5	0 25	2-1	Comstock	1891 87	114 7	0 21	6	Schiaparelli
1893 92	123 6	0 27	8	Schiaparelli	1895 62	107 9	0 17	6	Barnard

This remarkable double star was discovered with the 18-inch refractor of the Dearborn Observatory. Its extreme closeness led to the belief that it would prove to be binary,\* and accordingly it has been found to be in rapid revolution. DR ENGLEMAN of Leipzig succeeded in making one measure of the pair in 1883, which indicated a retrograde motion. BURNHAM's measures were continued at the Lick Observatory from 1888 to 1892, and the new data thus obtained enabled him for the first time to get the approximate period of revolution (*Monthly Notices*, March, 1891).

At the request of BURNHAM and the writer, BARNARD has since followed the star, and obtained additional measures which appear to be sufficient to give us a reasonably good approximation to the elements of the orbit. In his first examination of the motion of this pair, BURNHAM made the orbit nearly circular, but the recent observations show that the orbit has about the usual eccentricity prevailing among binaries, and that the inclination of the orbit is very high. In the *Monthly Notices* for November, 1891, Mr LEWIS has given a set of measures recently obtained with the Greenwich 28-inch refractor, and sketched an apparent orbit which would better satisfy the latest observations.

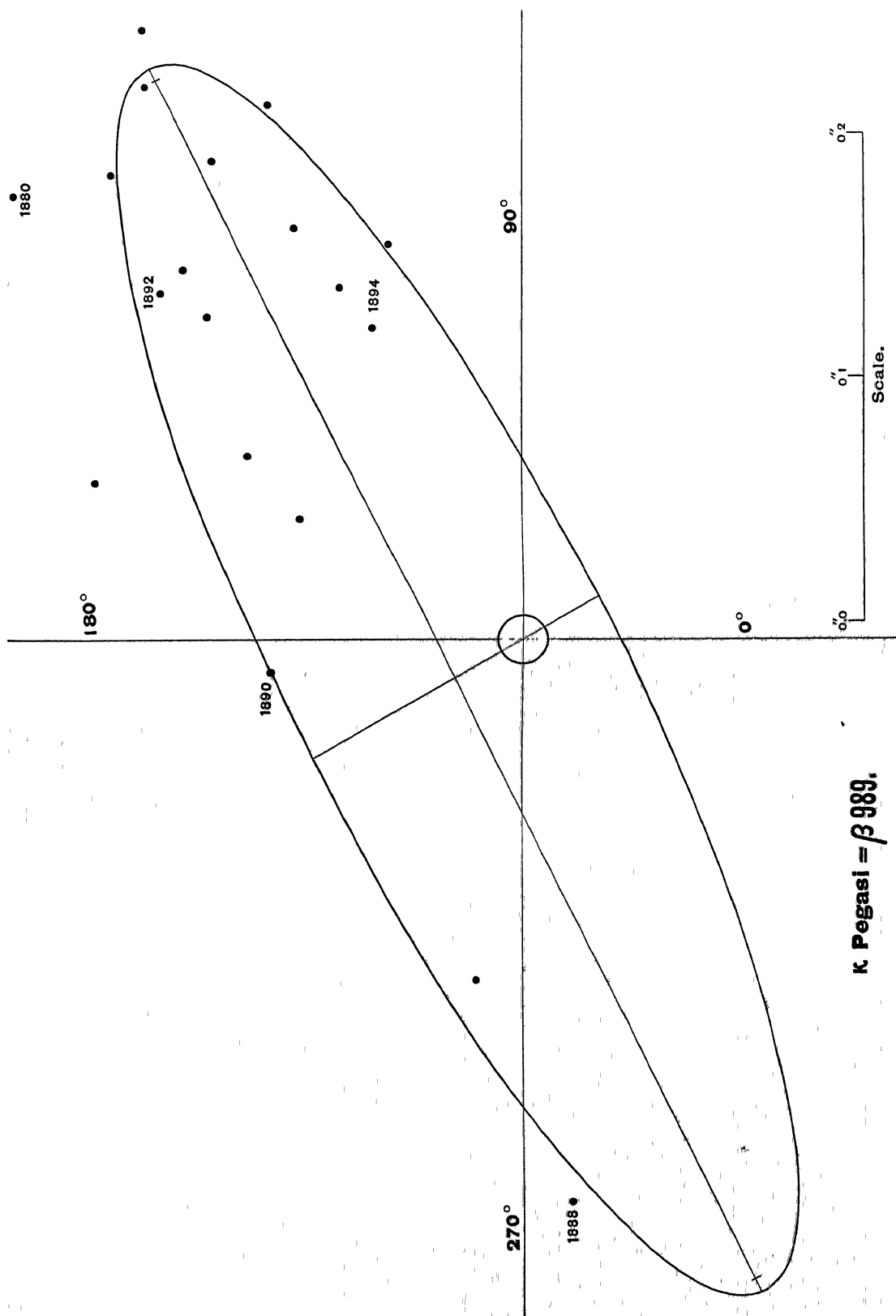
Having collected all the observations of this difficult star, including some unpublished measures kindly furnished by BARNARD last Autumn, we have investigated the orbit by the method of KLINKERFUES, and find the following elements:

$$\begin{array}{ll}
 P = 11.42 \text{ years} & i = 81^\circ 2' \\
 T = 1896.03 & \Omega = 116^\circ 25' \\
 e = 0.49 & \lambda = 89^\circ 2' \\
 a = 0''.4216 & n = -31^\circ 52'36''
 \end{array}$$

#### Apparent orbit:

$$\begin{array}{ll}
 \text{Length of major axis} & = 0''.555 \\
 \text{Length of minor axis} & = 0''.130 \\
 \text{Angle of major axis} & = 115^\circ 7' \\
 \text{Angle of periastron} & = 30^\circ 2' \\
 \text{Distance of star from centre} & = 0''.032
 \end{array}$$

\* *Astronomische Nachrichten*, 3285



$\kappa$  Pegasi =  $\beta$  989.



COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1880 68	137 9	136 7	0 27	0 22	+ 1 2	+0 05	4	Burnham
1883 02	116 0	119 5	0 16	0 27	- 3 5	-0 11	1	Englemann
1888 78	274 7	274 1	0 23	0 21	+ 0 6	+0 02	3	Burnham
1889 51	262 3	257 9	0 14	0 15	+ 4 4	-0 01	4	Burnham
1890 57	187 1	191 5	0 10	0 10	- 4 4	$\pm$ 0 00	4	Burnham
1891 61	150 0	145 0	0 10	0 18	+ 5 0	-0 08	3	Burnham
1891 81	144 6	140 2	0 13	0 20	+ 4 4	-0 07	4	Burnham
1891 92	159 0	139 2	0 18	0 20	+19 8	-0 02	3	Schiaparelli
1892 39	132 8	133 2	0 18	0 24	- 0	-0 06	4	Burnham
1892 88	131 0	129 1	0 20	0 26	+ 1 9	-0 06	1	Barnard
1892 96	135 1	128 2	0 20	0 26	+16 9	-0 06	4	Schiaparelli
1893 51	121 0	125 5	0 29	0 27	- 4 5	+0 02	3	Leavenworth
1893 77	127 5	123 2	0 20	0 28	+ 4 3	-0 08	2	Barnard
1893 82	130 5	123 0	0 25	0 28	+ 7 5	-0 03	2-1	Comstock
1893 92	123 6	122 2	0 27	0 28	+ 1 4	-0 01	8	Schiaparelli
1894 51	117 6	118 8	0 19	0 26	- 1 2	-0 07	7-6	Barnard
1894 83	114 8	116 7	0 14	0 25	- 1 9	-0 11	4	Lewis
1894 87	114 7	116 6	0 24	0 25	- 1 9	-0 01	6	Schiaparelli
1895 62	107 9	106 7	0 17	0 16	+ 1 2	+0 01	6	Barnard

## EPIHEMERIS

$t$	$\theta_o$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 80	299 4	0 21	1899 80	279 0	0 24
1897 80	292 6	0 27	1900 80	260 4	0 16
1898 80	287 0	0 28			

The agreement must be considered very satisfactory when account is taken of the extreme closeness of the components, and the high inclination of the orbit, which permits a small error in angle to have a marked effect on the distance. From an examination of all the measures it seems probable that most observers have underestimated the distances, and this certainly must have been the case with DR ENGLEMAN, who used only a 7.5-inch refractor, and therefore could not have divided the components at a distance of 0" 16. The computed distance is therefore much more probable, and especially since the elements are based principally upon the excellent measures of BURNHAM and BARNARD, made with the 36-inch refractor of the Lick Observatory.

BURNHAM has repeatedly called the attention of astronomers to the high importance of systematically following such extremely rapid binaries with large telescopes, so that we could in a few years derive orbits, which, in the case of most stars, would require the observations of centuries.

We would beg to add that it is not only important to observe  $\kappa$  *Pegasi* annually, but especially at certain critical parts of its orbit, where measures would enable us to fix the eccentricity and the inclination more accurately. Thus, according to the above elements, the minimum distance will occur just

after periastron passage in 1896 03, and measures made on either side of the periastron will be very valuable At the minimum distance (0".034) the star will be single in the largest telescope in the world, but it would be important to ascertain just when this disappearance takes place, and how long it lasts According to the above orbit, the companion ought to be visible in a 30-inch refractor until August, 1895, and hence we suggest that observers should watch for it during the Summer of 1895 and the Autumn of 1896 Good observations at these epochs will be of the greatest value in improving the elements of the orbit.

85 PEGASI =  $\beta$  733.

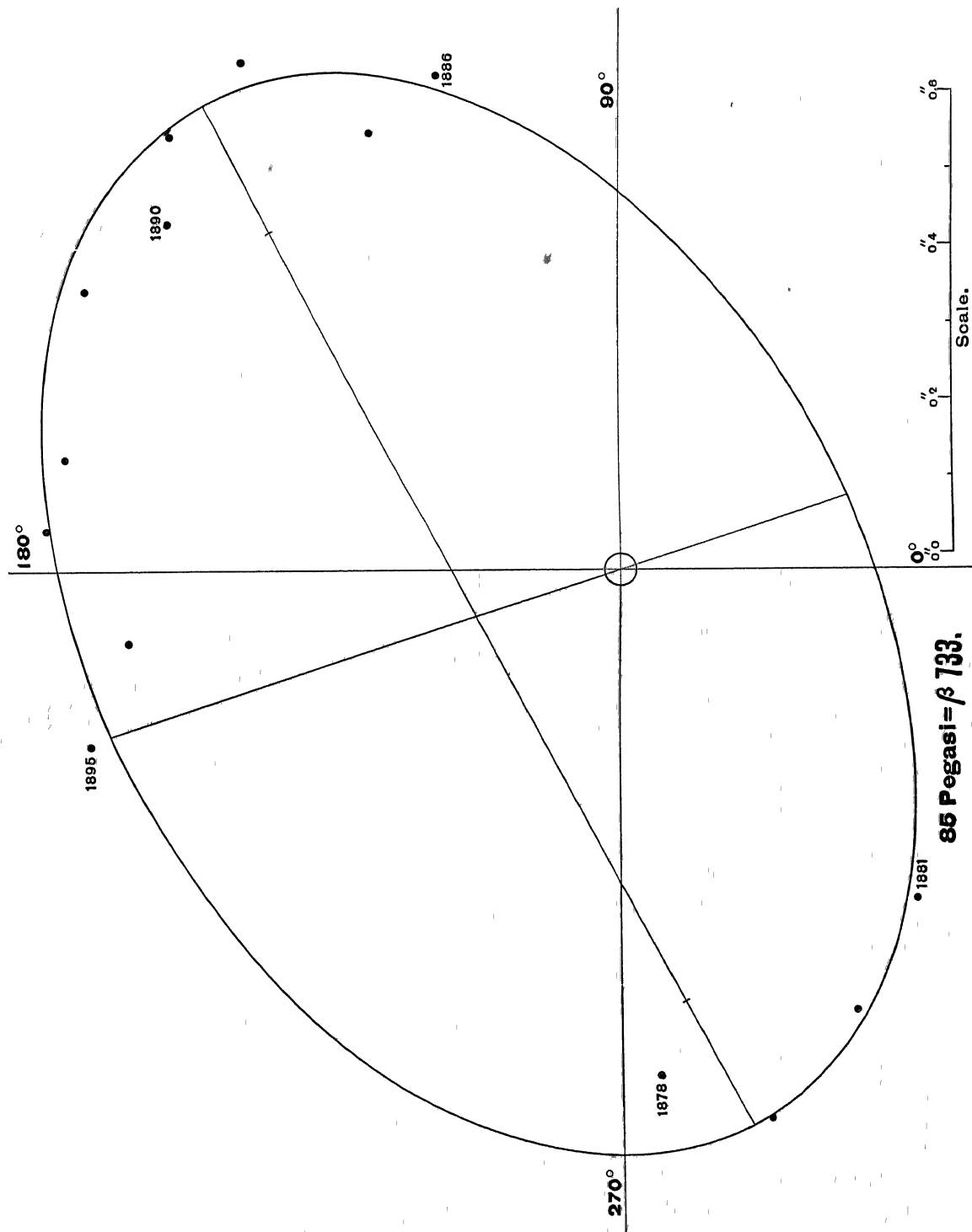
$\alpha = 23^{\text{h}} 56^{\text{m}} 9$  ,  $\delta = +26^{\circ} 34'$   
6, yellowish , 10, bluish

*Discovered by Burnham in 1878*

OBSERVATIONS									
<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers	<i>t</i>	$\theta_0$	$\rho_0$	<i>n</i>	Observers
1878 73	274 0	0 67	3	Burnham	1889 59	134 7	0 94	5	Burnham
1879 46	284 6	0 75	5	Burnham	1889 90	137 0	0 70	5	Schiaparelli
1880 59	298 3	0 65	5	Burnham	1890 55	139 0	0 78	4	Burnham
1880 79	297 2	0 66	3-2	Hall	1890 96	146 4	0 71	6	Schiaparelli
1881 54	311 5	0 58	1	Burnham	1891 56	151 8	0 79	3	Burnham
1882 62	89 4	0 64	1	O Struve	1891 94	152 7	0 78	3	Schiaparelli
1883 75	333 ±	—	1	Burnham	1892 75	169 7	0 57	1	Burnham
1886 91	109 1	0 79	3	Hall	1892 94	167 3	0 74	4	Schiaparelli
1886 98	111 0	0 58	1	Schiaparelli	1893 96	176 1	0 75	6-3	Schiaparelli
1887 91	119 3	0 66	1	Schiaparelli	1894 54	178 6	0 84	5	Barnard
1888 69	126 7	0 95	5	Burnham	1894 88	251 8	0 85	1	Lewis
1888 96	124 1	0 83	3	Hall	1894 93	188 6	0 65	2-1	Schiaparelli
1888 96	128 3	0 70	7	Schiaparelli	1895 65	190 2	0 80	10-9	Barnard
					1895 73	198 4	0 73	3	See
					1895 74	204 8	0 75	2	Moulton

Since BURNHAM'S discovery of this rapid binary, the companion has described an arc of 235°\* The components are of the 6th and 11th magnitude and so great an inequality in brightness combined with the closeness of the pair, renders exact measurement very difficult Therefore it is not strange that

\* *Astronomische Nachrichten*, 3339



85 Pegasi =  $\beta$  733.

the position-angles as well as the distances obtained by the same or by different observers should occasionally exhibit sensible discrepancies. Yet when the measures are properly combined into suitable yearly means we obtain a series of places which will give an orbit that is substantially correct

The first orbit of this pair was computed by PROFESSOR SCHAEBERLE in 1889; his elements are

$$\begin{array}{ll} P = 22.3 \text{ years} & \Omega = 306^\circ 1 \\ T = 1884.00 & i = 68^\circ 6 \\ e = 0.35 & \lambda = 70^\circ 3 \\ a = 0''.96 & n = +16^\circ 144 \end{array}$$

This orbit represents the measures prior to 1891 with the desired accuracy, but the error in angle rapidly accumulated and in 1892 surpassed  $20^\circ$ . Accordingly, PROFESSOR GLASENAPP attempted an improvement of the orbit (*A.N.* 3145), and obtained a set of elements which rendered the residuals in angle exceedingly small.

$$\begin{array}{ll} P = 17.487 \text{ years} & \Omega = 307^\circ 32 \\ T = 1884.21 & i = 66^\circ 74 \\ e = 0.164 & \lambda = 69^\circ 73 \\ a = 0''.80 & n = +20^\circ 586 \end{array}$$

Nevertheless the ephemeris computed by PROFESSOR GLASENAPP has signally failed of its purpose, as the error now amounts to about  $80^\circ$ . As the investigation was based wholly on angles of position we may infer that these coördinates were affected by sensible systematic errors, which might the more easily result from the inequality of the stars.

The careful measures which I recently secured at the Washburn Observatory (*A.J.* 359) have enabled me to make a new determination of the orbit based on all the material of a trustworthy character. We find the following elements of 85 *Pegasi*:

$$\begin{array}{ll} P = 24.0 \text{ years} & \Omega = 116^\circ 3 \\ T = 1883.80 & i = 55^\circ 6 \\ e = 0.388 & \lambda = 265^\circ 4 \\ a = 0''.8904 & n = +15^\circ 0 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 1''.52 \\ \text{Length of minor axis} & = 1''.00 \\ \text{Angle of major axis} & = 118^\circ 0 \\ \text{Angle of periastron} & = 18^\circ 2 \\ \text{Distance of star from centre} & = 0''.197 \end{array}$$

The accompanying table gives a comparison of the computed with the observed places

COMPARISON OF COMPUTED WITH OBSERVED PLACES

$t$	$\theta_o$	$\theta_c$	$\rho_o$	$\rho_c$	$\theta_o - \theta_c$	$\rho_o - \rho_c$	$n$	Observers
1878 73	274 0	275 5	0 67	0 77	-1 5	-0 10	3	Burnham
1879 46	284 6	282 2	0 75	0 76	+2 4	-0 01	5	Burnham
1880 69	297 7	294 4	0 66	0 69	+3 3	-0 03	8-7	Burnham 5, Hall 3-2
1881 54	311 5	309 0	0 58	0 58	+2 5	$\pm 0 00$	1	Burnham
1886 94	110 1	113 4	0 69	0 69	-3 3	$\pm 0 00$	4	Hall 3, Schiaparelli
1887 91	119 3	122 8	0 66	0 77	-3 5	-0 11	1	Schiaparelli
1888 87	126 4	130 8	0 83	0 81	-4 4	+0 02	15	$\beta$ 5, Hall 3, Schiaparelli 7
1889 74	135 8	137 8	0 82	0 83	-2 0	-0 01	10	Burnham 5, Schiaparelli 5
1890 76	142 7	146 0	0 75	0 83	-3 3	-0 08	10	Burnham 4, Schiaparelli 6
1891 75	152 2	154 7	0 79	0 81	-2 5	-0 02	6	Burnham 3, Schiaparelli 3
1892 85	168 5	165 0	0 74	0 78	+3 5	-0 04	5-4	Burnham 1-0, Schiaparelli 4
1893 96	176 1	176 4	0 75	0 75	-0 3	$\pm 0 00$	6-3	Schiaparelli
1894 93	188 6	187 5	0 65	0 72	+1 1	-0 07	2-1	Schiaparelli
1895 73	198 4	197 4	0 73	0 70	+1 0	+0 03	3	See

We are justified in predicting that the true period of 85 *Pegasi* will not differ from the value given above by more than one year, and that the error of the eccentricity will not surpass  $\pm 0.02$ . The good representation of the angles and distances shows that the other elements are equally satisfactory. The foregoing elements will therefore never be greatly changed, but some improvement is desirable, and observers with great telescopes should continue to give this important system regular attention. The following is an ephemeris for the next five years

$t$	$\theta_c$	$\rho_c$	$t$	$\theta_c$	$\rho_c$
1896 70	209 6	0 70	1899 70	245 8	0 74
1897 70	222 4	0 69	1900 70	256 1	0 76
1898 70	234 5	0 71			

## CHAPTER III.

### RESULTS OF RESEARCHES ON THE ORBITS OF FORTY BINARY STARS, WITH GENERAL CONSIDERATIONS RESPECTING THE STELLAR SYSTEMS

#### § 1 *Elements of the Orbits of Forty Binary Stars.*

IN THE preceding chapter we have presented detailed researches on the orbits of forty stars. To enable the reader to grasp readily the existing state of our knowledge, we have also included diagrams of the apparent ellipses, and of the mean observations from which the elements were derived. In many cases we have seen that the observations are relatively rough, and that while the errors are small absolutely, they are yet very large in comparison with the minute quantities measured. Under these circumstances it seemed useless to attempt a Least-Square adjustment of the residuals, and hence we have throughout employed graphical methods, and arrived at the adopted elements by successive approximations of an empirical character. Accordingly, the orbits are not definitive, but for reasons indicated in the several cases the changes which future observations may necessitate will be confined within narrow limits.

In the following Table we give a summary of the elements, with the probable uncertainty still attaching to the period and the eccentricity. From the variations of these elements it is easy to see about the extent of the alterations which may be required in the adopted values of the other elements. The final changes which future observations may produce in any given orbit can not yet be determined with certainty, and hence our variations may occasionally turn out somewhat too small; but as care has been exercised to avoid over-estimation of the accuracy of results, the values here indicated ought not to prove very deceptive.

In glancing over the apparent orbits of the preceding chapter the reader should remember that the adopted elements depend not only on the agreement of the observed distances with the apparent ellipses, but also on the accuracy with which the law of areas is satisfied. These two criteria seem to justify the comparatively small variations indicated in the Table of elements; but as

the orbits here presented depend essentially on the observations employed, and as our choice is to some extent a matter of judgement, it is not certain that we have always arrived at the best results

## RESULTS OF RESEARCHES ON THE

Star	$\alpha$	$\delta$	$P$	$T$	$e$	$a''$	$\Omega$	$i$	$\lambda$	
$\Sigma 3062$	<sup>h m</sup> 0 1	<sup>° ' "</sup> +57 53	<sup>hrs</sup> 104 61	$\pm 20$	1836 26	$0.450 \pm 0.02$	<sup>"</sup> 1 37 12	<sup>°</sup> 47 15	<sup>°</sup> 13 85	<sup>°</sup> 90 9
$\eta$ Cassiopeae = $\Sigma 60$	0 42 9	+57 18	195 76	$\pm 100$	1907 84	$0.514 \pm 0.03$	8 21 28	46 1	15 95	217 87
$\gamma$ Androm BC = $O\Sigma 38$	1 57 8	+41 51	54 0	$\pm 10$	1892 1	$0.857 \pm 0.02$	0 37 05	113 1	77 85	200 1
$\alpha$ Can Maj = Sirius	6 40 4	-16 34	52 20	$\pm 20$	1893 50	$0.620 \pm 0.02$	8 03 16	31 3	16 77	131 03
F 9 Argus = $\beta 101$	7 47 1	-13 38	22 0	$\pm 10$	1892 30	$0.700 \pm 0.02$	0 65 49	95 5	77 72	75 28
$\zeta$ Cancer AB = $\Sigma 1196$	8 6 2	+17 58	60 0	$\pm 05$	1870 40	$0.340 \pm 0.03$	0 85 79	88 7	7 4	261 0
$\Sigma 3121$	9 12 1	+29 0	34 0	$\pm 10$	1878 30	$0.330 \pm 0.03$	0 66 92	28 25	75 0	127 52
$\omega$ Leonis = $\Sigma 1356$	9 23 1	+9 30	116 20	$\pm 10$	1842 10	$0.537 \pm 0.01$	0 88 24	146 70	63 47	124 22
$\phi$ Urs Maj = $O\Sigma 208$	9 45 3	+54 33	97 0	$\pm 50$	1884 0	$0.440 \pm 0.03$	0 34 10	160 3	30 5	15 9
$\xi$ Urs Maj = $\Sigma 1523$	11 12 9	+32 6	60 0	$\pm 01$	1875 22	$0.397 \pm 0.005$	2 50 80	100 8	55 92	126 33
$O\Sigma 234$	11 25 4	+41 50	77 0	$\pm 50$	1880 10	$0.302 \pm 0.04$	0 34 67	157 5	50 8	206 8
$O\Sigma 235$	11 26 7	+61 38	80 0	$\pm 50$	1834 30	$0.324 \pm 0.05$	0 86 90	81 7	49 32	137 78
$\gamma$ Centauri = $H_2 5370$	12 36	-48 25	88 0	$\pm 30$	1848 0	$0.800 \pm 0.03$	1 02 32	4 6	62 15	191 3
$\gamma$ Virginis = $\Sigma 1670$	12 36 6	-0 54	194 0	$\pm 40$	1836 53	$0.897 \pm 0.005$	3 98 90	50 4	31 0	270 0
F 42 Com Bel = $\Sigma 1728$	13 5 1	+18 4	25 56	$\pm 01$	1885 69	$0.461 \pm 0.01$	0 64 16	11 9	90 $\pm$	280 5
$O\Sigma 269$	13 28 3	+35 46	48 8	$\pm 10$	1882 80	$0.361 \pm 0.05$	0 32 48	46 2	71 3	32 63
25 Can Ven = $\Sigma 1768$	13 33	+36 48	184 0	$\pm 250$	1866 0	$0.752 \pm 0.05$	1 13 07	123 0	33 5	201 0
$\alpha$ Centauri	14 32 6	-60 25	81 10	$\pm 03$	1875 70	$0.528 \pm 0.005$	17 70 0	25 15	79 30	52 0
$O\Sigma 285$	14 41 7	+42 48	76 67	$\pm 50$	1882 53	$0.470 \pm 0.05$	0 39 75	62 2	41 95	162 23
$\xi$ Bootis = $\Sigma 1888$	14 46 8	+19 31	128 0	$\pm 10$	1903 90	$0.721 \pm 0.02$	5 55 78	10 5	52 28	239 25
$\eta$ Cor Bor = $\Sigma 1937$	15 19 1	+30 39	41 60	$\pm 01$	1892 50	$0.267 \pm 0.01$	0 91 65	27 1	58 5	217 57
$\mu^2$ Bootis = $\Sigma 1938$	15 20 7	+37 43	219 42	$\pm 100$	1865 30	$0.537 \pm 0.03$	1 26 79	163 8	43 9	329 75
$O\Sigma 298$	15 32 4	+40 9	52 0	$\pm 10$	1883 0	$0.581 \pm 0.02$	0 79 89	1 9	60 9	26 1
$\gamma$ Cor Bor = $\Sigma 1967$	15 38 5	+26 36	73 0	$\pm 20$	1841 0	$0.482 \pm 0.05$	0 73 57	110 7	82 63	97 95
$\xi$ Scorpii AB = $\Sigma 1998$	15 58 9	-11 5	104 0	$\pm 40$	1864 60	$0.131 \pm 0.05$	1 36 12	9 5	70 3	111 6
$\sigma$ Cor Bor = $\Sigma 2032$	16 11	+34 7	370 0	$\pm 250$	1821 80	$0.540 \pm 0.04$	3 81 87	30 5	17 18	17 7
$\zeta$ Herculis = $\Sigma 2084$	16 37 6	+31 47	35 0	$\pm 03$	1864 80	$0.497 \pm 0.03$	1 43 21	37 5	51 77	101 7
$\beta 416$ = Lac 7215	17 12 1	-34 52	33 0	$\pm 10$	1891 85	$0.512 \pm 0.03$	1 22 12	111 6	37 35	86 1
$\Sigma 2173$	17 25 3	-0 59	46 0	$\pm 04$	1869 50	$0.200 \pm 0.03$	1 14 28	153 7	80 75	322 2
$\mu^1$ Herculis BC = A C 7	17 42 6	+27 47	45 0	$\pm 10$	1879 80	$0.219 \pm 0.02$	1 39 00	61 4	61 28	180 0
$\tau$ Ophiuchi = $\Sigma 2262$	17 57 6	-8 11	230 0	$\pm 150$	1815 0	$0.592 \pm 0.05$	1 24 95	76 4	57 6	18 05
F 70 Ophiuchi = $\Sigma 2272$	18 0 4	+2 33	88 39 54	$\pm 10$	1896 46 61	$0.500 \pm 0.02$	4 54 8	125 7	58 42	198 25
F 99 Herculis = A C 15	18 3 2	+30 33	54 5	$\pm 30$	1887 70	$0.781 \pm 0.02$	1 01 4	indeter	0 0	(*)
$\zeta$ Sagittarii	18 56 3	-30 1	18 85	$\pm 10$	1878 80	$0.279 \pm 0.02$	0 68 60	69 3	67 32	328 1
$\gamma$ Coronae Australis	18 59 6	-37 12	152 7	$\pm 50$	1876 80	$0.420 \pm 0.02$	2 45 3	72 3	31 0	180 2
$\beta$ Delphini = $\beta 151$	20 32 9	+14 15	27 66	$\pm 10$	1883 05	$0.373 \pm 0.03$	0 67 24	3 9	61 35	161 93
F 4 Aquarii = $\Sigma 2729$	20 46 1	-6 1	129 0	$\pm 50$	1899 40	$0.514 \pm 0.03$	0 73 20	177 7	72 53	68 63
$\delta$ Equulei AB = $O\Sigma 535$	21 9 6	+9 37	11 45	$\pm 02$	1892 80	$0.165 \pm 0.02$	0 45 2	22 2	79 0	0 00
$\kappa$ Pegasi = $\beta 989$	21 40 1	+25 11	11 42	$\pm 04$	1896 03	$0.490 \pm 0.1$	0 42 16	116 25	81 2	89 2
F 85 Pegasi = $\beta 733$	23 56 9	+26 34	24 0	$\pm 10$	1883 80	$0.388 \pm 0.02$	0 89 04	116 3	55 6	256 4

(\*) Angle Per =  $169^\circ 5$

In the course of the next twenty years a sensible improvement can be effected in the orbits of rapidly moving stars, such as  $\kappa$  Pegasi, but mean-

while the elements here adopted will give ephemerides sufficiently exact for the use of observers

Vigorous prosecution of the measurement of double stars will furnish the

## ORBITS OF FORTY BINARY STARS

$n$	Maj Axis App Orbit	Min Axis App Orbit	Angle of Maj Axis	Angle of Periastr	Star from Center	$\frac{\rho}{\alpha}$	$\pm \frac{\kappa}{\rho}$	Magnitude	Light-ratio	Colors	Proper Motion	$\Gamma$	$\Gamma'$
+ 3 4414	2 526	1 984	45 7	138 4	0 446	0 572	0 261	6 9, 7 5	1 1 75	yellowish bluish white	0 267	67 4	59 3
+ 1 8390	15 81	10 24	55 8	254 5	3 80	1 320	0 716	4 , 7	1 15 85	yellow purple	1 199	70 2	56 0
- 6 6667	0 706	0 084	109 9	289 0	0 298	0 611	0 232	5 5, 7	1 3 99	bluish bluish		13 1	32 2
- 6 8966	14 63	9 50	50 7	252 4	4 16	1 161	0 715	1 , 10	1 39 81	white yellow	1 306	83 0	70 7
+16 3636	0 941	0 267	99 2	134 5	0 152	0 595	0 977	5 7, 6 3	1 1 74	yellow yellow		54 0	68 3
- 6 0000	1 704	1 632	8 8	184 9	0 290	0 659	0 053	5 5, 6 2	1 1 91	yellow yellow	0 115	62 4	67 6
+10 5883	1 318	0 349	27 4	189 6	0 142	0 670	0 716	7 2, 7 5	1 1 32	yellowish white	0 523	81 1	56 2
+ 3 0981	1 576	0 738	141 1	293 4	0 317	0 460	0 387	6 , 7	1 2 51	yellow yellow	0 040	13 4	65 7
+ 3 7114	0 690	0 530	167 6	174 1	0 149	1 015	0 032	5 5, 5 5	1 1	yellowish yellowish	0 028	26 4	64 2
- 6 0000	4 760	2 700	104 6	318 0	0 750	0 665	0 062	4 , 5	1 2 51	yellowish yellowish	0 736	59 4	57 5
+ 4 6754	0 695	0 437	158 0	355 2	0 098	0 924	0 399	7 , 7 8	1 2 09	yellowish yellowish	0 115	42 1	64 6
+ 4 5000	1 682	1 020	72 8	231 1	0 242	0 912	0 577	6 , 7 8	1 5 25	yellowish yellowish	0 129	89 4	19 3
- 4 0911	2 100	0 580	0 1	177 8	0 794	0 198	0 438	4 , 4	1 1	yellowish yellowish		83 3	74 0
- 1 8557	6 824	3 530	140 4	140 4	3 062	0 172	0 403	3 , 3 2	1 1 20	yellow yellow	0 578	54 2	50 8
+14 0867	1 147	0 00	11 9	11 9	0 054	0 575	0 187	6 , 6	1 1	orange orange	0 488	84 9	84 9
+ 7 3771	0 64	0 20	47 7	57 8	0 102	0 779	0 849	7 3, 7 7	1 1 45	yellowish yellowish		69 3	60 5
- 1 9565	1 910	1 08	108 9	285 4	0 714	0 525	0 298	5 , 8 5	1 25 12	white blue	0 114	19 8	47 3
+ 4 4390	32 18	6 16	27 25	38 65	5 90	0 670	0 982	1 , 2	1 2 51	or yellow or yellow	3 685	47 8	88 1
- 4 6953	0 788	0 522	67 1	255 3	0 182	0 851	0 146	7 5, 7 6	1 1 10	yellowish whitish		48 0	47 7
- 2 8125	9 07	5 76	167 7	144 7	2 94	1 100	0 790	4 5, 6 5	1 6 31	yellow purple	0 161	80 4	24 7
+ 8 6538	1 804	0 934	28 7	229 0	0 209	1 152	0 661	5 5, 6	1 1 59	yellowish yellowish	0 217	30 0	89 5
- 1 6407	2 656	1 480	173 5	186 7	0 638	0 912	0 419	6 5, 8	1 3 98	white white	0 194	11 5	77 5
+ 6 9231	1 546	0 656	186 9	15 3	0 427	0 582	0 695	7 , 7 4	1 1 45	yellowish yellowish	0 500	25 5	83 1
- 4 9315	1 300	0 175	111 3	329 6	0 068	0 725	0 973	4 , 7	1 15 85	yellowish white	0 115	78 0	89 6
+ 3 4616	2 696	0 884	9 6	150 2	0 085	0 935	0 639	5 , 5 2	1 1 20	blue yellow	0 098	29 5	71 0
+ 0 9730	7 08	4 71	42 4	66 9	1 735	0 750	0 631	6 , 7	1 2 51	yellow bluish	0 342	18 5	88 7
-10 2843	2 498	1 752	43 1	289 0	0 455	1 180	0 559	3 , 6	1 15 85	bluish bluish	0 613	86 4	21 2
- 9 0908	2 76	2 38	142 5	59 5	0 61	1 042	0 449	6 4, 7 8	1 3 63	yellowish yellowish		71 9	86 0
- 7 8261	2 22	0 35	154 5	160 8	0 18	0 835	0 970	6 , 6	1 1	yellow yellow	0 185	53 6	60 6
+ 8 000	2 78	1 148	61 4	241 4	0 304	0 822	0 698	9 4, 10	1 1 74	bluish white bluish	0 811	48 3	83 1
+ 1 5652	2 46	1 09	80 0	85 8	0 712	0 475	0 781	5 , 6	1 2 51	yellowish yellowish	0 025	49 9	72 0
- 4 0728	9 00	4 17	122 9	295 8	2 198	1 475	0 848	4 5, 6 0	1 3 98	yellow purplish	1 123	76 7	88 9
+ 6 6055	2 028	1 278	169 5	169 5	0 792			6 0, 11 7	1 190 55	yellow purple	0 136	67 9	67 9
-19 098	1 300	0 423	74 8	82 8	0 168	1 223	0 430	3 9, 4 4	1 1 59	yellow yellow		44 6	59 5
- 2 3575	4 906	3 661	72 2	252 1	1 033	1 045	0 180	5 5, 5 5	1 1	yellowish yellowish		75 7	87 3
+13 015	1 060	0 477	2 5	176 6	0 194	0 645	0 828	4 , 6	1 6 31	yellowish yellowish	0 089	53 2	30 4
+ 2 7907	1 288	0 43	0 3	215 2	0 173	1 480	0 616	6 , 7	1 2 51	yellow yellow	0 064	29 8	60 5
-31 441	0 904	0 171	22 2	22 2	0 075	0 930	0 532	4 5, 5 0	1 1 59	yellow yellow	0 300	20 1	39 4
-31 5236	0 555	0 130	115 7	30 2	0 032	1 390	0 503	4 3, 5 0	1 1 91	yellowish yellowish		68 6	87 9
+15 0	1 52	1 00	118 0	18 2	0 197	0 610	0 062	6 0, 10	1 39 81	yellowish bluish	1 288	80 3	65 8

material for one hundred orbits at the end of another half century, and accordingly such effort is urgently demanded by the highest interests of science.



§ 2 *Relative Velocity of the Companion in the Line of Sight  
for the Epoch 1896 50*

When the elements of the orbit are known, the theory developed in §5, Chapter I, first published in the *Astronomische Nachrichten*, No 3314, enables us to predict the relative motion of the companion of a binary in the line of sight for any given time. The columns marked  $\frac{p}{a}$  and  $\pm \frac{\kappa}{\rho}$  in the foregoing Table contain the desired results for the epoch 1896 50. The numbers in the column  $\frac{p}{a}$  express the orbital velocities in units of the radius of the hodograph. As the scale of this radius is unknown, except in a very few cases, we are not able to express this velocity in kilometres or in other absolute units, but when the parallaxes are determined this may be readily accomplished. The column as it stands, however, shows the rate of orbital motion, compared to what is approximately the average velocity, and we are thus enabled to select those stars which have a rapid orbital motion. If the motion of any given pair be rapid, and also mainly in the line of sight, as in the case of 70 *Ophiuchi*, the system so circumstanced will be favorable for spectroscopic measurement. The column  $\pm \frac{\kappa}{\rho}$  shows what part of the orbital motion is in the line of sight, and this enables us to select for measurement with the Spectrograph those pairs which have a large orbital velocity with the major portion of it towards or from the earth.

The stars at present the most favorably situated for measurement of the relative motion in the line of vision are  $\eta$  *Cassiopeae*,  $\alpha$  *Canis Majoris*, 9 *Argus*,  $\xi$  *Bootis*,  $\gamma$  *Coronae Borealis*,  $\Sigma$  2173, 70 *Ophiuchi*,  $\beta$  *Delphini*, and  $\alpha$  *Centauri*.

Adopting parallaxes of  $0''.75$ ,  $0''.162$ , and  $0''.154$  for  $\alpha$  *Centauri*, 70 *Ophiuchi*, and  $\eta$  *Cassiopeae* respectively, we find the line-of-sight components for the several systems to be 6.66, 13.95, 8.89, where the unit is the kilometre. These quantities are well within the limit of spectroscopic measurement, and therefore an experimental determination offers an attractive problem to observers occupied with this branch of Astronomy.

It will be seen that several of the above stars are wide, while others are very close. If the two spectra can be photographed on the same plate, the lines being only slightly displaced by the relative motion of the stars, as in the case of spectroscopic binaries, the close pairs ought to be as easily measured as the wide ones, whose spectra could perhaps be photographed separately.

In any case the prosecution of these researches with the powerful spectroscopic appliances of the great telescopes of our time is an urgent *desideratum*.

of Astronomy. And until the relative motions of visible systems are thus determined there will remain some doubt as to the reality of the so-called spectroscopic binaries, not that any one doubts the theoretical validity of the DOPPLER-HUGGINS principle, but rather that other explanations of the phenomena interpreted as spectroscopic binaries are considered possible. Moreover, the great interest attaching to investigations which will give the absolute dimensions, parallaxes and masses of binary systems, as well as the possibility of testing the validity of the law of gravitation, ought to induce astronomers to prosecute these studies with a zeal commensurate with their real importance.

Owing to the small size of the earth's orbit, it seems that our principal hope for knowledge of the dimensions of the universe must be based upon this method. The change in wave-length due to motion in the line of sight was originally pointed out by DÖPPLER, but HUGGINS was the first to apply the Spectroscope to the heavenly bodies, and to reduce DÖPPLER'S principle to actual practice, and to assign it a place in modern Astronomy. The application of the principle to the determination of the dimensions of binary systems was first proposed by FOX TALBOT. But as his theory was restricted to the case of circular motion, it could not be applied to the eccentric orbits described by the stars, and accordingly it has since been somewhat varied and extended by others. The theory which we have developed is entirely general for ellipses of every possible eccentricity, and from the point of view of rigor and generality leaves nothing to be desired.

### §3 *Investigation of a Possible Relation of the Orbit-Planes of Binary Systems to the Plane of the Milky Way*

Owing to the well known arrangement of the stars and sharply-defined nebulae with respect to the Milky Way, it has been suggested that some relation might exist between the planes of the stellar orbits and this fundamental plane of the universe. An examination of this question is worthy of the attention of astronomers, and accordingly we shall compute the inclinations of the foregoing orbits by the formulae developed in the *Berliner Astronomisches Jahrbuch* for 1832. The method of transformation which ENCKE has employed enables us to refer the plane of a double-star orbit to any absolute plane in space.

Let us pass a plane through the central star parallel to the equator. The pole of this plane will meet the celestial sphere at the same point as the pole

of the heavens. Consider the triangle connecting the pole of the equator with the poles of the real and of the apparent orbit. The pole of the apparent orbit is determined by the right ascension and declination of the star ( $\alpha, \delta$ ). Let the coordinates of the pole of the real orbit referred to the same axes be  $A$  and  $D$ , and let  $\Omega'$  be the angle which the great circle passing through the poles of the real and apparent orbits makes with the meridian. The arc joining the poles of the orbits is the inclination,  $i$ , and this is one of the elements given in the foregoing Table. From the resulting spherical triangle we have

$$\begin{aligned}\sin D &= \cos i \sin \delta + \sin i \cos \delta \cos \Omega' = m \cos (M - \delta), \\ \cos D \sin (\alpha - A) &= \sin i \sin \Omega', \\ \cos D \cos (\alpha - A) &= \cos i \cos \delta - \sin i \sin \delta \cos \Omega' = m \sin (M - \delta),\end{aligned}$$

where  $\sin i \cos \Omega' = m \cos M$ ,  
and  $\cos i = m \sin M$ .

Then  $\tan M = \frac{1}{\tan i \cos \Omega'}$ ,  
 $\tan (\alpha - A) = \frac{\sin (M - \delta)}{\cos M \tan \Omega'}$ ,  
 $\tan D = \frac{\cos (\alpha - A)}{\tan (M - \delta)}$ .

When the right ascension and declination of the pole of the real orbit have been determined, we may pass a plane through the central star parallel to the Milky Way. In the spherical triangle which joins the pole of this plane with the pole of the real orbit and the pole of the heavens, the inclination of the real orbit to the plane of the Milky Way is given by the arc connecting their poles. Thus we have

$$\cos \Gamma = \sin D \sin \delta' + \cos D \cos \delta' \cos (A - \alpha'),$$

where  $\alpha'$  and  $\delta'$  denote the coordinates of the north pole of the Milky Way.

In our computations the coordinates of the north pole of the Milky Way are taken on the authority of SIR JOHN HERSCHEL, who found

$$\alpha' = 12^{\text{h}} 47^{\text{m}} \quad , \quad \delta' = +27^{\circ}$$

There are two solutions for  $\Gamma$ , owing to the two values of  $A$  and  $D$  incident to the indetermination of the ascending node, and the resulting inclinations to the Galaxy are tabulated as  $\Gamma$  and  $\Gamma'$ . Now, we do not know which of these two possible inclinations to the Milky Way is correct, but since it is impossible to select from either column any one prevailing angle, much less an evanescent inclination, we conclude that the orbits are not directly related to the Milky Way, or to any other fundamental plane of the heavens. Thus it is clear that the orbit-planes lie at all possible angles to the Milky Way, with no

marked relation to the general scheme which distinguishes the arrangement of the stars and well-defined nebulae. The consideration that the size of a stellar orbit is small compared to the dimensions of the Milky Way, and that the number of such systems is very great, might have enabled us to anticipate this result as probable *à priori*, since the condensation of nebulous matter to so many centres would almost of necessity have produced rotations in all possible planes, and even if confined originally to one plane the parallelism would have been disturbed by the action of foreign bodies during the ages required for the development of the visible universe.

#### §4. *High Eccentricities a Fundamental Law of Nature.*

It thus appears that the inclinations of the orbit-planes bear no definite relation to any given plane of the heavens, and an examination of the periods of revolution shows that this element likewise has no characteristic property. The periods are found to range from 11 to 370 years.

It is evident that such elements as  $T, a, \Omega, i, \lambda$ , can have no relation to physical causes, and an inspection of the Table shows no trace of such a connection. When, however, we came to deal with the eccentricity the case is different. The results given in the preceding Table establish a most remarkable law, which is of fundamental importance in our theory of the origin and development of the stellar systems, and is besides of practical value to working astronomers.

On glancing over the eccentricities it is found that while nearly all values exist, few, if any, are very small like those of the planets and satellites, nor are any very large like those of the long-period comets. The smallest eccentricity is that of  $\xi$  *Scorpii*,  $e = 0.131$ , the largest that of  $\gamma$  *Virginis*,  $e = 0.897$ , the mean value for the forty orbits,  $e = 0.482$ .

Let us take the  $x$ -axis as the axis of eccentricity, and the  $y$ -axis as the axis of number of orbits, and divide the interval from  $e = 0.0$  to  $e = 1.0$  into a convenient number of parts. Then, if we erect ordinates denoting the number of orbits falling in the given intervals, and connect the points thus determined, we shall be able to illustrate the distribution of orbits as regards the region of eccentricity.

We find no orbits between 0.0 and 0.1; two between 0.1 and 0.2; four between 0.2 and 0.3, eight between 0.3 and 0.4; nine between 0.4 and 0.5; nine between 0.5 and 0.6, two between 0.6 and 0.7, four between 0.7 and 0.8,

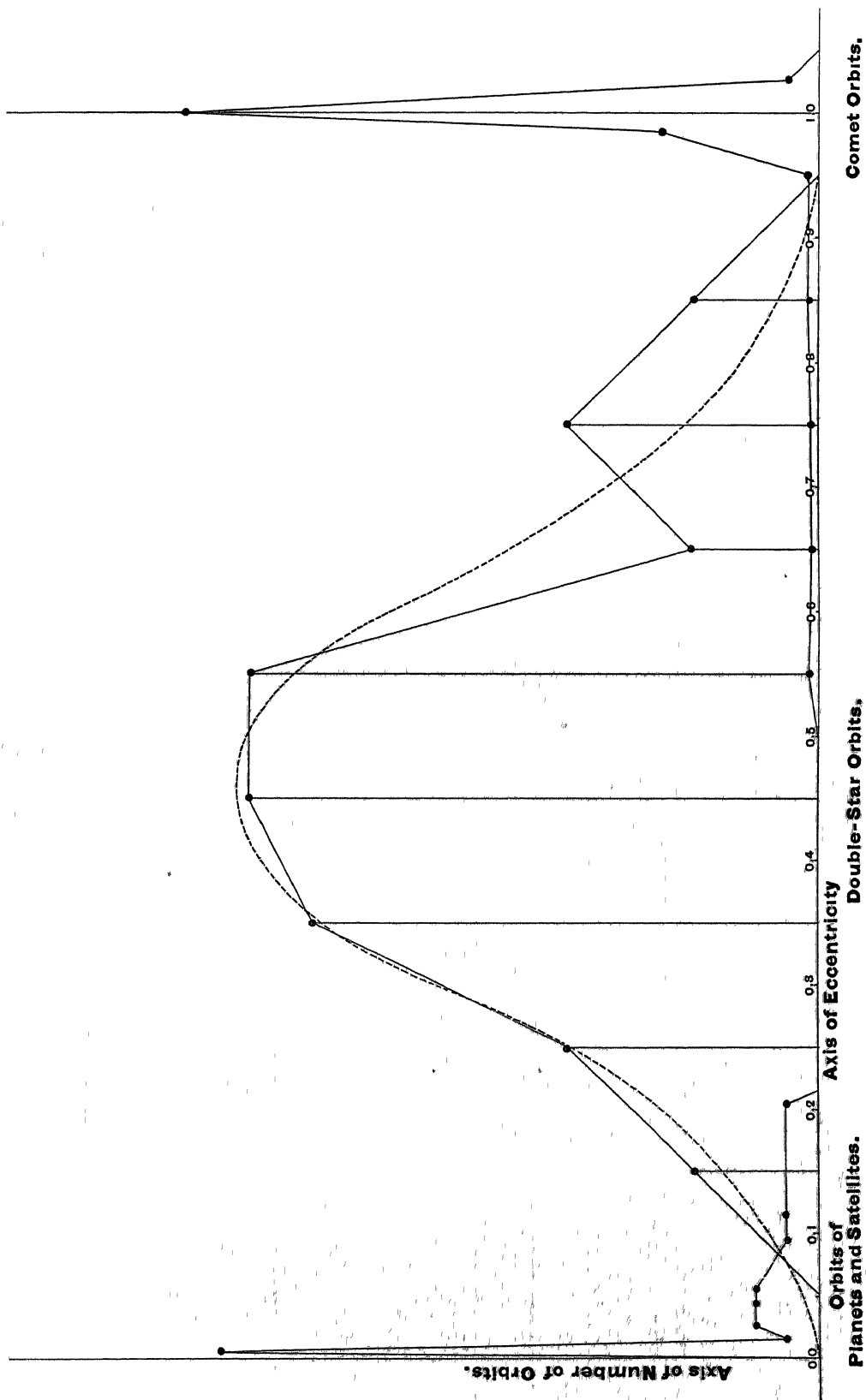
two between 0.8 and 0.9; none between 0.9 and 1.0. The distribution is illustrated by the broken line in the accompanying figure. Since the number of orbits is finite, the figure is an irregular line, if the number were indefinitely increased, the figure ought to become approximately a smooth curve.

It is evident, therefore, that the true curve of distribution of orbits resembles a probability curve with maximum near 0.482, the slope in either direction is gradual, but the curve vanishes before it reaches zero and unity. We have drawn a pointed curve to illustrate what is conceived to be the probability curve for the distribution of orbits, but it is based on forty orbits only, and therefore is necessarily provisional. We may observe, however, that forty is a number sufficiently large to realize the essential conditions underlying the theory of probability, and accordingly we are justified in the inference that the nature of the curve here indicated will never be greatly changed. There is an irregularity in the broken line between 0.6 and 0.7, which may be attributed to the effect of chance, if the number of orbits were greatly increased this gap would be filled up. In general, there will be irregularities in the distribution so long as the number of orbits is finite, but they ought to become less marked as the number is increased.

Thus, it is clear that in whatever intervals the axis of eccentricity be divided, and however the number of orbits be increased, there will remain in the curve of distribution a conspicuous maximum near 0.482, with a gradual slope in both directions. The following table shows the eccentricities of the orbits of the planets and satellites (*Inaugural Dissertation*, Berlin, 1893, p. 58).

Planet	Eccentricity	Mean Eccentricity	Planet	Eccentricity
<i>Venus</i>	0.00684	0.06026	<i>Jupiter</i>	0.04825
<i>Neptune</i>	0.00896		<i>Saturn</i>	0.05607
<i>Earth</i>	0.01677		<i>Mars</i>	0.09326
<i>Uranus</i>	0.04634		<i>Mercury</i>	0.20560

Satellite	Eccentricity	Mean Eccentricity	Satellite	Eccentricity	Mean Eccentricity
Satellite of <i>Neptune</i>	.	These orbits appear to be circular	<i>V</i> (BARNARD)		These orbits appear to be circular  0.0325
<i>Ariel</i>			<i>Io</i>		
<i>Umbriel</i>			<i>Europa</i>		
<i>Titania</i>			<i>Ganymede</i>		
<i>Obelion</i>			<i>Deimos</i>	0.0013	
<i>Mimas</i>			<i>Phobos</i>	0.0057	
<i>Enceladus</i>			<i>Calypso</i>	0.0066	
<i>Tethys</i>			<i>Iapetus</i>	0.0072	
<i>Dione</i>			<i>Titan</i>	0.0296	
<i>Rhea</i>			<i>Moon</i>	0.0299	
			<i>Hyperion</i>	0.05491	
				0.1189	





The orbits of several satellites appear to be circular, or rather the eccentricity is found to be insensible in consequence of the errors of observation. We shall not underestimate these unknown eccentricities if we assign to them the mean value of the known eccentricities of the satellite orbits (0.0325). Making this maximum assumption we find that the average eccentricity for the solar system—the eight great planets and their twenty-one satellites—cannot surpass 0.0389.

In these considerations we have omitted the comets and the asteroids, because the former have been drawn to our system from outer space, while the latter have originated by an anomalous process, and depart so radically from the other bodies of the system that they cannot be considered as a type of planetary evolution, but rather as an abnormal development. It is also to be remarked that the eccentricities of the orbits of the planets and satellites are still involved in some small degree of uncertainty, and moreover they will vary from century to century owing to the cumulative effects of the secular variations and of the long-period inequalities. Notwithstanding these changes it is clear that the values of the eccentricities given above represent the true nature of the solar system.

*It follows, therefore, that the average eccentricity among the double stars is more than twelve times that found in the planetary system, and this extraordinary result is manifestly the expression of a fundamental law of nature.*

The eccentricities of the orbits of the stars discussed in this work are still subject to slight changes, but there is reason to believe that the average value (0.482) will never be altered except by a very small quantity. The apparent orbits given in the preceding chapter enable the reader to make a direct inspection of the linear eccentricity, and he may thus judge of the magnitude of this element, as well as of the changes it is likely to undergo. In order to minimize the uncertainty in our final data, we have purposely restricted our researches to the forty orbits which were capable of the most exact determination. Since the orbits of the forty stars will undergo no sensible improvement, at least for a good many years, it seemed of interest to present also figures of the real orbits.

In the accompanying illustrations the orbits are arranged in the order of eccentricity, and the reader is thus enabled to examine the different degrees of elongation. Accordingly, it appears that while the orbits are much more eccentric than those of the planets and satellites, they are yet much less eccentric than those of the long-period comets.

In the preceding diagram we have drawn one broken line to illustrate the

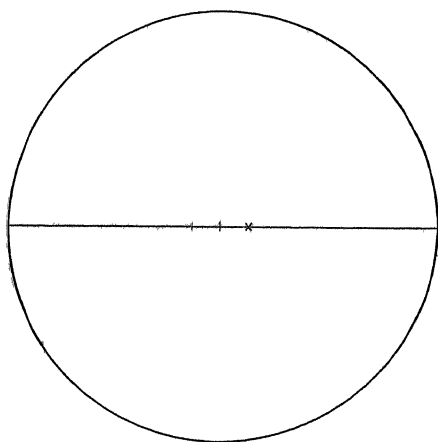


distribution of the orbits of comets, and another for the distribution of the orbits of the planets and satellites. The number of cometary orbits is so large that in this case the scale of ordinates had to be very much reduced. An inspection of these curves shows that the planetary orbits are heaped up about a very small eccentricity, while the cometary orbits cluster around the parabolic eccentricity. This characteristic of the orbits of comets indicates, as LAPLACE first pointed out, that these bodies have been drawn to our system from the regions of the fixed stars, and therefore their eccentricities surpass, equal or approximate unity. Some of the comets have passed near the larger planets, and thus suffered perturbations which have reduced their eccentricities, and hence the curve slopes down gradually on the side towards the origin. The right branch of the curve is but little known, since the great perihelion distance of hyperbolic comets enables them to pass through our system unnoticed, unless they happen to be very bright.

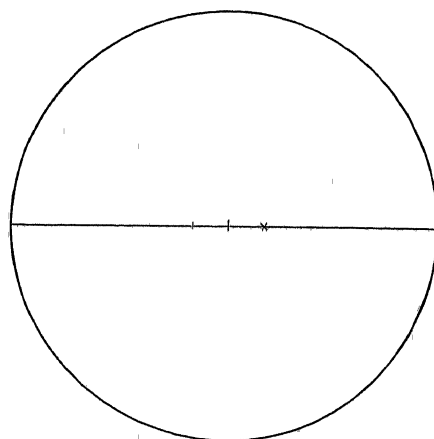
Thus it is evident that the tendency of double-star orbits is to group about a mean eccentricity which is almost equally removed from the two extremes presented in the solar system. Orbits which are so much elongated have no close analogy with those of the planets and satellites, on the other hand their lack of very great eccentricities excludes them from the category of comets, and does not permit us to assign to these systems a *fortuitous* origin. We shall see hereafter that the orbits were originally nearly circular, in the course of immeasurable ages they have been gradually expanded and elongated by the working of tidal friction in the bodies of the stars. The visible elongation of the orbits thus enables us to trace the changes of the stellar systems through millions of years, and to throw light upon the problems connected with their evolution.

In discussing the motion of  $\gamma$  *Virginis*, SIR JOHN HERSCHEL long ago remarked that "the eccentricity is, physically speaking, by far the most important of all the elements," and now we see that this element, which depends wholly on micrometrical measures, and is independent of the parallaxes and relative masses of the stars, gives the sole clue to the evolution of the stellar systems, and will some day enable us to lay a secure foundation for scientific Cosmogony.

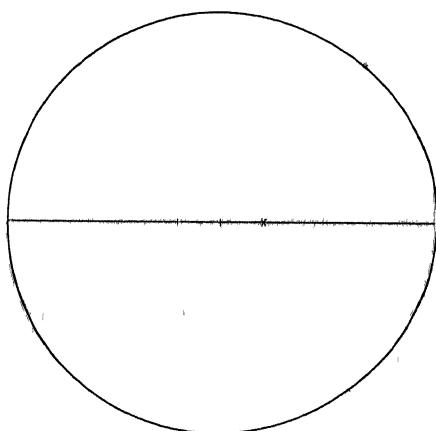
We may observe that besides throwing light upon the past condition of the universe the general law of the eccentricity here established will also be useful to practical astronomers. The eccentricity of any given orbit may depart considerably from the mean here indicated as the most probable value, yet the tendency towards this region will on the whole prove useful to computers.



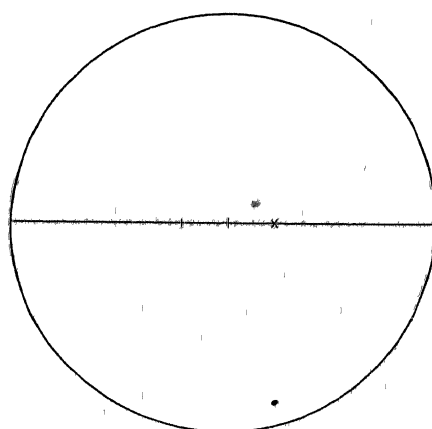
**$\xi$  Scorpii.**



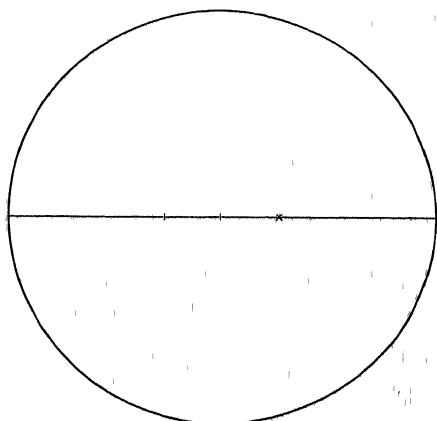
**$\delta$  Equulei.**



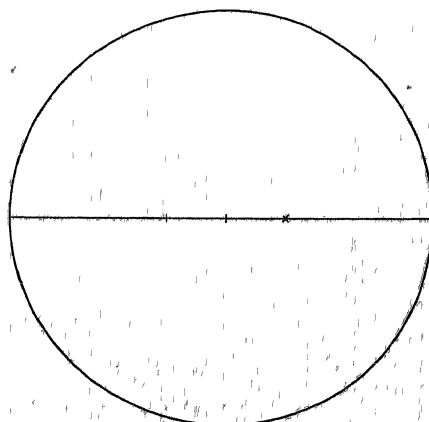
**$\Sigma$  2173.**



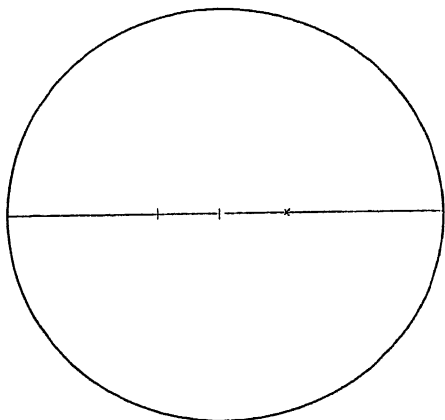
**$\mu'$  Herculis.**



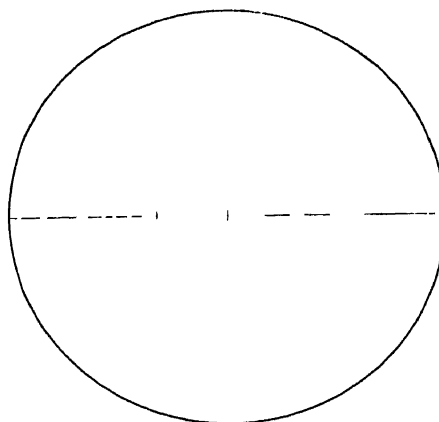
**$\eta$  Coronae Borealis.**



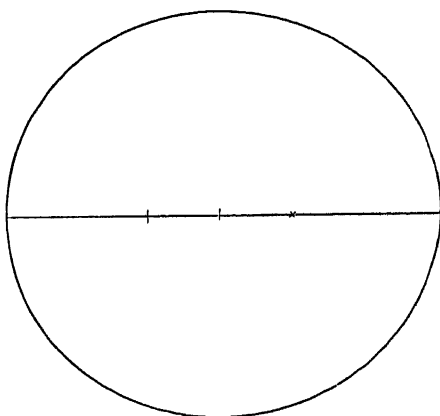
**$\zeta$  Sagittarii.**



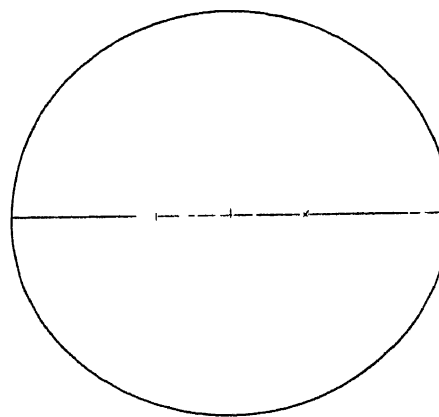
**0 $\lambda$  234.**



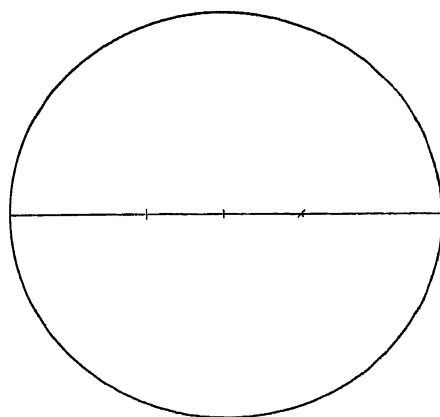
**0 $\lambda$  235.**



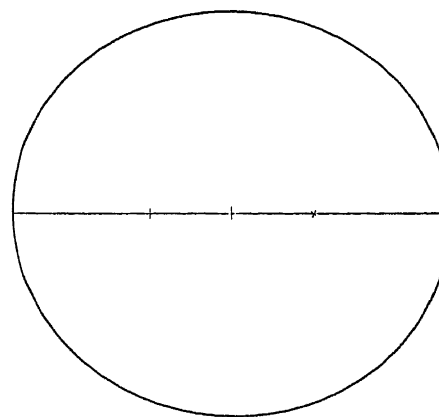
**$\lambda$  3121.**



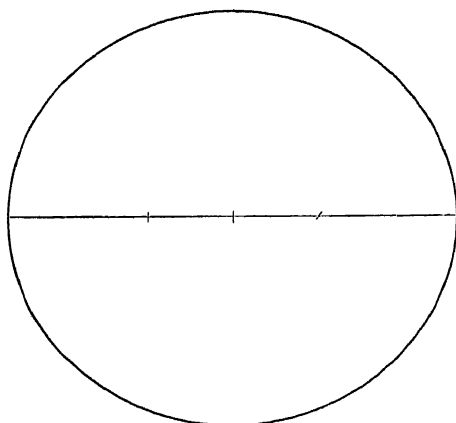
**$\xi$  Cancr.**



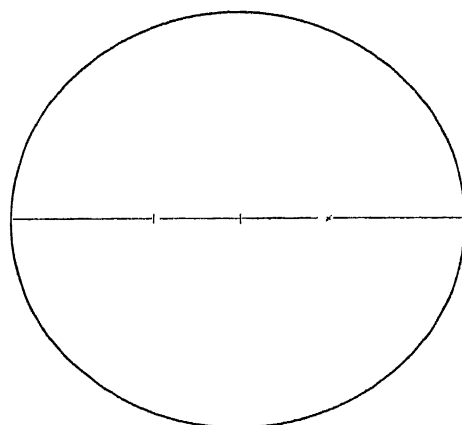
**0 $\lambda$  269.**



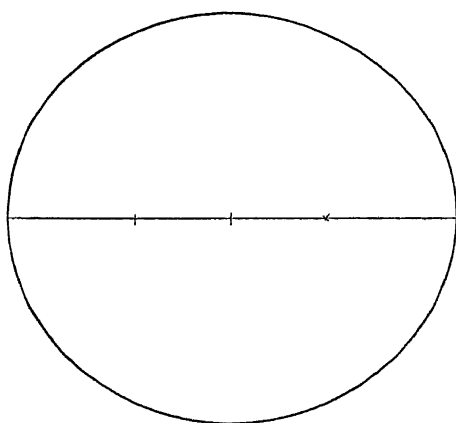
**$\beta$  Delphini.**



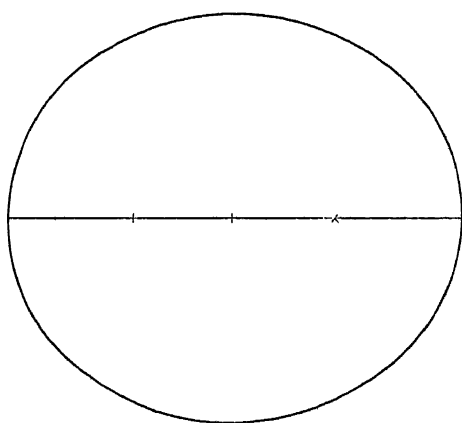
**85 Pegasi.**



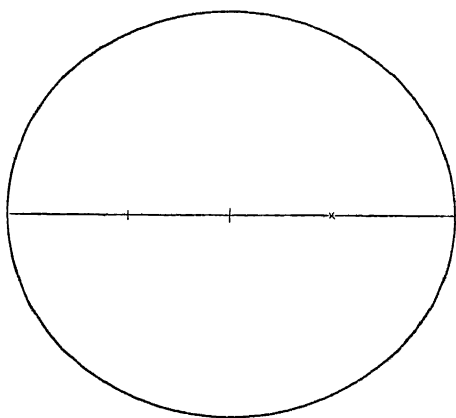
**ξ Ursae Majoris.**



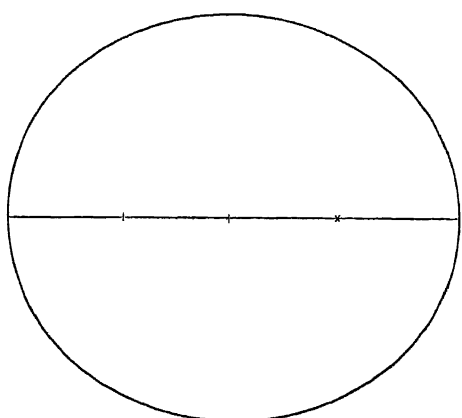
**γ Coronae Australis.**



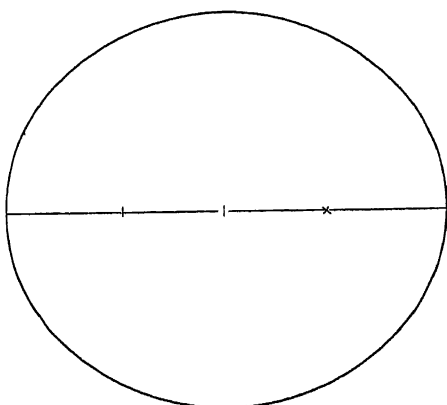
**φ Ursae Majoris.**



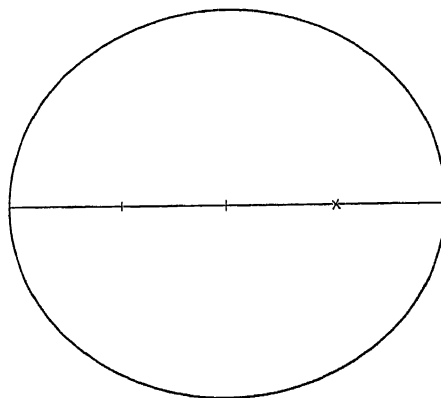
**M 3062**



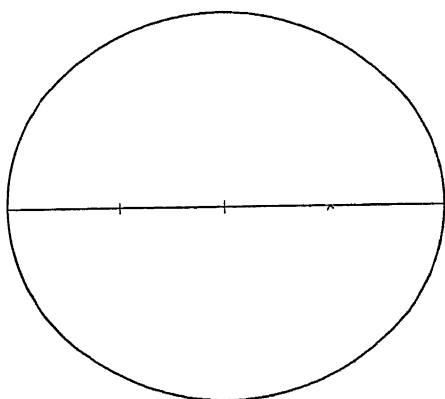
**42 Comae Berenices.**



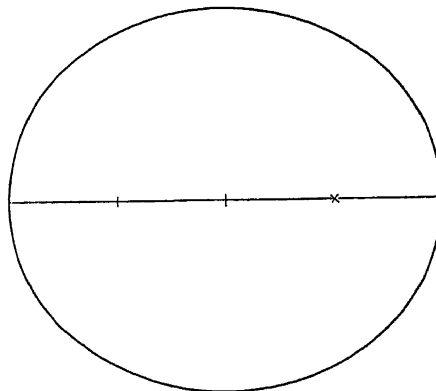
**0 $\chi$  285.**



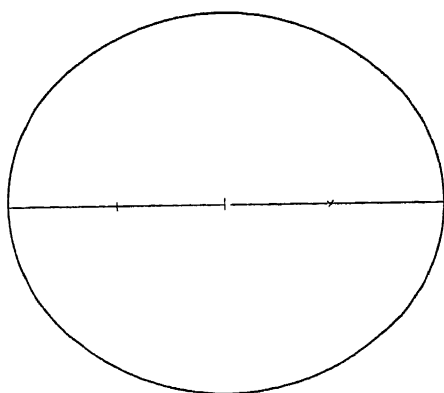
**$\gamma$  Coronae Borealis.**



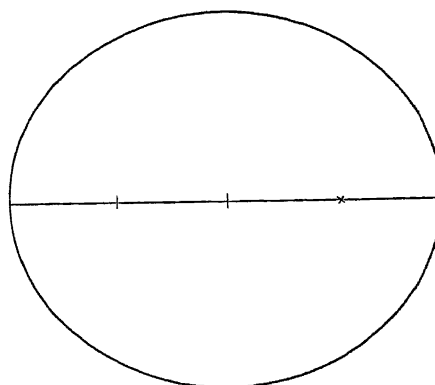
**$\kappa$  Pegasi.**



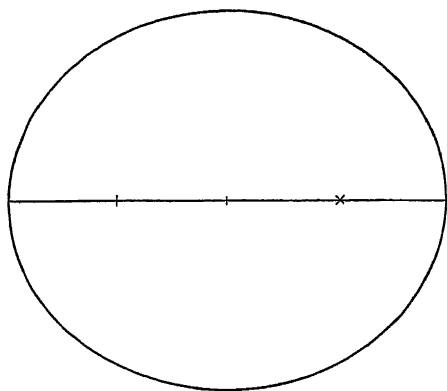
**$\zeta$  Herculis.**



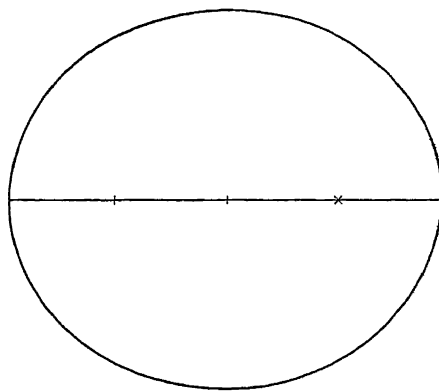
**70 Ophiuchi.**



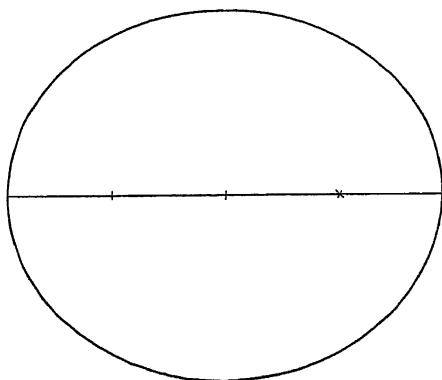
**$\beta$  416.**



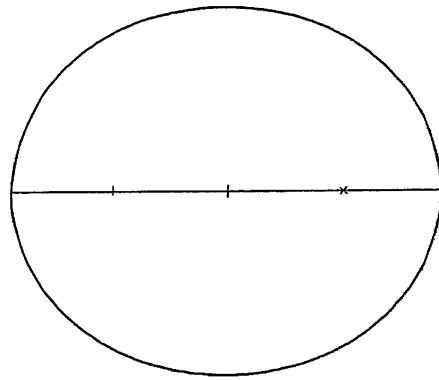
**$\eta$  Cassiopeae.**



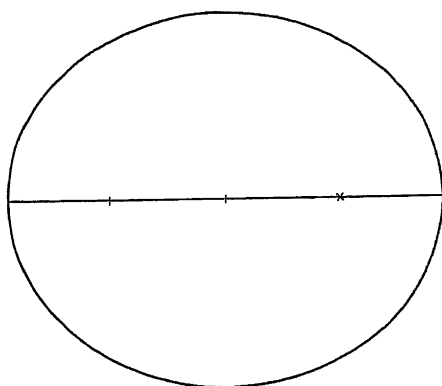
**4 Aquarii.**



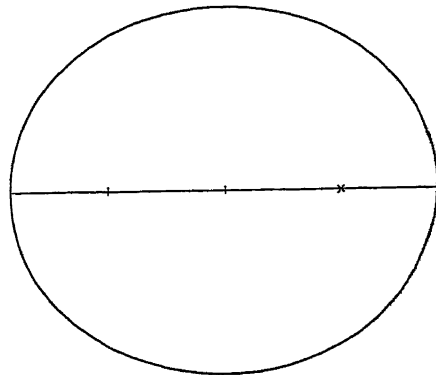
**$\alpha$  Centauri.**



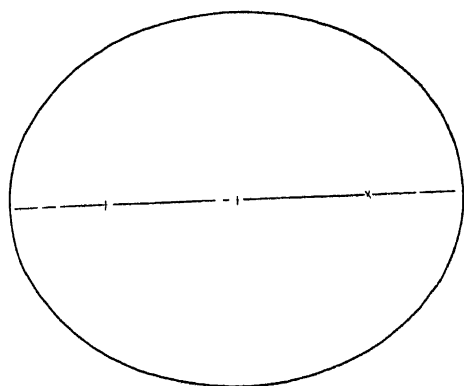
**$\omega$  Leonis.**



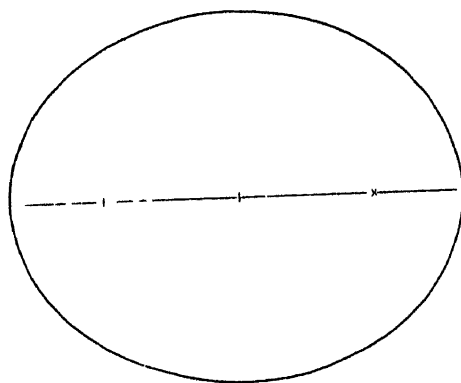
**$\mu^2$  Bootis.**



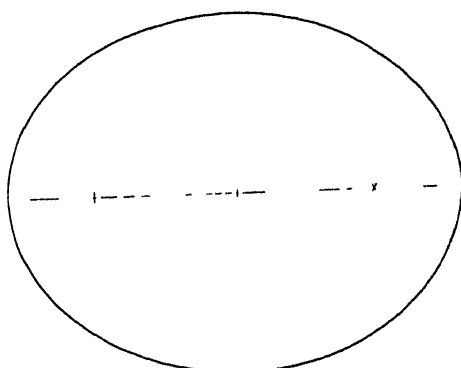
**$\sigma$  Coronae Borealis.**



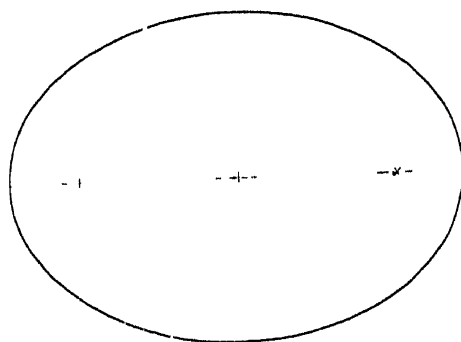
**0 Σ 298.**



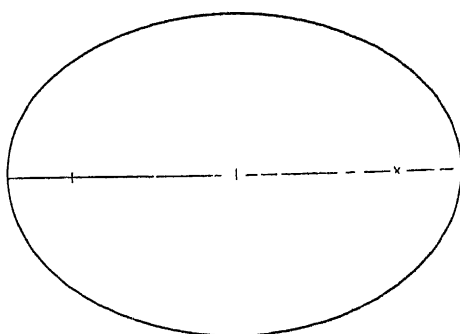
**τ Ophiuchi**



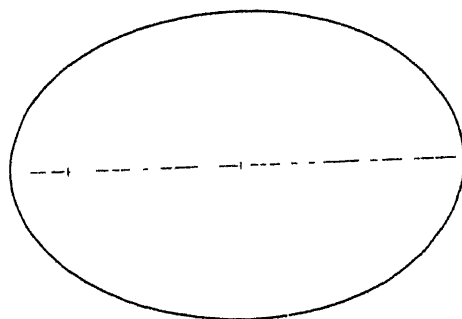
**Sirius**



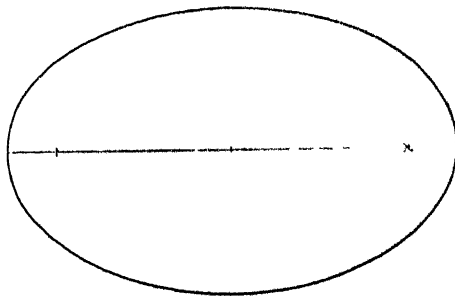
**9 Argûs.**



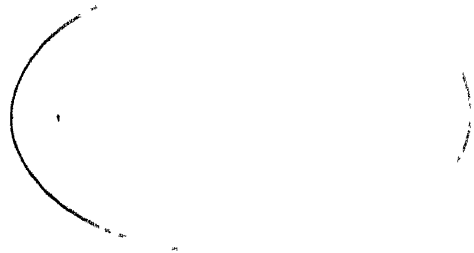
**ξ Bootis**



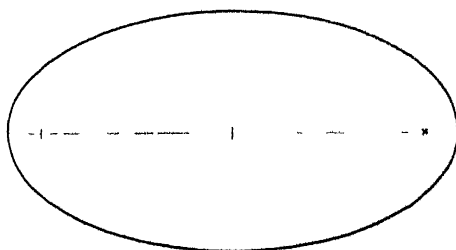
**25 Canum Venaticorum.**



**99 Herculis.**



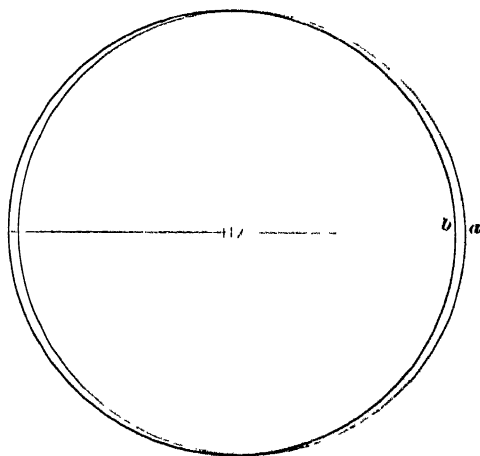
**7 Centauri.**



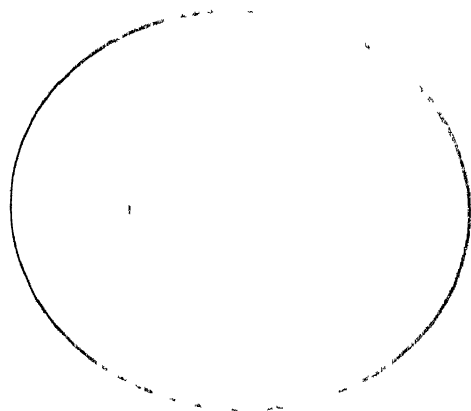
**7 Andromedae**



**1 Virginis.**



$\alpha$ =Circle  
 $\beta$ =Mean Planetary Orbit,  $e=0.0389$ .



**Mean Double-Star Orbit,  $e=0.482$ .**



The observer who is aware of the high eccentricities and different inclinations of the orbits will know that in many cases the length of the apparent radius-vector is subject to great variations, and as a shortening of the radius-vector corresponds to accelerated angular motion of the companion, he will never find it safe to assume that the motion is uniform. The forty stars treated in this work present several instances where the angular motion at certain epochs has been extremely rapid, and it is much to be regretted that more observations were not secured at such critical points of the orbits. These general results may prove of value to the observer of the future, and stimulate an increased interest in the systematic measurement of revolving binaries.

### §5 *Relative Masses of the Components in Stellar Systems*

A problem of fundamental importance in the study of the stars is the determination of the relative masses of the components of a system. Such determinations have been made heretofore in very few cases, and even when undertaken have been seriously embarrassed by the errors of observation. It has been customary to base the investigations upon absolute positions determined with the Meridian Circle. The errors of our absolute positions deduced in this way are so large in comparison with the delicate quantities depending on the irregularity of the proper motions of the individual components of a system whose centre of gravity moves uniformly on the arc of a great circle, that the results obtained are affected by large probable errors.

The systems in which such researches have been attempted are

(1)  *$\alpha$  Camis Majoris*, where AUWERS finds the masses to be in the ratio of 1.2119

(2)  *$\alpha$  Centauri*, in which STONE found the masses approximately equal, ELKIN made them as 1.1124, and ROBERTS finally concludes from a more elaborate investigation that they are in the ratio of 1.1041.

(3)  *$\eta$  Cassiopeae*, investigated in 1881 by LUDWIG STRUVE, who found the masses to be in the ratio of 1.3731

So far as we are aware these three wide systems are the only ones whose relative masses have been investigated, and we may remark that the condition of each star is favorable to a determination from the circumstance that the pairs are wide and tolerably rapid in their orbital motion, and therefore the

irregularity of the proper motions of the components is conspicuous in comparison with the errors of observation

There are other systems such as  $\gamma$  *Ophiuchi*,  $\xi$  *Boötis*, and  $\gamma$  *Virginis*, which are favorable for similar investigations, but none have yet been attempted. It would be all the more interesting to investigate the relative masses of  $\gamma$  *Ophiuchi* from the circumstance that the system contains a dark body which sensibly perturbs the visible components

In the case of  $\gamma$  *Virginis* we might infer that the masses are nearly equal, as in the system of  $\alpha$  *Centauri*

But even if the bright and widely-separated pairs were all investigated, it would still be difficult to reach any of the small, close stars whose distances are less than two seconds of arc. The investigation of the relative masses of the components of such systems by means of absolute positions determined with the Meridian Circle seems forever impossible, since the stars under such power would seldom be separated, and when separated the errors of observation would be larger than the quantities involved in the determination of the relative masses. The old method is therefore very limited in its application, and a new method must be invented if we are ever to have precise knowledge of the relative masses of the components of binary systems

We suggest the following method as much more general and also much more exact than the one depending on absolute positions. The distance and position-angle of each component with respect to a neighboring star should be determined at different epochs, the measures being taken with the Helometer if the distance is large, with the Micrometer if the neighboring star is close or very faint. A series of such relative positions would disclose the location of the centre of gravity by its uniform motion and the resulting conservation of areas with respect to the neighboring star. And since the measures are differential only, it ought to be possible to attain the desired degree of accuracy; the only difficulty likely to arise in practice would be one depending on the personal equations and the constant errors affecting the work of individual observers. Experience alone could determine how serious this difficulty would be, but it seems probable from the results obtained in the measurement of double stars that it would become considerable only in the case of pairs which have no near companion.

Indeed, this method for finding the relative masses of stars is exactly the same as that employed in parallax measurement, except that the observations must extend over the period of a revolution (or a large part of such a period) instead of over the period of one year

The proposed method therefore is as follows Let the differences in right ascension and declination with respect to either of the components at the epochs  $t, t', t''$  be

$$\begin{aligned}\Delta\alpha_0 &= \rho_0 \sin \theta_0 \sec \delta_0, & \Delta\delta_0 &= \rho_0 \cos \theta_0 \\ \Delta\alpha'_0 &= \rho'_0 \sin \theta'_0 \sec \delta'_0, & \Delta\delta'_0 &= \rho'_0 \cos \theta'_0 \\ \Delta\alpha''_0 &= \rho''_0 \sin \theta''_0 \sec \delta''_0, & \Delta\delta''_0 &= \rho''_0 \cos \theta''_0\end{aligned}$$

Let the differences in right ascension and declination of the components of the system in like manner be

$$\begin{aligned}\Delta\alpha &= \rho \sin \theta \sec \delta, & \Delta\delta &= \rho \cos \theta \\ \Delta\alpha' &= \rho' \sin \theta' \sec \delta', & \Delta\delta' &= \rho' \cos \theta' \\ \Delta\alpha'' &= \rho'' \sin \theta'' \sec \delta'', & \Delta\delta'' &= \rho'' \cos \theta''\end{aligned}$$

Then the coordinates of the centre of gravity of the system referred to the neighboring star will be given by the expressions,

$$\begin{aligned}\Delta\alpha_0 + \frac{M_1}{M_1+M_2} \Delta\alpha, & \quad \Delta\delta_0 + \frac{M_1}{M_1+M_2} \Delta\delta \\ \Delta\alpha'_0 + \frac{M_1}{M_1+M_2} \Delta\alpha', & \quad \Delta\delta'_0 + \frac{M_1}{M_1+M_2} \Delta\delta' \\ \Delta\alpha''_0 + \frac{M_1}{M_1+M_2} \Delta\alpha'', & \quad \Delta\delta''_0 + \frac{M_1}{M_1+M_2} \Delta\delta'',\end{aligned}$$

where the formulæ are arranged for the case of the smaller star, which is generally to be preferred, as the magnitude of the absolute orbital motion about the centre of gravity is in the inverse ratio of the masses of the components

In these expressions the only unknown quantity is the ratio  $\frac{M_1}{M_1+M_2}$ . The most natural condition for the determination of this unknown is furnished by the principle of the conservation of the motion of the centre of gravity of a system of bodies. When the arc described by the centre of gravity is small, the motion in right ascension and declination is uniform like that in the arc of a great circle. Thus we have

$$\frac{\Delta\alpha'_0 - \Delta\alpha_0 + \frac{M_1}{M_1+M_2} (\Delta\alpha' - \Delta\alpha)}{\Delta\alpha''_0 - \Delta\alpha_0 + \frac{M_1}{M_1+M_2} (\Delta\alpha'' - \Delta\alpha)} = \frac{\Delta\delta'_0 - \Delta\delta_0 + \frac{M_1}{M_1+M_2} (\Delta\delta' - \Delta\delta)}{\Delta\delta''_0 - \Delta\delta_0 + \frac{M_1}{M_1+M_2} (\Delta\delta'' - \Delta\delta)} = \frac{t' - t}{t'' - t}$$

When  $n$  sets of independent observations have been secured, the number of equations for the determination of the most probable value of the ratio  $\frac{M_1}{M_1+M_2}$  is  $2(n-2)$

If the precession is sensible, the observations of  $\theta_0, \theta'_0, \theta''_0$ , and  $\theta, \theta', \theta''$ , etc, must be referred to a common epoch. An independent formula for the determination of the ratio  $\frac{M_1}{M_1+M_2}$  may be deduced from the criterion that the motion of the centre of gravity is confined to the arc of a great circle.

While the method may not prove to be entirely general, owing to the occasional absence of suitable comparison stars, there is reason to think that the Heliometer and Micrometer together ought to prove very effective. Such measurements, if extended to groups of perspective involving two or more objects, will furnish the means also of detecting the existence of any possible irregularities in the proper motions of single stars. In the early days of star cataloguing it was difficult to believe that the proper motions were uniform and rectilinear, but as this has been found to be the general rule, it is now difficult for some to credit the existence of irregularities in the proper motions, or the presence of dark bodies perturbing the motions of the stars. The errors of observation are relatively so large that sound method of procedure requires caution in attributing anomalies to foreign causes, lest by undue credulity we be led to introduce all manner of vain fictions; yet it is certainly unphilosophical to doubt the existence of numerous dark companions which disturb the motions of the fixed stars. It will ultimately be a matter of great interest to determine the extent and the character of such perturbations. These considerations suggest fields of inquiry of the widest scope, and assure us that while exact Astronomy shall be cultivated, the Heliometer and the Micrometer are not likely to lose their present importance, through the introduction of any sort of mechanical methods.

It will be some years before the above method can be applied, and hence it is interesting to reach some general result as to the relative masses of binary stars. The determinations above spoken of, except in the case of *Sirius*, show that the masses are roughly in proportion to the brightness of the stars. This rule would doubtless lead to erroneous conclusions in a good many individual cases, yet in taking double stars as a class, it will give results which are not far from the truth, and hence the light-ratios of the forty stars given in the Table show that on the average the components of binaries are comparable, and frequently almost equal, in mass. This we may infer to be a general law for all binaries, and the corresponding relative masses accord perfectly with those of the double nebulae drawn by SIR JOHN HERSCHEL, and with the mass-ratios resulting from the rupture of the figures of equilibrium of rotating mass of fluid investigated by POINCARÉ and DARWIN.

### §6 *Exceptional Character of the Planetary System*

The fundamental result indicated in the foregoing section is in striking contrast with the phenomena presented in the solar system. The masses of the planets are very small compared to that of the Sun, and the masses of the satellites are very small compared to those of the planets around which they revolve. The mass-ratio in the case of the Earth and Moon amounts to  $\frac{1}{81}$ , and is by far the largest in the solar system. The mass of *Jupiter*,  $1047.35$ , is much larger than that of any other planet, and yet such a body is wholly insignificant compared to the Sun. If such inconsiderable companions attend the fixed stars, they would neither be visible, nor could they be discovered by any perturbations which they might produce. It is therefore impossible to determine whether the stellar systems include such bodies as the planets, and we are thus unaware of the existence of any other systems like our own. On the other hand the heavens present to our consideration an indefinite number of *double systems*, each of which is divided into comparable masses. These *double systems* stand in direct contrast to the planetary system, where the central body has 746 times the mass of all the other bodies combined. In binary stars the mass distribution is evidently *double*, while in the solar system it is essentially *single*. By a process extending throughout the universe it seems that the nebulae frequently divide into approximately equal or comparable masses, and develop into double stars, while in the case of our own nebula substantially all the matter has gone into the Sun.

Therefore while observation gives us no ground for denying the existence of other systems like our own, it does not enable us on the other hand to affirm or even to render it probable that such systems do exist. And in this state of insufficient evidence we are confronted by the undoubted existence of a great number of systems of an entirely different type. Whatever theories of Cosmogony are proposed, it is evident that in order to have any claim to acceptance, they must be based upon what is really known, not upon what may or may not exist. Those who have proceeded to deduce Cosmogonic processes from our own isolated and abnormal system, have therefore pursued an illogical course, and it is not remarkable that they have failed to throw much light upon the laws of Cosmogony.

The solar system is rendered abnormal by the great number and small masses of its attendant bodies and by the circularity of their orbits about the large central bodies which govern their motion. The system is throughout so

regular, and adjusted to such admirable conditions of stability, that among known systems it stands absolutely unique. Whether observation will ever disclose any other system of such complexity, regularity and harmony, is an interesting question for the future of Astronomy. It is certain that the number of double stars will be augmented in proportion to the diligence of observers and the improvement of our telescopes, and we may reasonably expect a sensible increase in the number of triple and quadruple stars and of stars attended by dark bodies.

Such systems as *Sirius*, *Procyon*,  $\zeta$  *Canceri* and 70 *Ophiuchi* are not likely to be isolated cases; but caution is required where the observations are not decisive, lest the number be unduly increased by imaginary bodies resulting from errors of observation. It seems probable that a number of double stars are likely to disclose perturbations which can be investigated, and we have already some indications that the motions of  $\zeta$  *Herculis*,  $\xi$  *Ursae Majoris*,  $\mu^1$  *Herculis* and  $\eta$  *Coronae Borealis* are not perfectly regular. But in the present state of the measures it seemed best to attribute the apparent irregularities to errors of observation.  $\zeta$  *Herculis* especially merits the most careful attention of observers; after its periastron passage a refined investigation will show whether the motion is really perturbed.

The question naturally arises whether the stars of these double systems are attended by small dark bodies of a planetary character. We have seen that most of the binaries have highly eccentric orbits, and hence if planetary bodies revolved around either component, they would experience great perturbations, besides the most violent changes of light and heat. It seems probable that planets could not be formed without developing very eccentric orbits, and if once in existence, it is questionable whether such bodies could endure under the violent perturbations to which they would be subjected at periastron passage. Even if a planet were very close to its central star, its motion would be affected by an inequality of enormous magnitude analogous to the annual equation in the moon's motion; and if not destroyed by collision with one of the stars or by disintegration under the tidal forces within ROCHE'S limit, in all probability it would sooner or later be driven from the system on a curve analogous to a parabola or an hyperbola. Thus, while the motion of a planet around one of the components could hardly be so stable as the corresponding phenomena of the solar system, it might yet continue for long ages if the orbit of the binary be not too eccentric; the final state of the system would depend upon the densities, relative masses and distances of the components, the mutual inclinations, and above all, the eccentricities, of their orbits.